Dynamic Competition in Deceptive Markets

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Abstract

In many deceptive markets, firms design contracts to exploit mistakes of naive consumers. These contracts also attract less profitable sophisticated consumers. I study such markets when firms compete repeatedly and gather usage data about their customers which is informative about the likelihood of a customer being sophisticated. I find that in sharp contrast to a model with only rational consumers, this customer information is of great value to its owner despite perfect competition. Formally, I introduce a two-period model in which all consumers are aware of a transparent price component. Naives additionally pay a hidden fee, e.g. for an add-on service, that they do not take into account. Competing firms cannot discriminate between new consumers, but in period 2 can employ their private information to offer type-dependent contracts to their first-period customer base. I find that in period 2, firms offer a transparent discount to continuing naives but not to sophisticates, thereby making the less profitable sophisticates more prone to switch to poaching competitors. Uninformed competitors therefore adversely attract unprofitable sophisticates, leading them to compete less vigorously. This allows firms to earn positive margins on continuing naives, while breaking even on sophisticates. Since the adverse attraction of sophisticates mitigates competition, margins from naives increase in the share of sophisticates and firms prefer an even mix of both customer types. I also show that if firms can educate (some) naives about hidden fees, competition is already mitigated when firms compete for customers in the first period with symmetric information. Intuitively, firms coordinate prices in period 1 to prevent education in period 2. As a result, total profits increase already before firms learn about their customers’ naiveté. I analyze a policy that discloses customer information to all firms and thereby increases consumer surplus, and illustrate the robustness of results through several extensions.

Keywords: Deceptive products, shrouded attributes, add-on pricing, price discrimination, industry dynamics, big data

JEL Codes: D14, D18, D21, D99, D89

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1 Introduction

Both intuition and extensive empirical evidence suggest that firms in many markets have a better understanding of consumer behavior and certain product features than consumers themselves. This allows firms to exploit consumer misunderstandings.\(^1\) Examples are markets for credit cards (Ausubel, 1991; Agarwal et al., 2008; Stango and Zinman, 2009, 2014) retail banking (Cruickshank, 2000; OFT, 2008; Alan et al., 2015), mortgages (Cruickshank, 2000), insurance (DellaVigna and Malmendier, 2004) or mobile-phone services (Grubb, 2009). In most of these markets, firms and consumers interact repeatedly. Yet the existing theoretical work—such as Gabaix and Laibson (2006), Grubb (2009), Armstrong and Vickers (2012), and Heidhues et al. (2014)—considers only static models. In these models, some naive consumers are unaware of shrouded or hidden attributes. I extend this literature by introducing a dynamic model of competition with deceptive products. This allows me to investigate an increasingly relevant aspect of reality: developments in the analysis of large amounts of data allow firms to predict consumer behavior with increasing precision. By evaluating their own customers’ usage data, firms have an informational advantage relative to their competitors. The main question I ask is how this informational advantage affects competition when some consumers are more prone to making mistakes than others.

While there are many reasons for firms to collect customer data, I find that when all consumers are rational, (perfect) competition severely limits the value of this information. In this setting, uninformed competitors offer first-best contracts at cost and rational consumers efficiently self-select into contracts. This self-selection renders private information of firms on their consumers’ preferences valueless. In contrast, I show that in a setting with some naive consumers, this information is highly valuable in spite of competition. Naives will select a suboptimal contract, which they perceive to be optimal. These naive consumers are more profitable than sophisticated consumers choosing the same contract. Private usage data enable firms to distinguish between these customers. This allows firms to offer a transparent discount to naives but not to sophisticates. Without the discount, sophisticated consumers are more prone to switch to offers made by rival firms. To avoid adversely attracting too many of these unprofitable sophisticated consumers, rival firms compete less vigorously.

The value of private information on naiveté of customers has further interesting implications. I find that firms prefer an even mix of naive and sophisticated consumers. If in addition firms can

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\(^1\)A naive credit-card consumer, for instance, might falsely believe that she will not pay late-payment or overlimit fees, tempting profit-maximizing firms to raise fees for these add-on services.
educate (some) consumers, my model highlights a novel competition impairing effect. The ability to educate leads to increased profits in a seemingly competitive market even before firms learn to distinguish their customers’ level of sophistication. Firms coordinate prices and do not compete fiercely even though market shares are valuable.

Formally, I study a two-period model with shrouded product attributes. $N$ firms sell a homogeneous good in each period to maximize total profit. They compete for a unit mass of consumers by setting a transparent price and a hidden fee, i.e. the shrouded attribute. There are naive and sophisticated consumers. Naives pay the hidden fee but do not take it into account. The sophisticated are aware of the hidden fee and avoid it. Thus, all consumers choose a product only based on transparent prices and firms cannot discriminate between new customers. The timing is as follows: firms first compete for market shares with symmetric information on customers. After observing which first-period customer paid the hidden fee, firms learn to distinguish between naives and sophisticated within their customer base. In the second period, firms then use this private information to discriminate between continuing customers based on their level of sophistication. Other firms do not observe which offer a customer received. I will later discuss implications when firms can educate naive consumers about hidden fees.

Credit cards are an example of a market close to this setting. The market is competitive by conventional measures, i.e the product is quite homogeneous and there are many firms. Consumers are usually aware of maintenance fees, cash rewards, introductory APRs or new-client bonuses. But many consumers ignore overlimit, overdraft or late fees, or underestimate their tendency to borrow money when choosing a credit-card contract. Firms condition offers on customer characteristics which are inferred from usage data, including naiveté. Firms can also take efforts to educate (some) consumers.

My main result is that competing firms benefit from private information on naiveté because

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2 I use the terms ‘educate about’ and ‘unshroud’ hidden fees interchangeably.
3 E.g. naives could be unaware of their demand for an add-on service.
4 This captures consumers who either have no demand for an additional service or the idea that consumers can take precautions to escape these fees. Credit-card holders, for instance, could avoid interest payments by paying their debt in time. Mobile phone owners can avoid roaming charges by purchasing extra packages or calling from a land-line etc. But even without precautions, Heidhues et al. (2014) show that sophisticated consumers that pay the ‘hidden fee’ can be screened into buying an alternative transparent product.
5 For empirical evidence, see Ausubel (1991); Agarwal et al. (2008); Cruickshank (2000); OFT (2008); Stango and Zinman (2009, 2014); Alan et al. (2015).
6 To see how firms can identify naive consumers from usage data, consider Stango and Zinman (2009) or Grubb (2009). Alternatively, recall that estimated elasticities represent revealed preferences when making a decision. Thus, everything else equal, naive consumers have a lower elasticity of demand. For the use of big data for demand analysis, see Shiller (2014).
7 Simply mentioning certain fees can raise consumer awareness. For examples, consider Stango and Zinman (2014) or Alan et al. (2015).
it allows them to reduce the intensity of competition. Intuitively, private information about their clients’ level of sophistication in period 2, i.e. via customer data, allows firms to offer different transparent prices to their continuing naive and sophisticated consumers. But since both types choose only based on transparent prices, a rival can only try to poach these consumers using a single transparent-price offer. In particular, firms offer their continuing sophisticates a larger transparent price than their naives to make them more responsive to poaching offers. Competitors who try to poach profitable naives then adversely attract unprofitable sophisticates. They respond by reducing the intensity of competition. This allows firms to break even on their existing sophisticates while maintaining positive margins on their naives. Firms profitably exploit the fact that customer data allow them to discriminate between continuing customers while competitors lack the data to do so, and naive consumers lack the sophistication to recognize better offers. These results do not depend on the ability of firms to unshroud hidden fees.

Naiveté is crucial for this result. With only rational consumers, competition severely limits the value of customer data. To establish this, I study a classic analog with only sophisticated consumers. Some purchase only a base product while others also value an add-on. Firms with private information on add-on demand can make targeted consumer-specific offers. But in competitive markets, uninformed rivals offer first-best contracts. Sophisticated consumers then self-select efficiently and reveal the information on their add-on demand by their product choice. This renders private information on preference for add-ons valueless in the absence of naive consumers.

My main result has important implications. First, companies in data-driven industries or firms specializing in data collection can profitably sell data to non-rival firms, even when these firms are active in competitive markets. In my model, the ability to distinguish naive and sophisticated customers is highly profitable despite competition. This implies that firms have a high willingness to invest in data and methods that improve predictions on their customers’ naiveté. But since these investments do not increase product value, they are inefficient despite their profitability. Selling consumer information to third parties can be important in the context of internet search engines, social networks, or loyalty programs such as those frequently used in retailing. In addition, these results suggest that even firms in perfectly competitive markets can profitably (but inefficiently) invest in big-data analysis to improve predictions about consumer naiveté.

Second, in contrast to the previous literature, firms prefer a mix of naive and sophisticated customers. As the share of sophisticated customers increases, competitors who try to poach naives adversely attract additional unprofitable sophisticates and they further reduce the intensity of com-
petition. This allows firms to earn a larger margin from existing naive customers. But since only naives pay a positive margin, firms earn the largest profits with an even number of naives and sophisticates in their customer base. I show that this enables firms to benefit from trading customer portfolios within the market. When firms have different shares of naives in their customer base, they can trade client portfolios to make them more balanced on average. This increases total market profits.

Third, my results predict price dispersion when firms are privately informed about the naiveté of their customers. One reason is that firms condition offers on privately observed characteristics, i.e. from analyzing customer data. Another reason is that firms play mixed strategies in period 2. Intuitively, firms offer two different prices to their sophisticated and naive customers. But since both types only consider transparent fees, rivals can compete for them only with a single transparent price. In equilibrium, this leads to mixed strategies for transparent prices to new- and naive customers. This price dispersion is consistent with empirical findings on credit-card products.

Schoar and Ru (2014) study offers of credit-card companies and find substantial variation based on unobserved characteristics. As in my model, this dispersion is larger in subpopulations where consumers are more likely to be naive. Stango and Zinman (2013) find that borrower risk and other observables explain only about half of the substantial dispersion in borrowing costs.

Fourth, I demonstrate that the ability to educate consumers about hidden fees mitigates competition for customer bases in period 1 and increases total profits. This holds despite price competition with homogeneous products and initially symmetrically informed firms. Intuitively, firms without customer base earn zero profits in period 2 and educate consumers to attract them. Thereby, they decrease profits for all competitors with a customer base. To prevent education in period 2, firms want to ensure in period 1 that each competitor gets a sufficient portion of the market. This induces firms to coordinate prices in period 1 and mitigates competition for customer bases. I establish in an extension that the same qualitative results hold when unshrouding is partial and arbitrarily weak, i.e. when firms can educate a share of naives and this share is arbitrarily small. This shows that transparency campaigns capable of educating consumers about shrouded fees can actually serve as a credible threat to competing rivals and increase profits.

Consumer education or a simplified product design are common policy suggestions for deceptive products. I suggest the following alternative: disclose consumer data to all firms in the market. Each competitor can then target each profitable naive consumer separately without adversely attracting unprofitable sophisticates. This effectively splits the market and induces marginal cost
prices for all consumers. The resulting sharp drop in profits also increases incentives of firms to unshroud hidden fees and makes transparent pricing schemes more likely. But the ability to educate consumers is not necessary for this policy to be effective. For example, when a credit-card company identifies a profitable client of a competitor, it could try to inform the client about hidden fees, e.g. late-payment or overcharge fees he would save by switching. But if this fails, the company could offer him a lower transparent price instead, e.g. a new-client bonus. In this way, disclosing customer data to competitors restores effective competition when private information on customer naiveté impede it. Of course, this is a rather radical intervention and should not be implemented lightly. Nonetheless, the effectiveness of such a policy suggests a conflict between antitrust- and privacy concerns in data-driven industries.

The following Section reviews the related literature. In Section 3, I introduce the basic setup and discuss in more detail how the model captures crucial features of important markets such as the credit-card industry, retail banking, markets for insurance or (mobile-)phone services. I analyze two benchmarks in Section 4. The classic analog described above and another one with naive consumers but firms have no usage data. In Section 5, I present the main results on the profitability of private customer information on naiveté and look at policy implications in Section 6. In Section 7, I establish robustness of the results to several extensions: (i) When naives cannot avoid unshrouded hidden fees, firms can profitably attract their competitors’ non-avoiding naives by unshrouding. This imposes stronger existence conditions on shrouding equilibria but leaves them qualitatively unaffected. (ii) Partial unshrouding to naive consumers. I discuss further extensions in the Appendix A on new customers arriving in period 2, learning of naives, and a model with more than two periods. Unless stated otherwise, proofs are in Appendix B.

2 Related Literature

To the best of my knowledge, I am the first to study dynamic competition with shrouded attributes and the first to analyze the impact of private information about consumer naiveté on competition. I discuss three areas of related literature. First, the literature on behavioral economics and exploitative contracting, second, the literature on adverse selection and worker poaching in labor markets, and third, the literature on customer poaching and switching costs.

Ausubel (1991) does an early empirical study of the credit-card market in the USA. Despite the market being highly competitive by conventional measures such as the number of firms and
product homogeneity, he finds large profits. But while he suggests that search- and switching costs could explain parts of this profitability, he also states that these costs would need to be huge to explain the observed profits. Though consumer misperceptions and misunderstandings seem an intuitive explanation for these observations, most papers that investigate these phenomena do not predict extraordinary profits under perfect competition: profits obtained from naive consumers are used to reduce transparent prices in order to attract more customers (e.g. see Gabaix and Laibson (2006), Armstrong and Vickers (2012) or Murooka (2013)).

The only paper I am aware of with an explanation for extraordinary profits in competitive markets with shrouded attributes is Heidhues et al. (2014). They study profitable deception and inferior products. Deception is profitable due to a price floor on transparent fees. Firms do not hand over all profits from hidden fees to consumers via reduced transparent prices because they cannot reduce them below the price floor. This price floor is motivated by adverse selection of unprofitable consumers who sign up for a new-client bonus but do not use the product, or suspicion of a “bad” product when prices are too low. They also argue that firms share a common interest in sustaining each others market shares. Their framework is close to mine, but I study a dynamic model, and I do not impose a price floor. Large profits result from private information on the naiveté of customers.

In retail-banking or credit-card applications, a price floor may not apply, since banks often require a minimal product use (e.g. minimal monthly cash-inflows on a bank account) for new-client bonuses. I also show that the firms’ interest in sustaining each others market shares has important dynamic, and competition-impairing effects, and increases total shrouding profits.

Gabaix and Laibson (2006) introduce a model where firms sell a transparent base good with an add-on price that can be shrouded and is ignored by naive consumers. Sophisticated consumers take these fees into account, and can avoid them by taking costly and inefficient precautions. Their main contribution is to show that in some market equilibria, firms do not want to unshroud hidden fees to consumers since profitable naive consumers become sophisticated and can therefore not be attracted in a profitable way. Building on that model and applying it to the UK retail-banking industry, Armstrong and Vickers (2012) analyze a model with contingent charges. They argue that the existence of naive customers can explain the common “free-if-in-credit” model that charges nothing for account maintenance but contingent charges like overdraft fees or interest payments to generate revenues.

I build on these models and extend them to a dynamic setting in which firms’ information

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8In particular, per outstanding balance of 1,000$, monthly search and switching costs in the range of 250$ were required to explain the observed profits.
on consumer naiveté plays a role. My results differ in crucial aspects: I find that shrouding increases equilibrium profits, and cross subsidization between customer types is limited. Additionally, shrouding occurs even if there is only a small share of naive consumers. Moreover, firms prefer a mix of naive and sophisticated customers to only naives.

Other papers study the role of information of firms with respect to consumers’ level of sophistication. Heidhues and Koszegi (2015) consider seller information on customer naiveté and establish that third-degree price discrimination can lower welfare when firms discriminate based on naiveté. But in contrast to my model, they consider symmetric information of firms on the consumers’ degree of sophistication. Though these symmetric information on naiveté have important welfare implications, they do not explain extraordinary profits. Kamenica et al. (2011) study asymmetric understanding of firms and consumers. They discuss the impacts of a regulation that informs consumers about their own behavior and thereby reduces this asymmetry. Such a regulation can increase consumer welfare when firms keep prices constant. But when firms adjust prices, consumer and producer surplus remain unchanged. My approach is different in that I consider information that help firms to identify customers based on their sophistication. This allows me to study the impact of price discrimination and the role of asymmetric information about customers on competition between firms. Consequently, I suggest a disclosure policy that does not rely on better informed consumers but rather on better informed competitors. I show that this eliminates the competitive advantage that firms have due to their superior knowledge about old consumers, and increases consumer welfare.

Murooka (2013) analyses the incentives of intermediaries to sell a deceptive product rather than a transparent one. Because intermediaries earn high commissions despite competition by selling deceptive rather than transparent products, shrouding equilibria exist in which only the deceptive product is sold. These shrouding equilibria can be eliminated by regulating commissions. This induces intermediaries to reveal hidden attributes to consumers.

A crucial feature of this article is that I consider naive and sophisticated customers that cannot be screened ex ante because naives and sophisticates consider only transparent fees. Eliaz and Spiegler (2006) discuss screening of adversely-naive agents in a setting where all agents are ex-post identical but differ in their beliefs about their realized ex-post type. In contrast, I study agents with different ex-post types but identical ex-ante beliefs.

The adverse-attraction effect established here is reminiscent of adverse selection and worker poaching in the labor market literature. Greenwald (1986) and Katz (1991) study labor markets
where current employers observe a worker’s productivity. Competing employers try to poach workers from other firms, but they neither know their productivity nor can they offer contingent contracts. In these models, employers let their low-productivity workers go to other firms but keep their high-productivity ones. Results crucially depend on the assumption that firms cannot make offers contingent on ex-post observable private information, i.e. the workers’ productivity. Otherwise, poaching firms could screen workers by offering a bonus in case a high productivity is observed and a punishment for low productivities. I emphasize this point because consumption of additional goods or services is frequently easy to verify. Therefore, one would expect adverse-selection effects similar to worker poaching to be less important when the asymmetric information of firms is not workers’ productivity but their customers’ demand for additional goods or services. But I find that adverse selection is important in retail-market settings when some consumers are naive and mistakenly make the same choices as sophisticates.

Deceptive products might seem reminiscent of products with switching costs. But there are fundamental differences. Markups due to switching costs are efficient when they prevent consumers from switching too often, whereas markups via hidden fees exploit naiveté. Additionally, basic dynamic models with switching costs exhibit a bargain-then-ripoff structure. Firms with small market shares price aggressively to attract customers while larger firms set higher prices to profit from their customers’ switching costs. In my model, though, while prices are also high for existing customers, they are high for new customers as well.

The literature of switching costs also considers history-based price discrimination. Chen (1997) studies a model with two periods and two firms selling a homogeneous product. After the first period, consumers draw uniformly distributed switching costs. Firms price discriminate between old and new customers, and earn profits from both. This prevents firms from competing aggressively in the first period and induces positive total profits. Taylor (2003) extends this framework to more than two firms. With three or more firms, the second-period monopoly in attracting customers vanishes and total profits reduce to zero. In contrast, I predict positive profits even with competition for existing customers.

Stole (2007), summarizes another strand of the literature on history-based price discrimination.
that focuses on customer poaching. The poaching literature studies price discrimination between old and new customers with horizontally differentiated products. A common finding is that even a monopolist does not benefit from history-based price discrimination. Forward-looking consumers take higher future prices into account and benefit from not consuming in the first period to get a lower price in the second one. In contrast, I find that firms benefit from price discrimination between their old customers even under perfect competition. Additionally, forward looking sophisticates cannot do better by not purchasing in the first period.

Note that both in the switching cost and poaching literature, firms usually discriminate between own and competitors’ customers but not between different types of own customers. This discrimination, however, is a crucial feature of my analysis.

3 The Basic Model

3.1 Setup

There are two periods. Firms sell a homogeneous product in each period to a unit mass of consumers. In each period consumers value the product at \( v > 0 \) and their outside option at zero.

There are two types of consumers. The share \( 1 - \alpha \in [0, 1] \) is sophisticated. They observe transparent and hidden price components, and can avoid paying the hidden one at no costs. The share \( \alpha \) is naive and takes only transparent prices into account when firms shroud hidden fees. Thus, with shrouded hidden fees, all consumers only consider transparent prices. Naive consumers who are educated about hidden fees become sophisticated, i.e. when a firm unshrouds hidden prices, naives take them into account for all firms, and can avoid them. In extensions I relax these assumptions and discuss partial education, and naives who can not avoid unshrouded hidden fees. Consumers maximize their perceived utility.

Naive consumers do not learn about their naïveté over time, except when educated by a firm. This is consistent with empirical evidence of consumers repeatedly triggering fees they

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14 Sophisticated consumers might simply have no demand for an additional service associated with the hidden fee. Alternatively, they could engage in precautionary behavior. In the credit-card example, many consumers pay overcharge fees or interests on their credit-balance despite having liquid funds available to reduce their balance to avoid these fees (see Stango and Zinman (2009)). Sophisticated consumers with liquid funds can easily avoid such fees. Mobile phone owners can avoid roaming charges by purchasing extra packages or calling from a land-line etc. But even without precautions, Heidhues et al. (2014) show that sophisticated consumers that pay the ‘hidden fee’ can be screened into buying an alternative transparent product.

15 Within each period, naive consumers perceived utility of purchasing from firm \( n \) is \( v - f_n \), while their actual utility is \( v - f_n - a_n \). They correctly perceive the former when unshrouding occurs. Sophisticated consumers’ perceived and actual utility of purchasing from \( n \) is \( v - f_n \). In period 1, consumers take their perceived continuation utility into account.
are unaware but—as I show in extensions—relaxing this simplifying assumption does not change results qualitatively.\textsuperscript{16} I assume that consumers, once educated about hidden fees, remain so for the subsequent period.

There are \( N \geq 2 \) firms, each with marginal cost \( c \geq 0 \). In each period \( t \), firm \( n \) sets hidden and transparent price components \( a_{nt} \) and \( f_{nt} \). I call the set of customers who purchased from a firm in period 1 its customer base. Firms learn their customers’ types by observing their consumption patterns in period 1, i.e. by noting that naives pay the hidden fees and sophisticates do not. Since firms alone observe the consumption of their own customers, customer data are private information to each firm. Firms can charge different prices to different consumers when they can identify their types. In period 2, this enables each firm \( n \) to charge \( f_{n2}^{\text{naive}} \) and \( f_{n2}^{\text{soph}} \) to naive and sophisticated consumers in its customer base, respectively. Competitors do not observe which of \( n \)'s customers received which offer. In order to attract new customers from competitors, firm \( n \) charges a new-customers price denoted \( f_{n2}^{\text{new}} \). Since all consumers only consider transparent fees, a single price to attract the two customer types is not a restriction. In period 1, firms cannot discriminate between consumers and set one price \( f_{n1} \). In each period, firms set a hidden fee \( a_{nt} \in [0, \bar{a}] \), \( t = 1, 2 \).\textsuperscript{17} I follow the literature by assuming a price cap \( \bar{a} \) on hidden prices.\textsuperscript{18} Additionally, firms choose whether to educate consumers about hidden prices in each period.

When consumers are indifferent between all firms, I employ a general tie-breaking rule: each firm gets a market share \( s_n > 0 \) with \( \sum_{n=1}^{N} s_n = 1 \). When indifferent between less than \( N \) firms, I impose for ease of exposition that market shares are assigned proportionally.

**Sorting Assumption:** Among firms that make them indifferent, consumers are sorted independently of their type. This simplifies the analysis by guaranteeing that—given shrouding—the distribution of types within a non-empty customer base is the same as in the overall population. This assumption is relaxed in Subsection 5.2.

The timing of the game is as follows:

**Period 1: Competition for a Customer Base**

- **Firms** simultaneously choose transparent prices \( f_{n1} \) and hidden fees \( a_{n1} \), and decide whether to educate consumers about hidden fees or not.

\textsuperscript{17}Not conditioning \( a_{n2} \) on observed types is w.l.o.g. since sophisticates avoid them and naives do not. Making an offer to sophisticates that has no hidden fee leads to the same payments as an offer with a hidden fee.
\textsuperscript{18}Gabaix and Laibson (2006) and Armstrong and Vickers (2012) make the same assumption. They justify the assumption by legal or regulatory restrictions on fees or consumers only noticing a fee when it is sufficiently large, i.e. above such a threshold. Another way to think about \( \bar{a} \) is as the maximal willingness to pay for an additional service that consumers require unexpectedly after signing the contract.
• **Consumers** buy from the firm they perceive cheapest among the firms preferred to their outside option. I.e. if hidden fees are shrouded, both types choose a firm \( n \), where \( n \in \arg\max_{n' \in N} v - f_{n'1} + V_{n'2} \), where \( V_{n'2} \) denotes the expected continuation utility in period 2 after consuming from firm \( n' \) in period 1. Naive types additionally pay \( a_{n'1} \). When a firm unshrouds hidden prices, naive consumers become sophisticates.

**Period 2: Asymmetric Information on the Firms’ Customer Bases**

• After observing which of their customers in period 1 payed the hidden fee, firms learn their old customers’ types. Customer-base information is private: a firm can only identify the type of its own customers.

• **Firms** choose a price to the sophisticated and naive consumers in their customer base, denoted \( f_{soph}^n \) and \( f_{naive}^n \), respectively. Additionally, they can set a price to attract customers from competitors, denoted \( f_{new}^n \). Firms choose hidden fees \( a_{n2} \), and whether to educate consumers about hidden fees or not.

• **Consumers** purchase from the firm they perceive to be the cheapest. If hidden fees are shrouded, a consumer of type \( \theta \in \{soph, naive\} \) who purchased from firm \( n \) in period 1 picks the smallest price in \( \{f_{soph}^\theta, f_{new}^n \}_{n \neq n} \) conditional on this price being smaller than \( v \). When hidden prices are unshrouded, naives become sophisticated and solve the same problem but without paying hidden fees.

I apply the concept of Perfect Bayesian Equilibrium. Even though firms have beliefs on the composition of their competitors’ customer bases, PBE is relatively straightforward here since the Sorting Assumption pins down these beliefs: after shrouding occurs in period 1, all firms with a non-empty customer base have the same type distribution in their customer bases. With unshrouding in period 1, all customers become identical, and type information and beliefs are obsolete. Hence, beliefs on the composition of customer bases of competitors only matter after shrouding in period 1 and are then identical to the distribution of types in the population. This is why I do not point out beliefs explicitly throughout the paper and focus on sequential rationality.

In what follows, I study the existence and properties of shrouding equilibria, that is, equilibria in which shrouding occurs with positive probability. In addition, “Bertrand equilibria” always exist where at least two firms unshroud hidden fees and all consumers pay marginal cost for consuming the product. Since those are less interesting, and arguably less robust, I do not focus on
Before continuing with the analysis, I discuss in more detail the concept of deceptive markets and common applications. Readers who are familiar with the literature might want to skip this subsection.

3.2 Applications

The goal of models on deceptive products is to understand markets where consumers do not take some product characteristics into account. These naive consumers might misperceive product- or pricing features, or misestimate their own future demand at the time of contracting/purchase. The reduced-form model analyzed in this paper covers both cases. Markets where a potential for deception has been established empirically include those for credit cards, insurances, mortgages, retail banking, (mobile-)phone services, printers, and casinos. The additional feature of my model in relation to the literature is repeated consumption of the product combined with the firms’ ability to infer the level of sophistication of their customers by observing past behavior. Potential applications for this model are markets for credit cards, retail banking, (mobile-)phone services, and insurance, since both deception and behavior/history based pricing occur, and information collection of customers’ purchase patterns is simple and pervasive.

Credit cards are a quite homogeneous products that mainly vary in fee structures. Many consumers do not take overlimit, overdraft or late fees into account or underestimate their tendency to borrow money when choosing a credit-card contract. In my model, every such fee that naives do not take into account is represented by the hidden fee. Prices that are taken into account, such as maintenance fees, cash rewards, introductory APRs or new-client bonuses, are transparent fees. For examples on how firms can learn to distinguish customers based on their naiveté, see Stango and Zinman (2009) or Grubb (2009). Alternatively, credit-card providers could measure

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19 Heidhues et al. (2014) argue that Bertrand equilibria are not reasonable in the context of deceptive products. Among other things, they are not robust to positive unshrouding costs—no matter how small.

20 In all of these markets, detailed information on consumption patterns are required to write bills to customers.

21 For empirical evidence, see the papers cited in the introduction and the literature section. Heidhues and Koszegi (2015) also discuss the application of the model to credit card markets. Consumers pay additional hidden fees because of their taste for immediate gratification that leads them to borrow more than they prefer ex ante. See also Meier and Sprenger (2010) for a discussion of existing evidence.

22 Though heterogeneity within these two blocks of fees is certainly important, it is not the focus of this analysis.

23 Stango and Zinman (2009) identify hidden fees by the savings in fees that a credit-card customer could have made by shifting liquidity between accounts. Grubb (2009) looks at how much telephone customers could have saved if they had the same consumption with a different contract. Though the latter approach might not distinguish customers who are naive from those purchasing a commitment device, it probably provides an informative signal, which is enough for the purpose of this paper.
naiveté indirectly by simply estimating a consumer’s elasticities of demand for the different fees, and check how these elasticities correlate with the clients’ purchase patterns. Firms can also use big-data analysis on the purchase details of customers or the usage-patterns on online accounts.

Besides this, some simple education policies of consumers w.r.t. hidden fees are effective, as shown by Stango and Zinman (2014). They observe that simply asking consumers about overdraft fees in a survey significantly reduces their probability of paying those fees relative to a control group that was not asked these questions. Similarly, Alan et al. (2015) inform some customers of a Turkish retail bank of the possibility of overdraft without mentioning prices while others are offered a discount. While mentioning overdraft without mentioning prices increases use of overdraft, offering a discount reduces it. This strongly suggests that overdraft prices are indeed shrouded to customers and that simple information campaigns can be effective in unshrouding hidden components to at least some customers.

In retail banking, empirical evidence suggests that customers underestimate their likelihood of overdraft when choosing a bank-account (see (OFT, 2008; Cruickshank, 2000)). Hence, fees and interest payments associated with overdraft are hidden fees to many consumers. Account maintenance fees are rather salient and more likely to be transparent fees.

Grubb (2009) studies the (mobile-)phone market. At the time of contracting, firms have better forecasts on consumers’ later demand for phone calls; i.e. when consumers underestimate the variation of their demand, firms can offer contracts with high payments in states that customers falsely perceive as unlikely. These unexpected payments also function as hidden fees.

Applications to insurance markets work in a similar way. Given the huge amount of data that insurance companies have over their customers, they are likely to have better estimates on at least some of their clients’ risks than these have themselves.

As these applications show, fees in observed contracts do not need to fit perfectly into the categories hidden or transparent fee. A more general way to think about transparent and hidden fees is as anticipated and unanticipated payments. E.g. take a credit-card customer. Assume he pays $10\text{€}$ maintenance fees and $0.10\text{€}$ for each Euro he does not pay back within 30 days. Say he believes he will borrow $50\text{€}$ for more than 30 days while he will actually borrow $100\text{€}$. Then in this model his transparent fee is $10\text{€} + 5\text{€} = 15\text{€}$ and his hidden fee the unanticipated $5\text{€}$. 
4 Benchmarks

To emphasize the impact of consumer naiveté and private customer data, I analyze two benchmark cases. First, a classic analog where all consumers are perfectly sophisticated and value a base product, but only some consumers value an add-on as well. For example, some consumers only buy a credit-card account to do transactions while others also borrow. Afterwards, I study the basic model absent customer data, i.e. where firms do not learn their old customers’ types. In both benchmarks, profits are zero and expected consumer surplus is maximized.

4.1 Private Customer-Base Information without Naive Consumers

In this benchmark all consumers value the base good with $v > c$, but the share $\alpha$ of consumers—called add—buy an add-on good for which they have valuation $\bar{a}$. The remaining consumers only buy the base good and are called base. There are two firms $A$ and $B$, which produce the base good at cost $c$, and the add-on without additional marginal costs. W.l.o.g., let firm $A$ know all customers’ types while firm $B$ knows only their distribution. Thus, firm $A$ can assign prices for each type, $f_{add}^A$, $f_{base}^A$, while $B$ can instead offer two products—the base product only, and a product with add-on—at different prices $f_{add}^B$, $f_{base}^B$.

A simple screening argument shows that firm $A$ cannot benefit from her information. To see this for pure strategies, first note that firm $B$ cannot earn positive margins from any customer type. Otherwise, firm $A$—being able to target each customer group—could increase profits by marginally undercutting prices for each customer group. Now suppose towards a contradiction that firm $A$ earns a positive margin from any customer group. Suppose $A$ profitably offers $f_{add}^A > c$ to types add. Then firm $B$ can earn strictly positive profits by setting $f_{add}^B = f_{add}^A - \epsilon$ and $f_{base}^B = f_{add}^A + \epsilon$ for some $\epsilon > 0$ small enough. add consumers prefer paying $f_{add}^B$, base consumers either stay with $A$ or profitably self select into paying $f_{base}^B$ and $B$ earns strictly positive profits—a contradiction. Similarly, we cannot have $f_{base}^A > c$, since $B$ could then profitably draw all bases by setting $f_{base}^B = f_{base}^A - \epsilon$ and $f_{add}^B = f_{base}^A - 2\epsilon$ for some $\epsilon > 0$ small enough. This leads to a contradiction as well. I extend the results to mixed strategies in this Proposition.

**Proposition 1.** [Private Customer Information with Sophisticated Consumers only]

When customers are sophisticated and have heterogeneous add-on demand, a firm that is privately
informed about add-on-demand types earns zero profits from each type in a competitive market.

In competitive markets, firms offer first-best contracts. When consumers are sophisticated, they make optimal choices and self-select into the first-best contract. Thereby, they reveal their information by their product choice such that private information of firms on willingness to pay for add-ons is unprofitable.

In the credit-card context this means that after correcting for non-demand heterogeneity such as risk levels of customers etc., profits from consumers that borrow with their credit-card account and from those that simply use their credit card for transactions should be similar. This prediction on margin levels extends to environments with switching or search cost, as long as these do not asymmetrically differ across the two customer groups.

Remark: Of course, also in models where all consumers are sophisticated, firms can have many reasons to gather information on their customers that are beyond the scope of this paper. But as I show in Section 5, the rational model strongly underestimates the benefits of information on customers if some consumers are naive.

4.2 No Customer Data

The next benchmark highlights the role of customer data in the presence of naive customers. I look at the two-period model for deceptive products when there are naive and sophisticated consumers but firms do not learn their customers’ types.

When customers cannot be distinguished, firms offer only one transparent price in each period. Discrimination between a firms’ own and competitors’ customers does not help since they have the same distribution of types.

**Proposition 2.** [Deceptive Markets without Customer Data]

Let \( v \geq c - \alpha \bar{a} \). Shrouding equilibria exist. In each shrouding equilibrium, firms earn zero profits. In each equilibrium in which shrouding occurs with probability one, consumers pay transparent prices \( f_{n1} = f_{n2} = c - \alpha \bar{a} \) and naives additionally pay hidden prices \( a_{n1} = a_{n2} = \bar{a} \).

Proposition 2 translates the results of Gabaix and Laibson (2006) to this setting. The main difference, aside from simplifications, is that there are two periods but—when firms are unable to distinguish customers by naiveté to price discriminate in period 2—there are no dynamic effects.

---

\(^{26}\)The assumption guarantees that sophisticated consumers want to buy in equilibrium even when the product is socially wasteful. For the case of \( v < c - \alpha \bar{a} \), firms do not sell to sophisticates anymore. This has been studied by Heidhues et al. (2012).
The equilibrium is simply a repetition of the one-period equilibrium discussed by Gabaix and Laibson (2006), in which shrouding profits from naive consumers cross-subsidize sophisticated ones in each period.

The intuition is as follows: given shrouding, hidden fees increase margins without affecting consumers’ decisions and firms consequently set them to $\bar{a}$. But since firms cannot discriminate between consumers, they use profits from hidden fees to lower transparent prices to compete for more consumers until the average customer is not profitable anymore. This reduces profits to zero in both periods.

Shrouding equilibria are not very stable since firms earn the same profits by unshrouding prices. In particular if some naive customers cannot avoid unshrouded hidden fees, as shown in Section 7.1, competitors can profitably attract these non-avoiding naives by unshrouding hidden fees so that a shrouding equilibrium never exists for socially valuable products ($v > c$).

For future reference, note that the continuation profits on any equilibrium path are zero in the second period, whether hidden fees are shrouded in the first period or not.

The two benchmarks establish that if either all consumers are sophisticated or firms do not have private usage data of their consumers, profits are zero. There is inefficient trade in the latter case if $c - \alpha \bar{a} < v < c$. In the next section, I analyze the main model introduced in Section 3.1.

5 The Benefits of Customer Data in Deceptive Markets

I now discuss the model introduced in Section 3. Before I look at the Propositions, I illustrate why in the second period of shrouding equilibria, firms earn positive profits and play mixed strategies. Proposition 3 establishes how firms benefit from informational advantages in distinguishing their old customers. Proposition 4 discusses why these profits might not be handed over to customers in period 1 when firms can educate consumers about hidden fees.

5.1 Exploiting Naiveté with Customer Data

To start, I look at period 2 after prices are shrouded in period 1 and all firms have a non-empty customer base. Firms set two different prices for their own old customers, $f_{n2}^{naive}$ and $f_{n2}^{soph}$, and one to poach customers from competitors, $f_{n2}^{new}$. Recall that firms do not benefit from making additional poaching offers since naives and sophisticates only consider transparent prices. Since firms
can identify their old customers, firms do not attract their own sophisticates with new-customer prices below \( c \).

To see that period 2 profits are positive in a shrouding equilibrium, consider the simple case with two firms \( A \) and \( B \), and suppose both firms shroud hidden fees. Take firm \( A \) and note first that it only wants to sell to its existing sophisticated customers at prices \( f_{A}^{\text{soph}} \geq c \). This implies that \( B \) always attracts sophisticates of \( A \) with prices \( f_{B}^{\text{new}} < c \). Since naive customers pay hidden fees of \( \bar{a} \) in each shrouding equilibrium, only \( f_{B}^{\text{new}} \geq c - \alpha \bar{a} \) can lead to non-negative profits for \( B \) from attracting new customers. All \( f_{B}^{\text{new}} < c - \alpha \bar{a} \) induce strictly negative profits for \( B \) from new customers, even when profitable naive customers are poached from firm \( A \). Thus, roughly speaking, prices \( f_{A}^{\text{soph}} < c \) and \( f_{B}^{\text{new}} < c - \alpha \bar{a} \) cannot occur in an equilibrium with positive probability. But with \( f_{B}^{\text{new}} \geq c - \alpha \bar{a} \), there is no reason for firm \( A \) to price its naives below \( c - \alpha \bar{a} \). This implies prices for existing naives \( f_{A}^{\text{naive}} \geq c - \alpha \bar{a} \). The same reasoning holds for firm \( B \) and can be generalized to any number \( N \geq 2 \) of firms (see Figure 1). Consequently, firms can always deviate to achieve profits of at least \( s_n \alpha (1 - \alpha) \bar{a} \) from consumers in its customer base by setting \( f_{n}^{\text{naive}} = c - \alpha \bar{a} \) and \( f_{n}^{\text{soph}} \geq c \). Charging \( f_{n}^{\text{new}} \geq c \) ensures that these profits are not wasted by unprofitably attracting new customers. Though this is not an equilibrium, it establishes the minimum profits firms can guarantee themselves in each shrouding equilibrium in period 2.

Next, let us see why there is no pure-strategy shrouding equilibrium in period 2. Take again two firms \( A \) and \( B \), and let \( f_{A}^{\text{naive}} > c - \alpha \bar{a} \). By marginally undercutting \( f_{A}^{\text{naive}} \) with \( f_{B}^{\text{new}} \), firm \( B \) can profitably attract \( A \)’s customers. Firm \( A \) can prevent this by charging \( f_{A}^{\text{naive}} = c - \alpha \bar{a} \). Then firm \( B \) attracts no naive consumer from \( A \) and charges some \( f_{B}^{\text{new}} \geq c \) to break even on new customers. But then, \( A \) is better off by increasing her naive-customer price to \( f_{A}^{\text{naive}} = c \). This, however, gives \( B \) an incentive to marginally undercut \( f_{A}^{\text{naive}} = c \) and the argument starts again.

![Figure 1: Support of prices in period 2. Firms only keep sophisticated consumers if they at least break even with them. Thus, there is no cross-subsidization. New customers’ prices are never below \( c - \alpha \bar{a} \), such that there is no need to price naive customers below this threshold.](image-url)
In the end, after shrouding in period 1 and when all firms have a positive customer base, consumers pay transparent naive and new-customer prices based on the following distributions:

\[
F_{\text{new}}(f_{\text{new}}) = \begin{cases} 
0, & \text{if } f_{\text{new}} \in (-\infty, c - \alpha \bar{a}] \\
1 - \frac{\sqrt{1 - (1 - \alpha \bar{a}) f_{\text{new}} - \alpha}}{\alpha (f_{\text{new}} - c)}, & \text{if } f_{\text{new}} \in (c - \alpha \bar{a}, c) \\
1, & \text{if } f_{\text{new}} \in [c, \infty)
\end{cases}, \forall n. \tag{1}
\]

The distribution has a mass point on \(c\) of weight \(\frac{N - 1}{\sqrt{1 - \alpha}}\). Firms mix naive customers’ prices according to

\[
F_{\text{naive}}(f_{\text{naive}}) = \begin{cases} 
0, & \text{if } f_{\text{naive}} \in (-\infty, c - \alpha \bar{a}] \\
\frac{f_{\text{naive}} - \alpha - c}{\alpha (f_{\text{naive}} - c)}, & \text{if } f_{\text{naive}} \in (c - \alpha \bar{a}, c) \\
1, & \text{if } f_{\text{naive}} \in [c, \infty)
\end{cases}, \forall n. \tag{2}
\]

These distributions are independent of market shares such that firms play identical strategies for the respective prices on \((c - \alpha \bar{a}, c)\). Intuitively, all firms \(j \neq n\) mix new-customer prices to make firm \(n\) indifferent between all \(f_{\text{naive}} \in (c - \alpha \bar{a}, c)\). This must be true for all \(n\) and therefore all new-customer prices must follow the same distribution. The same logic applies to distributions of naive-customer prices. For \(N > 2\) there are other equilibria with \(f_{\text{soph}} \geq c\) for some \(n\), or where \(F_{\text{new}}(\cdot)\) has less probability weight on the mass point at \(c\) and instead charges prices above \(c\). But since these prices are never actually paid by consumers, all shrouding equilibria lead to the same profits, purchase prices of consumers and welfare.
With these mixed strategies at hand, Proposition 3 summarizes the results for period 2.

**Proposition 3.** [Exploiting Private Information on Customer Data in Period 2]

Consider any equilibrium in which shrouding occurs with positive probability in the second period. Then, shrouding occurs with positive probability in period 2 if and only if hidden prices are shrouded in period 1 and each firm has a non-empty customer base.

In such equilibria, hidden prices are shrouded with probability one. For each firm $n$, profits are $\pi_{n2} = s_n \alpha (1 - \alpha) \bar{a}$. Hidden prices are $a_{n2} = \bar{a}$. Transparent prices are $f_{n2}^{soph} \geq c$, and consumers pay $f_{n2}^{new}$ and $f_{n2}^{naive}$ based on (1) and (2) respectively.

The supports of (1) and (2) show that information advantages on customer bases create an information-based price floor in the second period of shrouding equilibria at $c - \alpha \bar{a}$. Firms earn positive margins on their old naives, and break even on sophisticates and new customers. To see that effects on profits are indeed strong, note that the overall market revenue from hidden fees in period 2 is $\alpha \bar{a}$. Of this amount, firms keep the share $(1 - \alpha)$ despite competition. For example, if $\alpha = 0.5$, half of the hidden fees payed remain as profits to firms. Thus, Proposition 3 establishes that firms benefit strongly from their customer data by being able to distinguish naive and sophisticated customers.

The intuition behind the results in Proposition 3 is that firms use their customers’ usage data to reduce the intensity of competition. Private information on customer data allow firms to price their continuing customers differently based on their sophistication. But since naives and sophisticates only consider transparent prices, competitors can poach new customers with only a single offer. This creates a competitive asymmetry. Because firms keep their old sophisticates only if they are profitable, firms charge them at least marginal-cost prices. At the same time, naive customers remain profitable due to the hidden fee but pay transparent prices below marginal costs—they get a transparent discount. This renders unprofitable sophisticated customers more responsive to poaching offers than profitable naive ones: profitable naive customers are always attracted together with unprofitable sophisticates but sophisticates might come without naives. Rivals respond to this adverse attraction of unprofitable sophisticated customers by mitigating their poaching intensity. Consequently, firms can break even on continuing sophisticates and earn strictly positive margins from naives. Since firms earn zero profits by attracting new customers on average, overall profits are strictly positive.

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Note that if competitors could observe each customers contract, they could infer its type and each firm could make type-specific offers. This would split the market and lead to zero profits.
Importantly, these properties of shrouding continuation equilibria do not require the ability of firms to unshroud hidden fees. These insights carry over to cases where consumer education is infeasible or very costly. When unshrouding is not feasible, total continuation profits always equal $\alpha(1 - \alpha)\bar{a}$ and each firms’ customer base determines their share of this profit.

Proposition 3 sheds new light on the value of customer data in competitive environments. To see how, compare the results with Propositions 1 and 2. When consumers are aware of their demand for additional services, they optimally respond to poaching offers. Proposition 1 shows that this renders usage data unprofitable when consumers are sophisticated but differ in their demand for an add-on. When there are naive consumers but firms cannot distinguish customers’ types, naives pay more but these profits are handed over to sophisticated customers, inducing the cross-subsidization that other papers in the literature usually find.\textsuperscript{28} Firms earn zero profits and sophisticated consumers earn the benefits from exploitation. In contrast, firms are able to keep revenue from exploitation when they learn to distinguish customers based on their naiveté.

Another interesting finding is that shrouding profits are not monotone in the share of naive consumers $\alpha$. More precisely, firms have a preference for a balanced customer base. Firms earn a positive expected margin of $(1 - \alpha)\bar{a}$ from naives, but this margin decreases in the share of naives. Intuitively, with more naives, uninformed rivals adversely attract additional unprofitable sophisticates which makes poaching less profitable. Thus, with more sophisticates firms can keep a larger margin of its naives. This implies that firms might want to educate some customers about hidden fees but not too many. Moreover, firms with different shares of naive consumers can gain from trading consumer portfolios. This finding is discussed in more detail in Section 5.2.\textsuperscript{29}

Also note that each firm strictly prefers shrouding over unshrouding. In particular, since naives become sophisticated after unshrouding and can then avoid hidden fees, the most profitable deviation by unshrouding hidden prices is the same as in Proposition 2 and leads to zero profits. But in contrast to Proposition 2, firms earn positive profits here, giving them a strict incentive to keep prices shrouded when they learn to distinguish the naiveté of their customers. Thus, customer data make shrouding more stable.

Since naive consumers render usage data profitable despite (perfect) competition, these results suggest that firms have a high willingness to pay for data or technologies that improve predictions

\textsuperscript{28}This is discussed in more detail in the literature section.

\textsuperscript{29}Additionally, a common finding in the literature is that shrouding conditions require a sufficiently large amount of naive consumers. Shrouding conditions ensure that no profitable unshrouding deviation exists. For examples, consider Gabaix and Laibson (2006) or Murooka (2013). But when firms can distinguish customers based on their naiveté, shrouding equilibria can exist with arbitrarily small shares of naive customers in the population. This extends to the case in Section 7.1 where non-avoiding naives make unshrouding more profitable.
of customer naïveté. Since these information do not create any fundamental, they would be clearly inefficient.

In line with the mixed strategies for new-customer prices, Schoar and Ru (2014) find that credit-card companies have substantial variations in their offers to new consumers, even after controlling for observable characteristics. Similarly, Stango and Zinman (2014) observe substantial variation in borrowing costs for credit-card customers after controlling for observable characteristics. Both findings are in line with the mixed strategies in Proposition 3 and with firms conditioning offers on privately observed characteristics. Also in line with the supports of (1) and (2), Schoar and Ru (2014) find a larger price dispersion for subpopulations where consumers are more likely to be naive, i.e. have a lower level of education.

5.2 Gains from Trading Customer Portfolios

Before discussing how the possibility to educate customers mitigates competition already when firms compete for a customer base, I present an important implication of the firm’s benefit from a mixed customer base: firms with different shares of naive clients can increase profits by trading parts of their customer base.

For this purpose, I consider a simpler version of the second period of the previous model in this subsection with two firms that cannot unshroud hidden fees. Assume each firm already has a customer base \( s_n \) for \( n \in \{1, 2\} \) with \( s_1 + s_2 = 1 \). Firms have different shares of naives in their customer bases with \( \alpha_1 \neq \alpha_2 \). Since firms cannot unshroud hidden fees, they earn at least \( s_n\alpha_n(1 - \alpha_n)\bar{a} \). This follows from the same logic leading to Proposition 3.

A potential problem with selling naive customers is that firms might poach them back. Since they know these consumers’ types, they could target them directly and avoid the adverse attraction of naive consumers. Thus, to gain from trade, firms need to be able to commit not to poach consumers back. Potentially, this could be agreed upon in a contract. But more convincingly, firms and banks commonly leave the market, and sell their entire credit-card business. \(^{30}\)

Since second-period profits are strictly concave in \( \alpha_n \), firms with different shares of naives

benefit from trading customer portfolios:

\[ s_1 \alpha_1 (1 - \alpha_1) \bar{a} + s_2 \alpha_2 (1 - \alpha_2) \bar{a} \leq (s_1 \alpha_1 + (1 - s_1) \alpha_2) (1 - (s_1 \alpha_1 + (1 - s_1) \alpha_2)) \bar{a}. \]

The left-hand side denotes total market profit if both firms keep their initial share of customers. The right-hand side is total market profit if both firms have the same share of naives, i.e. the share of naives in the population. The inequality follows directly from the concavity of profits in \( \alpha \).

Note that this inequality is strict whenever \( s_n \in (0, 1) \) and \( \alpha_n \in (0, 1) \) for all \( n \in \{1, 2\} \). Thus, firms can increase the profitability of their customers by acquiring customer portfolios that make their own portfolio more balanced. By doing so, they make portfolios more balanced on average and increase total market profits in the second period.

For a closer look, suppose \( \alpha_1 < \alpha_2 \), i.e. firm 2 has more naives. Then total market profits maximize when firm 2 sells \( s_2 (\alpha_2 - \alpha_1) \) naives and buys \( s_1 (\alpha_2 - \alpha_1) \) sophisticates in return.

Firms and banks frequently trade credit-card portfolios that are worth several billion dollars. Due to the large number of firms active in the credit-card industry, these deals are unlikely to result from a monopolization strategy. In contrast to mergers or acquisitions with effects on market power, my model predicts that these deals increase profits for the trading parties but they have no impact on profits of uninvolved rivals. Nonetheless, they are anti-competitive. While there are other explanations for credit-card portfolio deals such as changing business strategies or liquidity constraints, my model suggests an alternative explanation based on the value of data on naive consumers to competing firms.

A caveat of these results is that firms who sell customer portfolios need to commit not to use information that they obtained previously about these accounts. A non-poaching clause could be agreed upon in the contract but clauses of this kind might be problematic due to their anti-competitive nature. Acquiring the whole (local) credit-card business merges two unbalanced portfolios and is a second-best solution. It might not optimally balance customer bases, but certainly avoids the commitment problem since one firm leaves the (local) market.

In the next section, I return to the analysis of the main model as introduced in Section 3 where firms are able to unshroud hidden fees to naive customers.
5.3 Mitigated Competition for Customer Bases

The ability of firms to educate consumers induces multiple continuation equilibria. Before looking at period 1 when firms can educate naive consumers, I discuss equilibrium selection of the continuation equilibria.

**Equilibrium Selection.** When unshrouding occurs in period 1 or at least one firm has an empty customer base, continuation profits are zero. The latter is true since firms without customer base have no naive customers to exploit and earn zero profits. But when all naives can avoid unshrouded hidden fees, unshrouding induces zero profits as well, and firms without customer base are indifferent between unshrouding or not. Thus, there are multiple continuation equilibria. But Proposition 6 below shows that whenever there is a positive share of naives that cannot avoid unshrouded hidden fees, the multiplicity disappears and unshrouding occurs with probability one when at least one firm has an empty customer base.\(^{31}\)

Intuitively, since non-avoiding customers are aware of these fees but still pay them, they can be profitably attracted by unshrouding hidden fees. While poaching educated avoiding naives requires undercutting transparent prices, competitors can profitably attract non-avoiding naives by unshrouding hidden fees and marginally undercutting their total price, i.e. transparent price plus hidden fee. But then firms without a customer base have a strict incentive to educate customers and multiplicity disappears. This allows me—plausibly—to focus on equilibria in which firms without customer base educate consumers with probability one, since other equilibria are not robust to the presence of non-avoiding naives.

This also shows that positive total profits do not result from the multiplicity of stage-game equilibria in finitely repeated games and a related collusion-type logic. And since non-avoiding naives make unshrouding strictly profitable for firms without a customer base, results are also robust to positive costs for unshrouding.

Another type of equilibrium that exists besides shrouding continuation equilibria are “Bertrand equilibria”: when at least two firms unshroud hidden fees, none of them can benefit from shrouding instead and all consumers pay a total price of marginal costs. But since each firm strictly prefers the shrouding continuation equilibrium over the Bertrand one, it is plausible that firms coordinate on the equilibrium that is more profitable for each of them. I therefore make an equi-

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\(^{31}\)The existence of non-avoiding naives is particularly realistic in the credit-card example: consumers that become aware of high overdraft fees and interest rates still have some amount of credit-card debt. In order to avoid those fees, they require other liquid assets to pay back their whole credit-card debt which is impossible for consumers with liquidity constraints.
librium selection assumption: Whenever a shrouding equilibrium exists in period 2, firms play it.\footnote{Heidhues et al. (2014) argue that this is the only reasonable equilibrium. Among other things, the Bertrand-type equilibrium is not robust to positive unshrouding costs.}

We saw in Proposition 3 that customer data on naiveté can be profitably exploited by competing firms. I show now how competition for customer bases is mitigated as well when firms can unshroud hidden fees to naive consumers. In the main text I maintain the rather extreme assumption that unshrouding reaches all naive customers to keep the exposition simple. In an extension I verify that results are robust to partial unshrouding that reaches only an arbitrarily small share of consumers.

Denote by \( s_{\min} = \min_n \{ s_n \} \) the smallest market share and the set of all firms that charge the lowest price in period 1 by \( M = \{ n \in \{ 1, 2, ..., N \} | f_{n1} = \min_n \{ f_{n1} \} \} \). Then we can write down the total profits given firms shroud in period 1 (Figure 3):

\[
\pi_{n1}(f_{11}, ..., f_{N1}) = \begin{cases} 
\frac{s_n}{\sum_{o \in M} s_o} (f_{n1} + \alpha \bar{a} - c) + 0, & \text{if } f_{n1} = \min_n \{ f_{n1} \} \leq v \text{ } \& \text{ } M < N \\
\frac{s_n (f_{n1} + \alpha \bar{a} - c) + s_n \alpha (1 - \alpha) \bar{a}}, & \text{if } f_{n1} = \min_n \{ f_{n1} \} \leq v \text{ } \& \text{ } M = N \\
0, & \text{if } f_{n1} > \min_n \{ v, \min_n \{ f_{n1} \} \} 
\end{cases}
\]

(3)

Total profits exhibit a new kind of discontinuity that stems from the dynamic nature of the game and the possibility to educate consumers about hidden fees. Positive continuation profits can only be achieved when prices are shrouded in period 1 and all firms attract customers in this period. That is, all firms charge \( \min_n \{ f_{n1} \} \) with positive probability. This results in a strong incentive to coordinate on the same transparent price.

**Proposition 4.** [Mitigated Customer-Base Competition in Period 1 in Shrouding Equilibria]

Shrouding equilibria with shrouding in both periods exist. In each equilibrium satisfying the selection criteria, all firms choose hidden fees \( a_{n1} = \bar{a} \). In equilibrium with pure strategies in period 1, all firms set the same transparent price \( f_1 \in \left[ c - \alpha \bar{a} - \alpha (1 - \alpha) \bar{a}, c - \alpha \bar{a} + \frac{s_{\min}}{1-s_{\min}} \alpha (1 - \alpha) \bar{a} \right] \).

Total profits \( \Pi_n = s_n (f_1 + \alpha \bar{a} - c) + s_n \alpha (1 - \alpha) \bar{a} \in \left[ 0, s_n \frac{s_{\min}}{1-s_{\min}} \alpha (1 - \alpha) \bar{a} + s_n \alpha (1 - \alpha) \bar{a} \right] \).

For all equilibria in which \( \Pi_n > 0 \), shrouding occurs with probability one.

Profitable shrouding in period 2 can occur only if all firms have a positive customer base. Firms with an empty customer base unshroud hidden fees to attract customers. This reduces second-
Figure 3: The dotted line depicts total profits of a firm that undercuts all others in period 1. The solid line depicts total profits of a firm when all firms choose the same price in period 1. Hence, for all $f_1 \in [c - \alpha \bar{a} - \alpha(1 - \alpha)\bar{a}, c - \alpha \bar{a} + \frac{s_{\min}}{1-s_{\min}}\alpha(1 - \alpha)\bar{a}]$, no firm has an incentive to undercut competitors.

period profits for each firm with a customer base. Therefore firms benefit from coordinating prices in period 1. This mitigates customer-base competition already in the first period. Future profits are not competed away ex ante, but instead total profits can increase above the second-period level.

First-period transparent prices are not uniquely pinned down. Firms have an incentive to coordinate on an interval of prices. The coordination incentive also gives rise to—arguably less plausible—mixed-strategy equilibria with some miscoordination. I discuss them in the appendix. The upper bound of the interval depends on the smallest market share. This is because the firm with this market share gains most from undercutting competitors in the first period.

In all but one shrouding equilibrium, firms earn strictly positive total profits. If in addition some naive consumers cannot avoid unshrouded hidden fees, under reasonable conditions, firms earn strictly positive total profits in each shrouding equilibrium. This renders shrouding equilibria even more stable.

From the firms’ perspective, shrouding equilibria with higher transparent prices in period 1 Pareto dominate equilibria with lower prices. This gives firms an incentive to coordinate on higher transparent prices in the first period. The following corollary summarizes the results of Propositions 3 and 4 when Pareto dominance is applied as an equilibrium-selection device.
Corollary 1 (The Firms’ Preferred Shrouding Equilibrium). In the most profitable shrouding equilibrium, firms charge $f_{n1} = c - \alpha \bar{a} + \frac{s_{min}}{1 - s_{min}} \alpha (1 - \alpha) \bar{a}$, $\forall n$ and second-period prices are as in Proposition 3. Hidden fees are $a_{nt} = \bar{a}$ in both periods and total profits are $\Pi_n = s_n \frac{s_{min}}{1 - s_{min}} \alpha (1 - \alpha) \bar{a} + s_n \alpha (1 - \alpha) \bar{a}$. Shrouding occurs with probability one.

The comparisons with Propositions 1 and 2 show that private information on existing customers have a strong impact on the properties of shrouding equilibria. Total prices increase in both periods for all customer types. Shrouding becomes more stable since profits can be positive in each period. These results are driven mainly by two effects. First, second-period shrouding profits increase since firms can profitably use their customers’ usage data to reduce the intensity of competition. Second, the ability to educate naives mitigates competition for customer bases and period 2 profits might not be handed over to consumers in the first period.

The model highlights new and important dynamic effects in markets for deceptive products. While there are no dynamic effects in Proposition 2, they become crucial when firms learn about their customers. The competition for the market in period 1 works very different from competition within the market in period 2, and the results differ in crucial aspects from known properties of markets for deceptive products: information on customer naiveté become a valuable asset for firms. Additionally, the results suggest that firms have a reason not to become informed about competitors’ customers since this would intensify future competition and decrease shrouding profits.

I show in Section 7.1 how these results are robust when firms can undercut but cannot serve the entire demand. Intuitively, with non-avoiding naives, unshrouding in period 2 earns positive profits. But then firms do not need to have an empty customer base in period 2 to unshroud, a sufficiently small one suffices. In this sense, firms have a stronger incentive to unshroud in period 2 when they loose customers in period 1.

Let me point out again that the high profits of firms in the second period of shrouding equilibria do not depend on the firms ability to educate consumers about hidden fees, but positive profits in the first period do. Though such transparency policies are possible, they are very unlikely to reach all naive consumers. I discuss extensions with imperfect or partial unshrouding are in Section 7.2 and show that total profits remain positive when unshrouding is arbitrarily weak.
6 Policy Implications

Before discussing policy implications, note that despite concerns about safety-in-markets or consumer surplus, efficiency can be a concern as well. In a richer model with a smoothly decreasing demand curve, the price distortions away from marginal costs induce overconsumption of naive customers as well as for sophisticated ones. At the same time, large profits could lead to inefficient investments in exploitative technologies or excessive entry. The policies discussed below move prices closer to marginal costs and can improve efficiency in such frameworks.

A natural policy suggestion derives from the results above: firms should disclose their private information on their customers, i.e. their relevant customer data, to competitors. I discuss the impacts of such a policy in this Section.

Consumption data are usually not only accessible by firms but also by consumers. Especially since firms are required to write a bill to consumers—phone bills depend on how much and which network was called, credit-card bills depend on payments made with the card and the resulting overall balance—many usage data are in principle available to consumers as well and can therefore be given to competing firms.

In the context of this model, disclosing consumer data to all firms enables each firm to charge different prices to each customer type, whether it is in the firms’ customer base or not. I summarize the impact of such a policy in the next Proposition.

**Proposition 5.** [Deceptive Markets with Disclosed Customer Info in Period 2]

Firms earn zero profits in each second-period continuation equilibrium and in period 1. Equilibria exist where shrouding occurs with probability one. In these equilibria, consumers pay total prices equal to marginal costs in period 2 and transparent prices \( c - \alpha \bar{a} \) in period 1. Hidden prices are \( a_{n1} = a_{n2} = \bar{a} \). If shrouding does not occur with probability one in period 2, it occurs with probability zero.

With consumers’ types disclosed to all firms in period 2, the market is effectively split, and firms compete for naives and sophisticates separately. This induces marginal cost pricing even in shrouding equilibria and zero profits. Thereby, the disclosure policy triggers a rent shift from firms to consumers. Note that lower profits also imply larger incentives for firms to unshroud hidden fees.

\[^{33}\text{For more on this, see Heidhues and Koszegi (2015).}\]

\[^{34}\text{Thaler and Sunstein (2008) discuss a policy that aims at simplifying consumer data and make them available easily to consumers in order to help them make better decisions. In contrast to this, the policy I discuss below aims at sharing customer information with competing firms, not customers.}\]
Besides the regulatory benefits, Propositions 5, 2, and 3 together highlight that indeed the asymmetric information on customer data cause high profits of firms when some consumers are naive.

This policy increases consumer surplus also when firms are unable to educate consumers. To see this, consider any second period of the game after shrouding in period 1. Then the logic behind Proposition 5 says that profits are competed away, even when consumers are not educated about hidden fees. The extension with partial unshrouding strengthens the importance of this observation even more: when firms can only educate some consumers about hidden fees, the remaining naives can still be exploited, leaving positive profits to firms even after unshrouding. With customer data disclosed to all firms, however, profits still go to zero.

A nice feature of this policy is that it is not based on educating customers or providing them with tools that help them to make better decisions. Policies that go in this direction are discussed by Thaler and Sunstein (2008) and Kamenica et al. (2011). Despite the fact that empirical findings strongly suggest that consumers are not aware of product or contract features in some markets, it is not always clear how exactly those features are misunderstood. Before inducing an effective simplification or education policy, regulators would have to understand first the psychological process underlying consumers’ misunderstandings. Thus, such policies require deep regulatory knowledge, a feature they share with well-designed price regulations. In contrast, disclosing customer data to competitors is much less sensitive to regulatory knowledge. The policy simply limits the ability of firms to profitably price discriminate customers, since competitors can now target each customer group specifically.

These results suggest a potential conflict between antitrust and privacy concerns. A regulator might want to disclose a firm’s consumer data to break her market power and reduce profits. On the other hand, consumers might not want their data to be handed around the market. Naturally, such a policy could have many unforeseen consequences and should not be implemented lightly. E.g., disclosing customer data to all competitors might induce firms to enter the market just to get the data and then use them somewhere else. Nonetheless, these results highlight that exclusive information on customers can be harmful when some consumers are naive, and that there is a potential conflict between antitrust and privacy concerns.

Banning price discrimination is beneficial to consumer surplus as well and leads to the same outcomes as in Proposition 2. But banning price discrimination in credit-card or retail-banking markets would probably have many unintended consequences. Especially since discriminating
consumers along other dimensions, e.g. their risk behavior, is likely to increase welfare.

7 Extensions

7.1 Non-Avoiding Naives

When hidden fees are unshrouded, naive consumers could either be able to avoid them and become like sophisticates, or become aware of hidden fees without being able to avoid them. Both assumptions can be reasonable. As an example, consider the availability of external funds for credit-card or retail-bank borrowing. Consumers with external funds can easily avoid costs of borrowing or late payment fees by paying debt immediately. Consumers with liquidity constraints cannot.\footnote{In alternative examples consumers can avoid hidden fees by precautionary behavior that is not available in the short term. Expensive roaming charges can be partially avoided by booking additional packages or purchasing a local phonecard. But when an urgent phone call has to be made, those preparations cannot be done quickly. Note that sophisticated consumers that cannot avoid hidden fees can be screened into another product as in Heidhues et al. (2014), so I do not consider these types here.}

Assume that a share $\eta \in [0, 1)$ of the naives cannot avoid unshrouded hidden fees, though they take them into account. The remaining $1 - \eta$ naives can avoid them and become like sophisticated consumers after unshrouding.\footnote{The case $\eta = 1$ is ruled out to avoid that firms are indifferent between shrouding or not when only considering their own customer base.}

The qualitative properties of shrouding equilibria do not change in period 2 when some naive consumers cannot avoid unshrouded hidden fees. This is because both types of naives are identical when firms shroud hidden fees. But incentives to unshroud hidden fees change and the existence of shrouding equilibria becomes an issue. When all firms shroud hidden fees, non-avoiding naive consumers pay a total prices above marginal cost. Now when a firm unshrouds hidden fees, non-avoiding naives still pay the hidden fee and can therefore be profitably attracted. This renders shrouding conditions more restrictive. In particular, unshrouding and marginally undercutting the smallest total price of shrouding competitors for their naive customers attracts all non-avoiding naives, and makes them pay $\min\{c + (1 - \alpha)\bar{a}, v\}$. Overall, this gives $\alpha\eta \min\{(1 - \alpha)\bar{a}, v - c\}$ as profits of deviating from a shrouding equilibrium by educating customers in period 2. In the following Proposition, I summarize the general existence conditions of shrouding equilibria for the results in Sections 4, 5, and 6.

Proposition 6. [Shrouding Conditions with Non-Avoiding Naives]

Assume the share $\eta \in [0, 1)$ of naive consumers cannot avoid unshrouded hidden fees while the others can avoid them costlessly.
1. When firms do not learn their customers’ types, the shrouding equilibrium as in Proposition 2 exists if and only if

\[ 0 \geq \alpha \eta \min\{(1 - \alpha)\bar{a}, v - c\}. \]  

(4)

When this condition is violated, unshrouding occurs with probability one.

2. When firms learn their customers’ types, shrouding continuation equilibria as in Proposition 3 exist if and only if

\[ s_n \alpha (1 - \alpha)\bar{a} \geq \alpha \eta \min\{(1 - \alpha)\bar{a}, v - c\}, \forall n. \]  

(5)

When this condition is violated for at least one firm, unshrouding occurs with probability one in the first period. If (5) holds, shrouding equilibria with shrouding in both periods exist.

If total prices are below \( v \), i.e. \( f_1 + \bar{a} \leq v \), in each pure-strategy equilibrium, all firms charge \( f_1 \in c - \alpha \bar{a} + \left[ -\alpha (1 - \alpha)\bar{a}, \frac{s_n}{1 - s_{\min}} - \alpha (1 - \alpha)\bar{a}\right] \), giving rise to total profits \( \Pi_n = s_n (f_1 + \alpha \bar{a} - c) + s_n \alpha (1 - \alpha)\bar{a} \in [0, s_n \frac{s_{\min}}{1 - s_{\min}} - \alpha (1 - \alpha)\bar{a} + s_n \alpha (1 - \alpha)\bar{a}] \). Shrouding occurs with probability one.

If total prices are above \( v \), i.e. \( f_1 + \bar{a} > v \), in each pure-strategy equilibrium, all firms charge \( f_1 \in c - \alpha \bar{a} + \left[ \eta \alpha (v - c), \frac{s_{\min}}{1 - s_{\min}} - \alpha (1 - \alpha)\bar{a}\right] \), giving rise to total profits \( \Pi_n = s_n (f_1 + \alpha \bar{a} - c) + s_n \alpha (1 - \alpha)\bar{a} \in [\eta \alpha (v - c), s_n \frac{s_{\min}}{1 - s_{\min}} - \alpha (1 - \alpha)\bar{a} + s_n \alpha (1 - \alpha)\bar{a}] \). Shrouding occurs with probability one.

3. The results of Proposition 5 remain unchanged.

Note that for \( \eta = 0 \), the deviation profits of unshrouding become zero and the special case depicted in Section 5 applies. Again, there are mixed-strategy equilibria for first-period prices which are discussed in the appendix.

The most important result in this section is that with non-avoiding naives, firms with an empty customer base strictly prefer to unshroud hidden prices. This motivates the equilibrium selection discussed before Proposition 4 and implies that results are robust to positive unshrouding costs.

Another interesting result of Proposition 6 is that in case 2, for \( \eta > 0 \), total profits are strictly positive in each shrouding equilibrium when \( f_1 + \bar{a} > v \).\footnote{This condition is particularly interesting as it always holds at the margin in a richer setup in which consumers}

30
can be profitably attracted, unshrouding is now a deviation strategy that leads to strictly positive profits. In these deviations, firms undercut total prices to attract non-avoiding naives, and set transparent prices at marginal cost to at least break even with sophisticated and educated avoiding naive customers. Now distinguish two cases.

First, if \( f_1 + \bar{a} > v \), unshrouding firms can maximally earn the total valuation of non-avoiding naive consumers. Maximal profits from unshrouding are therefore \( \eta \alpha(v - c) \) which is independent of \( f_1 \). But when shrouding occurs transparent prices must be large enough to earn at least \( \eta \alpha(v - c) \). If firms do not earn at least these constant unshrouding profits, they unshroud with probability one. As a result, total profits must be strictly positive.

Second, if \( f_1 + \bar{a} \leq v \), unshrouding firms can maximally earn \( \eta \alpha(f_1 + \bar{a} - c) \) when all competitors shroud. This is not a constant but depends itself on \( f_1 \). Thus, the incentive to unshroud hidden fees decreases in \( f_1 \) as well such that shrouding equilibria with lower prices can be maintained.

The profitability of unshrouding in period 2 also renders results robust to firms undercutting in period 1 and not serving the entire demand. Since unshrouding is profitable, firms do not need to have an empty customer base in period 2 to unshroud, but a sufficiently small one suffices. Thus, firms have stronger incentives to unshroud in period 2 when they loose customers in period 1.

When customer information of firms on naiveté is symmetric—i.e. the cases of Proposition 2 and 5 when firms cannot distinguish their customers sophistication or naiveté is fully disclosed—Proposition 6 shows that shrouding equilibria either earn zero profits or do not exist. In particular for \( \eta > 0 \), they do not exist for socially beneficial products \((v \geq c)\). When customer-base information is private, however, positive profits induce the existence of shrouding equilibria. Thus, the results on stability of shrouding equilibria become even sharper in the presence of non-avoiding naives: firms have an incentive to shroud hidden fees only when firms can distinguish customers based on their sophistication.

Remark: If I allow firms to offer multiple contracts to customers, there would be additional equilibria. In particular, firms could make unshrouding unprofitable in period 2 to competitors for any value of \( \eta \) by offering an additional product to their existing naive customers with \((\hat{f}_{n2}, \hat{a}_{n2}) = (c, 0)\). If shrouding occurs, no consumer prefers this product to the one she gets in the equilibrium discussed in Proposition 3. But if unshrouding occurs in period 2, non-avoiding naive consumers are better off by choosing \((\hat{f}_{n2}, \hat{a}_{n2})\) instead of switching to a competitor. Thus, in the second period and for any \( \eta > 0 \) firms with zero market shares are indifferent between unshrouding or valuations are continuously distributed.
not. In addition to the shrouding equilibria in Propositions 3 and 4, this would induce equilibria in which firms always shroud in the second period, i.e. also when they have an empty customer base, and total profits are zero. This reasoning, however, relies on the fact that \((\hat{f}_{n2}, \hat{a}_{n2}) = (c, 0)\) is never chosen on the equilibrium path. It is, thus, not robust to consumers wrongly choosing this contract. To illustrate this, suppose naive consumers of firm \(n\) wrongly choose this contract with probability \(\epsilon > 0\). Since these naives would also pay a hidden fee, firm \(n\) is strictly better off by increasing \(\hat{a}_{n2}\) to \(\bar{a}\). As a consequence, naives of firm \(n\) pay a total price above \(c\) such that competitors of \(n\) can unshroud hidden fees and profitably attract these non-avoiding naive consumers. This argument shows that such offers that render unshrouding unprofitable are not robust to being chosen by mistake.

In the rest of the paper, I return to the case \(\eta = 0\) to simplify the exposition.

### 7.2 Partial Unshrouding

When firms start a transparency campaign or simplify their pricing scheme to make consumers aware of hidden fees, they are unlikely to affect all naive consumers. At the same time, such policies are unlikely to be without any effect at all. Examples for the effectiveness of simple interventions are given by Stango and Zinman (2014) and Alan et al. (2015). In this Section, I show that the main results of Propositions 3 and 4 are robust to partial unshrouding. To this end, I assume that unshrouding turns only the share \(\lambda \in (0, 1]\) of naives into sophisticates while the others remain naive.

**Proposition 7.** [Partial Unshrouding]

- For \(\lambda \in (0, 1]\), shrouding equilibria with shrouding in period 2 exist if and only if hidden prices are shrouded in period 1, and each firm has a non-empty customer base. Prices and profits under shrouding are as in Proposition 3.

- Shrouding equilibria with shrouding in both periods exist. In each equilibrium satisfying the selection criteria of Proposition 4, all firms choose hidden fees \(a_{n1} = \bar{a}\). In equilibria with pure strategies in period 1, each firm sets the same transparent price

\[
f_1 \in \left[ c - \alpha \bar{a} - \alpha(1 - \alpha)\bar{a}, c - \alpha \bar{a} + \frac{s_{\text{min}}}{1 - s_{\text{min}}} \alpha(1 - \alpha)\bar{a} - \frac{1 - \lambda}{1 - s_{\text{min}}} \alpha(1 - \alpha)\bar{a} \right].
\]

Total profits are

\[
\Pi_n \in \left[ 0, s_n \left[ \frac{s_{\text{min}}}{1 - s_{\text{min}}} \alpha(1 - \alpha)\bar{a} - \frac{1 - \lambda}{1 - s_{\text{min}}} \alpha(1 - \alpha)\bar{a} \right] + s_n \alpha(1 - \alpha)\bar{a} \right].
\]

For all these equilibria in which \(\Pi_n > 0\), shrouding occurs with probability one.

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38 These examples are discussed in more detail in Section 3.
The first bullet point states the robustness of second period of shrouding equilibria for the case of partial unshrouding. Naives are unchanged when shrouding occurs, which is why the properties of shrouding equilibria in period 2 remain unaffected. Since some consumers remain naive firms could earn positive profits conditional on unshrouding in period 2, but these profits are strictly smaller than profits conditional on shrouding.

Intuitively, when unshrouding occurs in period 2, some old naives become sophisticated and observing a customer’s naiveté after period 1 is less informative. Firms award transparent fees below marginal cost also to these old naive customers who turned sophisticated and do not pay hidden fees. This renders customer data less profitable after unshrouding in period 2.

The second bullet point states that for each \( \lambda > 0 \), shrouding remains both possible and profitable in period 1. Firms can achieve positive total profits for each \( \lambda > 0 \). At first, this seems to contradict the earlier observation that firms have a preference for a balanced customer base. If there are many naives and unshrouding reaches only some consumers, unshrouding can result in a more balanced customer base and increase continuation profits. But each naive consumer educated about hidden fees in period 1 pays the whole hidden fee—\( \bar{a} \)—less in this period.

Additionally, undercutting competitors in period 1 is more profitable to firms than unshrouding in period 1, mostly because more naives remain in the market that can be exploited in period 1. Undercutting competitors in period 1 still triggers unshrouding in the second period. Since unshrouding is partial, continuation profits decrease but they remain positive. This is why partial unshrouding reduces the firms’ Pareto-dominated equilibrium price and the largest total profits. But nonetheless, they remain positive for all \( \lambda > 0 \), leaving results qualitatively unchanged.\(^{39}\)

Note also that the effects of the disclosure policy are unchanged and all profits are competed to zero, though some consumers will always remain naive.

I discuss further extensions in Appendix A: when new customers arrive in period 2 or when some naive customers learn about hidden fees after period 1, results do not change qualitatively. In both cases, observing naiveté in period 1 remains an informative signal on naiveté in the second period such that firms earn positive expected profits from old naive customers while breaking even on all others. In another extension I look at a model with two firms and T periods. Shrouding equilibria exist where shrouding occurs with probability one in each period. Intuitively, firms benefit from not learning to distinguish their competitors’ customers because this induces firms to

\(^{39}\)Note that firms face the same coordination issue as inherent in Proposition 4 and discussed in Appendix A.4.
compete more aggressively on naive consumers and reduces continuation profits to zero.

8 Conclusion

I investigate the role of customer data in markets in which firms can employ their customers’ usage data to predict the likelihood of customer mistakes. While customer data can also be valuable in rational models, my results suggest that these model severely underestimates the firms’ benefits of private customer data. This paper, therefore, gives a novel explanation for high profits—excluding any fixed cost of operation—in seemingly competitive markets such as the credit-card industry.

Beyond consumption data, any kind of usage data can be informative with big amounts of data. Big-data analysis becomes more and more relevant for firms to better predict their customers’ behavior. In particular when it allows firms to predict their customers’ degree of sophistication, the results of this paper shed new light on the role of big data in competitive markets. When firms manage to get hold of their customers usage data, e.g. via cookies, search histories, or by requiring them to create an online account that facilitates the observation of usage patterns, firms can gather a lot of usage data that can be used to distinguish customers based on their naiveté. This paper therefore offers a new explanation on how big data related to such services or search engines can be profitably used or sold even to firms active in competitive markets. As my benchmarks show, this is not obvious without naive consumers since sophisticated consumers optimally self-select into efficient offers made by competing firms.

In addition, big-data analysis has the potential to introduce a novel form of asymmetric understanding to market settings when consumers are unaware of the informational traces they leave behind. Shiller (2014), for example, finds that the number of websites visited on Tuesdays and Thursdays predict demand for Netflix accounts while surfing on a Wednesday seems to carry little information. He also simulates that Netflix could have raised profits by only 0.8 percent when using price discrimination based on usual demographic characteristics. By using data on browsing behavior, such as website visits on Tuesdays and Thursdays, profits could have been increased by 13 percent. It is hard to imagine that many consumers take the effect on prices into account when browsing the internet.

I show that disclosing consumer data to competitors can be beneficial in breaking the competitive asymmetry created by the interaction of consumer naiveté and private information thereof. Even if this policy does not trigger firms to offer more transparent products, it reduces transparent
prices to naive consumers. When implementing such a policy, however, consumers might be concerned about their data privacy. This suggests a potential conflict between antitrust and privacy concerns when dealing with consumer data. In principal, there could be ways to avoid this conflict. Since consumers with the same characteristics are offered the same (expected) price, firms could be given data about customers in an anonymous way without affecting the results of such a policy. A potential drawback is that in a richer model with a heterogeneous participation decision of consumers, this policy can lead to excessive participation in the market by naive consumers. It might decrease transparent prices to naive customers, and when these prices are below marginal cost, naives might overparticipate in the market.\footnote{Heidhues and Koszegi (2015) analyze this participation distortion in detail.} But at the same time, lower profits reduce incentives of firms for excessive entry, and to inefficiently invest in exploitative innovation.

In many industries with naive consumers, another crucial dimension of heterogeneity consumers is their riskiness. In consumer borrowing consumers usually differ in their likelihood of paying their debt, and in insurance markets consumers have different levels of risk against which they want to be insured. The analysis in this paper can be viewed as conditional on a realization of such a risky dimension. Combining the two heterogeneities is left for future research.

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A More Extensions

A.1 New Customers Arriving in Period 2

Let $\gamma > 0$ be the share of customers that arrive in period 1. They stay for both periods. The share $1 - \gamma$ arrives in period 2. For simplicity, assume that old and new customers are naive with probability $\alpha$, and sophisticated with $1 - \alpha$. The whole analysis can easily be extended to new and old customers following different distributions. To make the extension interesting, assume that a firm $n$ cannot distinguish new period-two customers from old ones that did not buy from $n$ in period 1. The results of this extension are summarized in the following Proposition:

**Proposition 8.** [New Customers in Period 2]

Shrouding equilibria exist. In each shrouding equilibrium that is Pareto dominant for the firms, consumers pay transparent prices $f_{n1} = c - \alpha \bar{a} + \frac{s_{\min}}{1 - s_{\min}} \gamma \alpha (1 - \bar{a})$ in the first period; $f_{n2}^{\text{soph}} \geq c$ and hidden prices are $a_{n1} = a_{n2} = \bar{a}$. If $N \geq 3$, $f_{n2}^{\text{new}}$ and $f_{n2}^{\text{naive}}$ are mixed on $\left[ c - \alpha \bar{a}, c - \frac{(1 - \gamma)}{(1 - \gamma + (1 - s_{\min}) \gamma(1 - \alpha))} \alpha \bar{a} \right]$. If $N = 2$, $n \neq \hat{n}$ choose prices such that $f_{n2}^{\text{new}}$ and $f_{n2}^{\text{naive}}$ are mixed on $\left[ c - \alpha \bar{a}, c - \frac{(1 - \gamma)}{(1 - \gamma + (1 - s_{n}) \gamma(1 - \alpha))} \alpha \bar{a} \right]$. Overall, firms earn profits $\Pi_n = \frac{s_{\min}}{1 - s_{\min}} s_n \gamma \alpha (1 - \bar{a}) + s_n \gamma \alpha (1 - \alpha) \bar{a}$. Shrouding occurs with probability one.

First of all, note that the nature of the support of the mixed prices changes from $N = 2$ to $N > 3$ due to a coordination problem: for $N = 2$, each firm has to make the other firm indifferent with her price choice. For $N > 3$, all $n \neq \hat{n}$ choose new-customer prices to make $\hat{n}$ indifferent in choosing naive-customer prices. This must be true for all $n$ so that firms need to mix on the same supports. This support requires that no firm benefits from attracting the newly-arriving customers in equilibrium.

Comparing Proposition 3 with Proposition 8 shows that new customers in period 2 lower the upper bound of the interval on which new-customer and naive-customer prices are mixed in period 2. Overall, the profitability of old customers is unchanged, but they are fewer due to the normalization of the customer base. Competition for new customers is more fierce due to the newly arriving customers in the second period, driving down the upper bound of the interval on which new-customer prices (and therefore naive-customer prices) are mixed. But this leaves the profitability of old customers unaffected.

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41If firms could make this distinction, markets for old and new costumers would be split. For the old customers, Corollary 1 would apply and the new ones would all get a transparent price of $c - \alpha \bar{a}$ in a shrouding equilibrium while only the naives pay hidden fees.
A.2 Learning about Hidden Fees in Period 2

To study the effects of learning by customers, assume that a share $\sigma$ of naives remain naive while $1 - \sigma$ become sophisticated in period 2.

**Proposition 9.** [Learning about hidden fees in Period 2]

Shrouding equilibria exist. In each shrouding equilibrium that is Pareto dominant for firms, consumers pay transparent prices $f^t_n = c - \alpha \bar{a} + \frac{s_{\min}}{1-s_{\min}} \sigma \alpha (1 - \sigma) \bar{a}$ in the first period. Second period transparent prices are $f^soph^t_n \geq c$ and $f^n_{new}^t$, and $f^n_{naive}^t$ are mixed on $[c - \sigma \alpha \bar{a}, c]$. Hidden prices are $a^n_1 = a^n_2 = \bar{a}$. Total profits are $\Pi_n = \frac{s_{\min}}{1-s_{\min}} s_n \sigma \alpha (1 - \alpha) \bar{a} + s_n \sigma \alpha (1 - \alpha) \bar{a}$. Shrouding occurs with probability one.

Proposition 9 establishes that learning of some naive customers increases the second-period price floor and therefore the level of total prices for naive customers. The share of profitable naive customers decreases and firms, not knowing which first-period naive customer learns or remains naive, pay the naive-customer price to some old naives who are now sophisticated. This reduces the margin on old naives by the factor $\sigma$. Thus, there is cross-subsidization between the old naives. But as long as $\sigma > 0$, customer data are informative and profits remain positive.

Overall, effects on profits are the same as in the case with new customers arriving. Intuitively, customer data give a signal on which customers can be profitably exploited and which ones cannot. From this perspective, learning customers or new customers blur the informativeness of customer data in a similar way.

A.3 T Periods

Take $N = 2$ and denote by $\pi_{nt}$ the profit in a shrouding equilibrium in period $t$ of firm $n$. Similarly, denote by $V_{nt}$ the continuation profit of such a firm when shrouding occurs in all forthcoming periods. Let $\hat{V}_{nt}$ be the continuation profit of the firm with the smallest market share.

**Proposition 10.** [Deceptive Markets with Private Information about Customer Bases, $N = 2$ and $T > 2$ Periods]

A shrouding equilibrium with shrouding in each period exists. In this shrouding equilibrium, $f^n_1 = \min \{ v, c - \alpha \bar{a} + \frac{\delta}{s_{\min}} V_t \}$ and $f^n_{naive} = f^n_{soph} = f^n_{new} = \min \{ v, c - \alpha \bar{a} + \frac{\delta}{s_n} \cdot V_{nt+1} \}$, and $\pi_{nt} = \min \{ s_n (v + \alpha \bar{a} - c), \delta V_{nt+1} \}$ for all $T > t > 1$. Prices and profits in $T$ are the same as in the Proposition 3.\(^{42}\)

\(^{42}\)With last-period profits given, continuation profits and prices can be computed recursively.
Profits remain positive on the shrouding equilibrium path if no firms learns about their competitor’s customers, i.e. if no customer type switches. Thus, firms adjust prices for sophisticates and new-customers in order to prevent switching. These results might be quite stark, but they establish that profitable shrouding equilibria can be robust to models with more than two periods.

A.4 Mixed Strategies in Period 1

I emphasize in Proposition 4 that when hidden fees can be unshrouded, firms have an incentive to coordinate on prices in period 1. Proposition 7 establishes that this coordination incentive extends to the case when unshrouding of hidden fees is recognized by only some naives. For simplicity, I focus on pure-strategy equilibria in both cases, but mixed equilibria exist as well. In these equilibria, firms play the same finite number of first-period transparent prices with positive probability. In the case of Proposition 4, these prices must be within

\[
\left[ c - \alpha \bar{a} - \alpha (1 - \alpha) \bar{a}, c - \alpha \bar{a} + \frac{\bar{s}_{\min}}{1 - \lambda} \alpha (1 - \alpha) \bar{a} \right],
\]

and with partial unshrouding within

\[
\left[ c - \alpha \bar{a} - \alpha (1 - \alpha) \bar{a}, c - \alpha \bar{a} + \frac{\bar{s}_{\min}}{1 - \lambda} \alpha (1 - \alpha) \bar{a} - \frac{1 - \lambda}{1 - \bar{s}_{\min}} \alpha (1 - \alpha) \bar{a} \right].
\]

Intuitively, if a firm would play a price with positive probability that no other firm sets with positive probability, shrouding does not occur when these prices realize and continuation profits are always reduced. By shifting probability mass from this price to another one which is played by all firms with positive probability, the firm can earn larger continuation profits and increase total expected profits. If firms would mix on an interval, coordination on the same price occurs with probability zero and large shrouding-continuation profits do not occur for these prices. But then, standard Bertrand arguments apply and drive prices downwards.

Since in each mixed-strategy equilibrium firms play the same finite number of transparent prices with positive probability, each price is played by all firms with positive probability as well. Thus, the coordination incentive to achieve large future shrouding profits prevails in mixed-strategy equilibria.

B Proofs

B.1 Proof of Proposition 1

Proof. I argue in the text that consumers buy at marginal cost in any pure-strategy equilibrium. The argument extends to mixed strategies here by a standard Bertrand argument as in the proof for Lemma 1, Case (i).

The proofs for Propositions 2, 3, and 4 are done for the more general setup of Proposition 6.
where unshrouding is more attractive. In addition to the basic framework, a share $\eta \in [0,1)$ of the naive consumers cannot avoid unshrouded hidden fees while the others can avoid them costlessly. The case $\eta = 1$ is ruled out to avoid that firms are indifferent between shrouding or not when only considering their own customer base. Thus, after unshrouding of hidden fees, the share $\alpha \eta$ of consumers still pays hidden fees while the share $1 - \alpha \eta$ does not. The special case presented in the text is obtained by setting $\eta = 0$.

### B.2 Proof of Proposition 2

Since neither firm learns about consumers’ types nor consumers about themselves, there is no updating of beliefs from any type; so the equilibrium is a SPNE. The relevant state variables are customer bases, represented by market shares in $t = 1$, and whether shrouding occurred in $t = 1$ or not.

#### Step 1: Period 2:

In the first step, I determine Nash equilibria of all period-2 subgames for all states.

**Lemma 1** (Nash Equilibria in Period 2 Subgames).

1. **After shrouding in period 1, a shrouding equilibrium exists if and only if**

   $$0 \geq \eta \alpha \cdot \min\{(1 - \alpha)\bar{a}, v - c\}.$$  \hspace{1cm} (6)

   *Consumers pay hidden fees of $a_{n2} = \bar{a}$ and transparent prices $f_{n2} = c - \alpha \bar{a}$. Profits are zero. When $\eta \alpha \cdot \min\{(1 - \alpha)\bar{a}, v - c\} > 0$, hidden fees are unshrouded with probability one and consumers pay total prices equal to marginal costs.*

2. **After unshrouding in period 1, all consumer types pay total prices equal to marginal costs, and hidden fees are zero.**

**Proof of Lemma 1.** Case (i): In a first step, I derive the strategies of firms given all firms shroud hidden prices. In a second step, I derive conditions under which firms do not deviate from these strategies by unshrouding.

   *Given all firms shroud, two firms must set $f_{n2} = c - \alpha \bar{a}$ and $a_{n2} = \bar{a}$. Given all firms shroud, all firms with positive market share optimally set $a_{n2} = \bar{a}$ since this does not reduce demand but raises profits. I use a standard Bertrand-type argument to show that $f_{n2} = c - \alpha \bar{a}$ with probability*
one for at least two firms. One cannot have \( f_{n2} \in (c - \alpha \bar{a}, \bar{f}_n) \) with positive probability for all firms for the supremum of transparent prices of firm \( n \) of \( \bar{f}_n > c - \alpha \bar{a} \). Towards a contradiction, assume \( \bar{f}_n > c - \alpha \bar{a} \forall n \). First note that \( \bar{f}_n = \bar{f} \forall n \). Otherwise, a firm setting prices above the lowest supremum, say at \( \bar{f} \), earns zero profits whenever these prices occur but could earn strictly positive profits by moving this probability mass to \( \bar{f} - \epsilon \) for some \( \epsilon > 0 \) since \( \bar{f} > c - \alpha \bar{a} \). Thus, if all firms have a supremum strictly above \( c - \alpha \bar{a} \), they must have the same supremum. If all firms play \( \bar{f}_n \) with positive probability, each firm earns non-negative profit when this occurs. But by taking the probability mass from \( \bar{f} \) to \( \bar{f} - \epsilon \), a firm could win the whole market when all others play \( \bar{f} \) and therefore strictly increase her profit. If at least one firm does not play \( \bar{f} \) with positive probability, all firms that do so earn zero profit with positive probability and could earn strictly positive profits by moving the probability mass somewhere below \( \bar{f} \) instead. Therefore \( f_{n2} < \bar{f} \forall n \) with probability one. But then profits go to zero as \( f_{n2} \) approaches \( \bar{f} \) whereas expected profits are strictly positive by playing \( c - \alpha \bar{a} + \epsilon \), for some \( \epsilon > 0 \), since all others play a larger price with positive probability when \( \bar{f} > c - \alpha \bar{a} \). Thus, firms could do better by shifting probability mass from marginally below \( \bar{f} \) to \( c - \alpha \bar{a} + \epsilon \), for some \( \epsilon > 0 \). This is a contradiction. Hence, we get \( \bar{f}_n = c - \alpha \bar{a} \) for at least two firms, since trivially, it is no equilibrium when only one firm sets \( \bar{f}_n = c - \alpha \bar{a} \). Thus, firms earn zero profit when shrouding occurs.\(^{43}\)

Given firms play a candidate shrouding equilibrium in which two firms set \( \bar{f}_n = c - \alpha \bar{a} \) and \( a_{n2} = \bar{a} \), unshrouding and setting \( f_{n2} = c \) and \( f_{n2} + a_{n2} = \min\{v, c + (1 - \alpha)\bar{a}\} \) attracts all educated naives that cannot avoid hidden fees. Thus, optimal deviation profits by unshrouding are given by \( \alpha \eta \cdot \min\{v - c, (1 - \alpha)\bar{a}\} \). When \( v - c > (1 - \alpha)\bar{a} \) unshrouding is profitable if \( \eta > 0 \) and a shrouding equilibrium does not exist; if \( \eta = 0 \), optimal deviation profits by unshrouding are zero and a shrouding equilibrium exists. When \( v - c < (1 - \alpha)\bar{a} \), shrouding occurs as long as profits in a shrouding equilibrium are larger than profits from unshrouding, that is if \( 0 \geq \alpha \eta(v - c) \). If \( v < c \), optimal deviation profits are negative and a shrouding equilibrium exists. Conversely, a shrouding equilibrium does not exist if \( 0 < \alpha \eta(v - c) \).

Next, I show that hidden fees are unshrouded with probability one when
\[
\eta \alpha \cdot \min\{(1 - \alpha)\bar{a}, v - c\} > 0
\]
in three steps. Towards a contradiction, assume shrouding occurs with positive probability and \( \eta \alpha \cdot \min\{(1 - \alpha)\bar{a}, v - c\} > 0 \).

Step (I): \textit{Firms earn positive profits.} When shrouding occurs, firms could unshroud and earn
\[
\eta \alpha \cdot \min\{(1 - \alpha)\bar{a}, v - c\} > 0,
\]
but since shrouding occurs with positive probability and firms

\(^{43}\)When I say below that a standard Bertrand type argument applies, I refer to this kind of reasoning.
must be indifferent between shrouding and unshrouding, firms must earn positive profits when
shrouding occurs.

Step (II): Firms earn zero profits whenever shrouding. Let \( \hat{t} \) be the supremum of total prices,
including hidden fees when unshrouding, payed by educated naives that cannot avoid hidden fees.
Then by playing \( \hat{t} \), a firm earns positive profits only if it is the only one that unshrouds and \( \hat{t} < f_{n2} + a_{n2} \) with positive probability. Thus, for all total prices above \( \hat{t} \), firms earn positive profits
only when shrouding occurs and they charge the smallest transparent price. But then, a standard
Bertrand-type argument implies that total prices are competed downwards until \( f_{n2} = c - \alpha \bar{a} \) for
all firms that attract customers and \( \hat{t} \leq \min\{(1 - \alpha)\bar{a}, v - c\} \). Thus, firms earn weakly less than
zero profits whenever shrouding.

Step (III): Unshrouding occurs with probability one. Since firms earn zero profits whenever
shrouding, they are strictly better of by unshrouding instead since they can then earn \( \eta \alpha \cdot \min\{(1 - \alpha)\bar{a}, v - c\} > 0 \). Thus firms are better off by unshrouding with probability one, contra-
dicting the assumption that shrouding occurs with positive probability whenever \( \eta \alpha \cdot \min\{(1 - \alpha)\bar{a}, v - c\} > 0 \).

Note that the case depicted in Proposition 3 is for \( \eta = 0 \). Thus, unshrouding hidden prices
can earn a firm maximally zero profits. Therefore, if \( \eta = 0 \) and shrouding occurs with positive
probability, firms must earn zero profits when shrouding. If shrouding occurs with probability
one, the result has been shown above. Suppose shrouding occurs with positive probability less
then one. We know that unshrouding earns firms maximally zero profits. If at least one firm earns
strictly positive profits when shrouding occurs, such a firm must have a supremum of transparent
prices when shrouding of \( \bar{f} > c - \alpha \bar{a} \). But then, a competitor could shift all probability mass from
unshrouding to shrouding and earn strictly positive profits by setting a transparent price \( \bar{f} - \epsilon \)
for some \( \epsilon > 0 \) and hidden fees of \( \bar{a} \). If all firms earn strictly positive profits when shrouding occurs,
shrouding would occur with probability one since unshrouding gives zero profits. But then we are
in the case from the beginning of this proof which contradicts positive profits. Thus, if \( \eta = 0 \) and
shrouding occurs with positive probability, expected profits must be zero.

Case (ii): The market is effectively split: when unshrouding occurred in \( t=1 \), firms compete
in transparent prices for sophisticated consumers and in total prices for non-avoiding naives. By
essentially the same Bertrand argument as above, firms that attract consumers charge \( f_{n2} = c \) and
\( a_{n2} = 0 \) and earn zero profits.

\( \square \)
Step 2: Period 1:

All consumers face the same price-schedule in period 2, irrespective of the firm they purchase from. Thus, consumers maximize their total payoff by maximizing their first-period payoff. Knowing that firms earn no profits in any second-period subgames, firms simply maximize their per-period profit in period 1. Thus, the same Bertrand-type argument as in Case (i) of period 2 applies.

B.3 Proof of Proposition 3

I am looking for a Perfect Bayesian Equilibrium. In the first step, I argue that updating of beliefs only matters for the firms’ customer base after shrouding in period 1. After such histories, firms learn only their own first period customers’ types. Thereafter, I determine conditions for shrouding to occur in equilibrium in period 2 and pin down consumers’ payments and firms’ profits. Those are summarized in Lemma 3.

Step 1: After shrouding occurred in period 1, firms update only about consumers in their customer base.

Assume shrouding occurred in period 1. When consumers are not educated about hidden fees, both consumer types solve the same problem: \( \max_n v - f_{n2}, \text{s.t. } v - f_{n2} \geq 0 \). Hence, both consumer types will always be indifferent between the same set of firms. Therefore the Sorting Assumption implies that the distribution of customers in each customer base is the same as in the population. Hence from observing her own customer base, a firm cannot learn anything about the distribution outside of her own customer base.

Recall that after unshrouding in period 1, all consumers are sophisticated in period 2, and this is known to firms.

Step 2: Period 2

To derive second-period equilibria, I begin by establishing some characteristics of the firms’ second-period pricing distributions.

Lemma 2 (Supports of Transparent Prices in Period 2). In each equilibrium in which prices remain shrouded in period 2 with probability one, \( f_{n2}^{naive} \in [c - \alpha \bar{a}, c] \) with probability one and sophisticates pay a price below \( c \), i.e. \( \min \{ f_{n2}^{soph}, (f_{n2}^{new})_{n \neq n} \} \leq c \forall n \) with probability one. \( F_{n2}^{new}(.) \) and \( F_{n2}^{naive}(.) \) are continuous on \( (c - \alpha \bar{a}, c) \), and on each subinterval on \( (c - \alpha \bar{a}, c) \)
at least one firm plays naive- and one firm plays new-customer prices with positive probability. Additionally, all firms play marginally undercut \(c\) with the new-customer price with positive probability, i.e. for all \(\epsilon > 0\) and for all \(n\), \(\bar{f}_{n^2}^{\text{new}} \in (c - \epsilon, c]\) with positive probability. Furthermore, \(F_n^{\text{new}}(c - \alpha\bar{a}) = F_n^{\text{new}}(c - \alpha\bar{a}) = 0, \forall n\).

**Proof of Lemma 2.**

\(f_{n^2}^{\text{naive}}\) is in \([c - \alpha\bar{a}, c]\) and \(\min\{f_{n^2}^{\text{soph}}, (f_{n^2}^{\text{new}})_{\hat{n} \neq n}\} \leq c \forall n\) with probability one. I have argued in the main body that in each equilibrium in which prices remain shrouded in the second period, \(f_{n^2}^{\text{soph}} \geq c, f_{n^2}^{\text{new}} \geq c - \alpha\bar{a}\) and \(f_{n^2}^{\text{naive}} \geq c - \alpha\bar{a}\). First, I show that in equilibrium no firm \(n\) sets a price \(f_{n^2}^{\text{naive}} > c\) with positive probability. A firm \(n\) can guarantee itself strictly positive expected profits from its naive customers by setting \(c - \alpha\bar{a}\). Thus, it must earn strictly positive expected profits for almost all prices it charges, and any price it charges with positive probability. Let \(\bar{f}_{n^2}^{\text{naive}}\) be the supremum of those prices and suppose \(f_{n^2}^{\text{naive}} > c\) with positive probability. Then, all rivals \(\hat{n} \neq n\) must set prices \(f_{\hat{n}^2}^{\text{new}} \geq \bar{f}_{n^2}^{\text{naive}}\) with positive probability. If all rivals do so, each firm \(\hat{n} \neq n\) can deviate and move probability mass from weakly above \(f_{n^2}^{\text{naive}}\) to \(\bar{f}_{n^2}^{\text{naive}} - \epsilon\), and for sufficiently small \(\epsilon\) increase its profits. We conclude that \(f_{n^2}^{\text{naive}} \leq c \forall n\).

To show that \(\min\{f_{n^2}^{\text{soph}}, (f_{n^2}^{\text{new}})_{\hat{n} \neq n}\} \leq c \forall n\), I first establish that firms earn zero expected profits from new-customers. Towards a contradiction, suppose a firm \(n\) makes positive expected profits from new customers and take its supremum of new-customer prices \(\bar{f}_{n^2}^{\text{new}}\). To be profitable at \(\bar{f}_{n^2}^{\text{new}}\), \(\bar{f}_{n^2}^{\text{new}} > c - \alpha\bar{a}\). In addition, there must be a firm \(\hat{n} \neq n\) such that \(f_{n^2}^{\text{soph}} > \bar{f}_{n^2}^{\text{new}}\) or \(f_{n^2}^{\text{naive}} > \bar{f}_{n^2}^{\text{new}}\) with positive probability. If \(f_{n^2}^{\text{naive}} > \bar{f}_{n^2}^{\text{new}}\) with positive probability, \(\hat{n}\) gets zero profits from naives whenever playing \(f_{n^2}^{\text{naive}} > \bar{f}_{n^2}^{\text{new}}\). By moving this probability mass to \(\bar{f}_{n^2}^{\text{new}} - \epsilon\) for sufficiently small \(\epsilon > 0\) instead, \(\hat{n}\) could make strictly positive profits, a contradiction. The same argument applies if \(f_{n^2}^{\text{soph}} > \bar{f}_{n^2}^{\text{new}}\) with positive probability. Hence, new-customer prices earn zero expected profits in equilibrium. This directly implies that firms earn zero profits on their old sophisticates as well: otherwise, by the same reasoning as above, a firm could move the probability mass of its new-customer prices from above the supremum of sophisticates’ prices of the positive-profit firm to minimally below it, and thereby increase its profits. It follows that \(\min\{f_{n^2}^{\text{soph}}, (f_{n^2}^{\text{new}})_{\hat{n} \neq n}\} \leq c \forall n\) with probability one. If \(f_{n^2}^{\text{soph}} > c\) with positive probability, then at least one firm \(\hat{n} \neq n\) must set \(f_{n^2}^{\text{new}} \leq c\) with probability one, since otherwise a competitor of \(n\) would get strictly positive expected profits from new-customer prices. Similarly, if all \((f_{n^2}^{\text{new}})_{\hat{n} \neq n} > c\) with positive probability, then \(f_{n^2}^{\text{soph}} \leq c\) with probability one for \(\hat{n}\) not to earn strictly positive profits with new-customer prices. Hence, \(\min\{f_{n^2}^{\text{soph}}, (f_{n^2}^{\text{new}})_{\hat{n} \neq n}\} \leq c\) for all \(n\),
and we established that the support of $f_{n2}^{\text{naive}}$ is $[c - \alpha \bar{a}, c]$.

On each subinterval on $(c - \alpha \bar{a}, c)$, at least one firm plays naive, and at least one firm plays new-customer prices with positive probability. All firms play new-customer prices arbitrarily close to $c$ with positive probability. I prove the claim in three steps: first, I establish that in any arbitrarily small interval $(c - \epsilon, c)$ at least two firms play naive- and all firms play new-customer prices with positive probability. Second, I show the same for any arbitrarily small interval $[c - \alpha \bar{a}, c - \alpha \bar{a} + \epsilon)$ for at least two firms’ naive- and two firms’ new-customer prices. Third, I prove that on each interval in-between these prices occur with positive probability.

Step (i): First, I show that for all $n$ and any $\epsilon > 0$, $f_{n2}^{\text{new}} \in (c - \epsilon, c]$ with positive probability. Suppose otherwise, i.e. for at least one firm there exists an $\epsilon > 0$ such that $f_{n2}^{\text{new}} \in (c - \epsilon, c]$ with probability zero. Of all of these firms, select a firm $n$ that has the smallest supremum $\hat{f}_{n2}^{\text{new}}$. If there are many such firms select one that sets the supremum with probability less than one. Since $\hat{f}_{n2}^{\text{new}} < c$, at least one firm $\hat{n} \neq n$ must set $f_{n2}^{\text{naive}} > \hat{f}_{n2}^{\text{new}}$ with positive probability for $n$ to break even. But then, $\hat{n}$ makes zero profit for all $f_{n2}^{\text{naive}} > \hat{f}_{n2}^{\text{new}}$ with probability one, a contradiction. Thus, for any $\epsilon > 0$ all firms set $f_{n2}^{\text{new}} \in (c - \epsilon, c]$ with positive probability. It follows that for every $\epsilon > 0$ and every $n$, some $\hat{n} \neq n$ sets $f_{n2}^{\text{naive}} \in (c - \epsilon, c]$ with positive probability: otherwise, firms could not break even when setting $f_{n2}^{\text{new}} \in (c - \epsilon, c]$ with positive probability. Since this holds for every $n$ and $\epsilon > 0$, at least two firms set naive-customer prices in any interval $(c - \epsilon, c]$. Thus, for all prices in $(c - \alpha \bar{a}, c)$, every firm sets larger new-customer with positive probability, and at least two firms set larger naive-customer prices with positive probability.

Step (ii): First I show that for every $\epsilon > 0$, at least two firms set $f_{n2}^{\text{naive}} \in [c - \alpha \bar{a}, c - \alpha \bar{a} + \epsilon)$ with positive probability. Suppose otherwise and take a firm $n$ and her competitors $\hat{n} \neq n$. Assume towards a contradiction that there exists an $\epsilon > 0$ such that for all $\hat{n}$, $f_{n2}^{\text{naive}} \in [c - \alpha \bar{a}, c - \alpha \bar{a} + \epsilon)$ with probability zero. Then the infimum of the naive-customer prices of $n$’s competitors $\underline{f}$ satisfies $\underline{f} > c - \alpha \bar{a}$. For naive-customer prices above this infimum to be profitable, all new-customer prices must be larger with positive probability. But then firm $n$ can earn strictly positive profits from new-customers by choosing $f_{n2}^{\text{new}} \in (c - \alpha \bar{a}, \underline{f})$ with probability one. But this contradicts the finding that firms earn zero expected profits from new-customers. Since this is true for all $n$, I conclude that for every $\epsilon > 0$, at least two firms set $f_{n2}^{\text{naive}} \in [c - \alpha \bar{a}, c - \alpha \bar{a} + \epsilon)$ with positive probability. To show that the same is true for new-customer prices, suppose towards a contradiction that there exists an $\epsilon > 0$ such that a firm $n$ plays $f_{n2}^{\text{naive}} \in [c - \alpha \bar{a}, c - \alpha \bar{a} + \epsilon)$ with positive probability but all $\hat{n} \neq n$ play greater new-customer prices with probability one. But then, $n$ could move its
probability mass from below $c - \alpha \bar{a} + \epsilon$ onto this point to strictly increase profits. Thus, we get a contradiction if for any $\epsilon > 0$, less then two firms play $f_{n2}^{new} \in (c - \alpha \bar{a}, c - \alpha \bar{a} + \epsilon)$ with positive probability.

Step (iii): On each subinterval on $(c - \alpha \bar{a}, c)$, at least one firm sets naive- and at least one other firm sets new-customer prices with positive probability. Suppose the opposite for some interval $(\tilde{r}, \tilde{s})$. Then there are three cases: either no naive- and new-customer price on $(\tilde{r}, \tilde{s})$ occurs with positive probability, or only naive-customer prices, or only new-customer prices. Take the largest interval containing $(\tilde{r}, \tilde{s})$, in which either no firm sets new- or no firms sets naive-customer prices with positive probability, and denote it by $(r, s)$; i.e., some new- or naive-customer prices are played with positive probability arbitrarily close below $r$ and arbitrarily close above $s$. Note that due to step (ii), we know that $r > c - \alpha \bar{a}$.

In the first case, no naive- or new-customer price occurs on $(r, s)$ with probability. But by construction, some naive- or new-customer price occurs on $(r - \epsilon, r]$ with positive probability. Note that there can be no mass point on $r$. If more than one firm had a mass-point on $r$, they could strictly increase profits by shifting probability mass from this mass point to slightly below it. If one firm had a mass point on $r$, it could shift this mass point upwards into $(r, s)$ and increase margins without affecting expected market shares since $(r, s)$ is empty. But when there is no mass point on $r$, then for some $\epsilon > 0$ small enough, a firm playing prices in $(r - \epsilon, r]$ with positive probability is strictly better off by shifting this probability mass to slightly below $s$, a contradiction.

Now consider the second case. Towards contradiction, assume only naive-customer prices are set on $(r, s)$ with positive probability. But by shifting probability mass of naive-customer price from within $(r, s)$ to $s$, firms can discretely increase margins on naives while leaving the probability to gain these margins unaffected, a contradiction.

Third, assume towards a contradiction that only new-customer prices are played on $(r, s)$ with positive probability. If only one firm plays new-prices on $(r, s)$ with positive probability, this firm could strictly increase its profits by moving this probability mass to slightly below $s$, a contradiction. Now suppose at least two firms play new-customer prices on $(r, s)$ with positive probability. Take a firm $n$ playing price $f \in (r, s)$ and $f' \in (r, s)$ with positive probability where $f \neq f'$. Recall that both prices are the smallest new-customer price with positive probability due to Step (i), and earn zero expected margins in this case, as shown in the beginning of this proof. Since no naive-customer prices occurs with positive probability on $(r, s)$, both prices induce exactly the same probability of attracting naives when being the smallest new-customer price. But since
one of these prices is strictly larger, they cannot both have zero expected margins when being the smallest new-customer price, a contradiction.

The CDFs are continuous in the interior of the support, i.e. $F_n^{\text{new}}$ and $F_n^{\text{naive}}$ have no mass point on $(c - \alpha \bar{a}, c)$, $\forall n$. Take $F_n^{\text{new}}$ and suppose otherwise. Pick the lowest mass-point of all firms. Say $n$ has this mass point at $f$. We know from above that larger naive-customer prices occur with positive probability, so that prices at this mass point are payed with positive probability. Then there exists some $\epsilon > 0$ such that no rival $\hat{n} \neq n$ charges a price $f_{n2}^{\text{naive}}$ in $[f, f + \epsilon)$. For otherwise, a firm $\hat{n}$ that sets $f_{n2}^{\text{naive}} \in [f, f + \epsilon)$ could charge $f - \epsilon$ instead; as $\epsilon \to 0$, the price difference goes to zero but $\hat{n}$ wins with higher probability. But when no rival charges a naive-customer price in $[f, f + \epsilon)$ and only $n$ sets a mass-point of new-customer prices at $f$, then $n$ can increase profits by moving the mass point upwards, a contradiction. Alternatively, another firm but $n$ has a mass point on new-customer prices at $f$ as well. Recall that profits from new-customers are zero in expectation. Thus, by shifting the mass point upwards, $n$ looses more often, gaining zero profits in this case; but due to Step (i), $n$ still has the lowest new-customer prices with positive probability and therefore earns a strictly positive margin when attracting customers, a contradiction. This shows that $F_n^{\text{new}}$ has no mass point on $(c - \alpha \bar{a}, c)$. A similar argument applies to $F_n^{\text{naive}}$: to see why, suppose otherwise that $F_n^{\text{naive}}$ has a mass point on $(c - \alpha \bar{a}, c)$. Pick again the lowest mass point of all firms. Say firm $n$ has this mass point at $f$. By the same argument as above, there exists some $\epsilon > 0$ such that no rival $\hat{n} \neq n$ sets a price $f_{n2}^{\text{new}} \in [f, f + \epsilon)$ with positive probability. And since $n$ only competes with these new-customer prices for its naive customers, $n$ can strictly improve profits by shifting the mass point upwards, a contradiction.

One has $F_n^{\text{new}}(c - \alpha \bar{a}) = F_n^{\text{new}}(c - \alpha \bar{a}) = 0$, $\forall n$. Suppose otherwise, i.e. $F_n^{\text{new}}(c - \alpha \bar{a}) = p > 0$ for some $n$. Then no rival $\hat{n} \neq n$ charges $f_{n2}^{\text{naive}} \in (c - \alpha \bar{a}, c - \alpha \bar{a} + \epsilon)$ for some $\epsilon > 0$, or otherwise $\hat{n}$ could strictly increase profits by moving this probability-mass on $c - \alpha \bar{a}$ instead. But then, by the same argument as in the last paragraph, $n$ can earn strictly positive profits by shifting the mass-point upwards, a contradiction.

Now suppose $F_n^{\text{naive}}(c - \alpha \bar{a}) = p > 0$ for some $n$ and take firms $\hat{n} \neq n$ that play new-customer prices on $(c - \alpha \bar{a}, c - \alpha \bar{a} + \epsilon)$ with positive probability. We already know that such firms exist. Then $\hat{n}$’s profits from $f_{n2}^{\text{new}} = c - \alpha \bar{a} + \epsilon$ converge to some profit-level below $p[s_n(1 - \alpha)(c - \alpha \bar{a} - c) + (1 - s_n)0] + (1 - p)0 = -ps_n a \bar{a} < 0$. This is a contradiction since firms can guarantee themselves at least zero profits from new-customer prices.
In the next Lemma, I summarize the properties in each shrouding equilibrium in period 2 for each state.

**Lemma 3** (Second Period Continuation Equilibria). There always exists the standard Bertrand equilibrium in which at least two firms unshroud and each consumer pays marginal costs. In addition to this equilibrium, there exist second-period continuation equilibria in which shrouding occurs with positive probability under the following conditions:

(i) If shrouding occurred in \( t=1 \) and all firms have positive customer bases, shrouding occurs with positive probability if and only if

\[
s_n \alpha (1 - \alpha) \bar{a} \geq \alpha \eta \min \{(1 - \alpha) \bar{a}, v - c\}, \quad \forall n.
\]

In such a shrouding equilibrium, profits are \( s_n \alpha (1 - \alpha) \bar{a} \) and shrouding occurs with probability one. \( f_{naive} \) is mixed as in (2). Switching naive-customers of firm \( n \)'s customer base pay the smallest new-customer prices of \( n \)'s competitors based on (1). Sophisticated customers in the customer base of firm \( n \) pay a price equal to the smallest new-customer price of \( n \)'s competitors based on (1). When the above shrouding condition is violated, unshrouding occurs with probability one and all consumers pay a price of \( c \).

(ii) If shrouding occurred in \( t=1 \) and some firm has an empty customer base, consumers are educated about hidden fees with probability one if and only if the good is socially desirable and \( \eta > 0 \). In this case, prices equal marginal costs and firms make zero profits. If the product is socially wasteful, prices are as in (i), but firms without customer base make zero profits. If \( \eta = 0 \), firms without customer base are indifferent between shrouding or unshrouding.

**Proof of Lemma 3.**

(i) First, I derive shrouding conditions and pin down the level of equilibrium profits in a shrouding equilibrium in which firms have a positive customer base. Then, I construct the mixed equilibrium strategies for period 2 in the shrouding equilibrium based on (1) and (2).

If \( s_n \alpha (1 - \alpha) \bar{a} \geq \eta \alpha \min \{(1 - \alpha) \bar{a}, v - c\} \forall n \), in all equilibria in which shrouding occurs with positive probability it occurs with probability one. If \( s_n \alpha (1 - \alpha) \bar{a} < \eta \alpha \min \{(1 - \alpha) \bar{a}, v - c\} \) for some \( n \), shrouding occurs with probability zero. If shrouding occurs with probability one, firms earn expected profits of \( s_n \alpha (1 - \alpha) \bar{a} \) from naives and zero from sophisticates and new customers.
Suppose that shrouding occurs with positive probability. I show that this implies Step (I) - (III) below. Using these facts Step (IV) proves the above.

**Step (I): Firms earn positive profits.** When shrouding occurs, firms can get positive profits of at least $s_n\alpha(1 - \alpha)\bar{a}$ by setting $f_{n2}^{\text{soph}} = f_{n2}^{\text{new}} = c$ and $f_{n2}^{\text{naive}} = c - \alpha\bar{a}$. I have established in the text that when shrouding occurs, new-customer prices below $c - \alpha\bar{a}$ are never played as they lead to strictly negative profits for at least one firm. When unshrouding, the share of consumers paying a hidden fee reduces to $\eta\alpha$ and this threshold shifts upwards to $c - \eta\alpha\bar{a}$. Thus, firms can indeed be sure to profitably keep its naive customers when shrouding occurs by setting the above prices. Since shrouding occurs with positive probability, firms make positive expected profits.

**Step (II): New-customer prices earn zero expected margins in equilibrium conditional on both shrouding or unshrouding occurring.** Sophisticated consumers never pay positive margins in equilibrium. Towards a contradiction, suppose a firm $n$ profitably attracts customers with her new-customer price in expectation. Then firm $n$ must earn positive expected margins with each new-customer price that is played with positive probability. Take the supremum of these prices $\bar{f}_{n2}^{\text{new}}$. Then prices that minimally undercut $\bar{f}_{n2}^{\text{new}}$, i.e. prices on $(\bar{f}_{n2}^{\text{new}} - \epsilon, \bar{f}_{n2}^{\text{new}}]$ for some sufficiently small epsilon > 0, profitably attract either sophisticates or naives from another firm, say $\hat{n} \neq n$. We therefore have to distinguish these two cases.

Suppose $n$ profitably attracts sophisticates conditional on shrouding in any interval of new-customer prices that marginally undercut $\bar{f}_{n2}^{\text{new}}$. Then $f_{n2}^{\text{soph}} \geq \bar{f}_{n2}^{\text{new}}$ with positive probability. Note that the inequality must be strict for some $f_{n2}^{\text{soph}}$ when $n$ sets $\bar{f}_{n2}^{\text{new}}$ with positive probability. Then $\hat{n}$ earns zero profits from sophisticates with probability one whenever $f_{n2}^{\text{soph}} \geq \bar{f}_{n2}^{\text{new}}$, though $\hat{n}$ could earn strictly positive profits from sophisticates when shifting this probability mass to $\bar{f}_{n2}^{\text{new}} - \epsilon$ for some small enough $\epsilon > 0$, a contradiction. The exact same argument applies conditional on unshrouding occurring.

Now suppose $n$ profitably attracts naives in any interval of new-customer prices arbitrarily close below $\bar{f}_{n2}^{\text{new}}$. They are profitable when shrouding occurs or when unshrouding occurs so that I have to distinguish these two cases. If they are profitably attracted under shrouding, we must have $f_{n2}^{\text{naive}} \geq \bar{f}_{n2}^{\text{new}}$ with positive probability. Note that the inequality must be strict for some $f_{n2}^{\text{naive}}$ when $\bar{f}_{n2}^{\text{new}}$ occurs with positive probability. Then $\hat{n}$ earns zero profits when shrouding occurs on prices $f_{n2}^{\text{naive}} \geq \bar{f}_{n2}^{\text{new}}$ that occur with positive probability. W.l.o.g. let $\bar{f}_{n2}^{\text{new}}$ be among the largest such suprema. If this was not the case, then another firm would have a larger supremum that earns zero profits for prices that marginally undercut it. But then this firm could do strictly better.
by shifting this probability mass to \( \tilde{f}^\text{new}_{n2} \). Thus \( \tilde{f}^\text{new}_{n2} \) can be taken among the largest suprema w.l.o.g.. But then moving probability mass from \([\tilde{f}^\text{new}_{n2}, f^\text{new}_{n2} + \epsilon]\) to \( f^\text{new}_{n2} - \epsilon \) increases \( \hat{n} \)'s profits discretely when shrouding occurs and reduces them by maximally \( 2\epsilon \) when unshrouding occurs. This is profitable for some small enough \( \epsilon > 0 \), a contradiction. If \( n \) profitably attracts naives when unshrouding occurs, the same argument can be applied to total prices, i.e. by taking \( f^\text{naive}_{n2} = f^\text{naive}_{n2} + a_{n2} \) and \( t^\text{new}_{n2} = f^\text{new}_{n2} + a_{n2} \) with \( \tilde{f}^\text{new}_{n2} \) as the supremum to total new-customer prices of firm \( n \).

I conclude that if shrouding occurs with positive probability, new-customer prices earn zero expected profits conditional on shrouding or unshrouding. To show that sophisticated consumers never pay a price \( f^\text{soph}_{n2} > c \), suppose otherwise. Since I have established that sophisticates never pay a new-customer price \( f^\text{new}_{n2} > c \), they must pay the positive margin to their old firm, i.e. with \( f^\text{new}_{n2} > c \). But then, a competitor can earn strictly positive profits with new-customer prices by offering \( f^\text{new}_{n2} = f^\text{soph}_{n2} - \epsilon \) for some \( \epsilon > 0 \) small enough, a contradiction.

Step (III): The profits of firms that shroud are weakly smaller than \( s_n\alpha(1 - \alpha)\bar{a} \) \( \forall n \) and zero when unshrouding occurs. To show that firms’ profits are weakly smaller than \( s_n\alpha(1 - \alpha)\bar{a} \) when shrouding occurs, suppose otherwise, i.e. there exists a firm \( n \) that earns strictly larger profits when shrouding occurs. Step (II) has established that firms earn zero profits from new- and sophisticated customers, therefore positive profits have to be earned from naive customers from a firm’s customer base. Let \( \tilde{f}^\text{naive}_{n2} \) be the supremum of \( n \)'s naive-customer prices that are payed with positive probability. Then all \( \tilde{n} \neq n \) must set \( f^\text{new}_{n2} \geq \tilde{f}^\text{naive}_{n2} \) with positive probability. I.e. for all \( \epsilon > 0 \), some \( \tilde{n} \neq n \) sets \( f^\text{new}_{n2} \in [\tilde{f}^\text{naive}_{n2}, \tilde{f}^\text{naive}_{n2} + \epsilon] \) with positive probability. But by moving probability mass from this interval to \( \tilde{f}^\text{naive}_{n2} - \epsilon \), \( \tilde{n} \) can make strictly positive profits: if some other firm than \( \tilde{n} \) sets a smaller new-customer price, \( \tilde{n} \) earns zero profits from new customers. But since all \( \tilde{n} \neq n \), \( \tilde{n} \neq \tilde{n} \) set \( f^\text{new}_{n2} \geq \tilde{f}^\text{naive}_{n2} \) with positive probability, \( f^\text{new}_{n2} = \tilde{f}^\text{naive}_{n2} - \epsilon \) is the smallest new customer price with positive probability. In this case, \( \tilde{n} \) earns profits strictly above \( s_n\alpha(1 - \alpha)\bar{a} \) in expectation from \( n \)'s naives and looses weakly below \( s_n\alpha(1 - \alpha)\bar{a} \) from \( n \)'s sophisticates. Note that we know from Step (I) that \( f^\text{new}_{n2} \geq c - \alpha\bar{a} \) and therefore \( \tilde{f}^\text{naive}_{n2} \geq c - \alpha\bar{a} \) for all \( \tilde{n} \), which is why losses from attracting sophisticates from firm \( n \) are weakly below \( s_n\alpha(1 - \alpha)\bar{a} \). From all other sophisticates that \( \tilde{n} \) attracts with this price, it looses maximally \( 2\epsilon \).

Thus, for some \( \epsilon > 0 \) small enough, \( \tilde{n} \) can discretely increase profits by shifting some probability mass from \( f^\text{new}_{n2} \in [\tilde{f}^\text{naive}_{n2}, \tilde{f}^\text{naive}_{n2} + \epsilon] \) to \( \tilde{f}^\text{naive}_{n2} - \epsilon \), a contradiction.

To show that shrouding firms earn zero profits conditional on unshrouding, suppose otherwise.
for at least one firm, say $n$. Step (II) implies that these profits must be earned from naive customers of firm $n$’s customer base. Thus, $n$ must keep some non-avoiding naives at a positive total prices $f_{n2}^{\text{naive}} + a_{n2} > c$. But then, a competitor $\hat{n} \neq n$ can earn strictly positive profits from new-customer prices conditional by unshrouding and setting $f_{\hat{n}2}^{\text{new}} + a_{\hat{n}2} = c + \epsilon$ for some sufficiently small $\epsilon > 0$, which contradicts Step (II). Thus, shrouding firms earn zero profits conditional on unshrouding. Since firms’ profits are weakly below $s_n \alpha (1 - \alpha) \bar{a}$ when shrouding but by Step (I) they can guarantee themselves these profits when shrouding occurs, we know that firms must earn profits of $s_n \alpha (1 - \alpha) \bar{a}$ in expectation when shrouding occurs.

Step (IV): If $s_n \alpha (1 - \alpha) \bar{a} \geq \eta \alpha \min\{(1 - \alpha) \bar{a}, v - c\} \forall n$, in all equilibria in which shrouding occurs with positive probability, it occurs with probability one. If $s_n \alpha (1 - \alpha) \bar{a} < \eta \alpha \min\{(1 - \alpha) \bar{a}, v - c\}$ for at least one $n$, shrouding occurs with probability zero. Steps (I)-(III) establish that expected profits from new customers are zero, whether shrouding or unshrouding occurs, and whenever shrouding, firms’ expected profits are $s_n \alpha (1 - \alpha) \bar{a}$ when shrouding occurs and zero when unshrouding occurs. Thus, in any candidate equilibrium in which shrouding occurs with positive probability, it occurs with probability one. Consequently, when $s_n \alpha (1 - \alpha) \bar{a} \geq \eta \alpha \min\{(1 - \alpha) \bar{a}, v - c\} \forall n$, no firm has an incentive to unshroud with probability one and set a total price of $\min\{c + (1 - \alpha) \bar{a}, v\}$. But when this condition is violated for at least one firm, this firm has a strict incentive to unshroud with probability one and set the above total price.

Now that I established that in any second-period continuation equilibrium where shrouding occurs, it occurs with probability one, I can use the properties on new- and naive-customer distributions derived in Lemma 2 and the profit levels pinned down above to construct equilibrium price-distributions.

Mixed strategies for new-customer prices. Recall that firms do not compete for their own old customers with the new-customer price. When a firm $n$ sets her naive-customer price lower than all her competitors’ new-customer prices, it keeps her naive customers. Otherwise, it looses them. Thus, expected profits are

$$(1 - \prod_{j \neq n} (1 - F_j^{\text{new}}(f_{n2}^{\text{naive}}))) \cdot 0 + \prod_{j \neq n} (1 - F_j^{\text{new}}(f_{n2}^{\text{naive}})) \cdot s_n \alpha (f_{n2}^{\text{naive}} + \bar{a} - c) = \text{const.}, \forall n.$$

We know from Lemma 2 that all new- and naive-customer prices on $(c - \alpha \bar{a}, c)$ occur with positive
probability and that $F_{j}^{\text{new}}(c - \bar{\alpha}a) = 0$ for all $j$. We also know that expected profits from naive-customer prices must be equal to $\text{const.} = s_n \alpha (1 - \alpha) \bar{a}$ for all prices on the interval. Thus, I can rewrite the above to get

$$\prod_{j \neq n}(1 - F_{j}^{\text{new}}(f_{n}^{\text{naive}})) = \frac{(1 - \alpha) \bar{a}}{f_{n}^{\text{naive}} + \bar{a} - c}, \forall n$$

(9)

In particular, for each $\hat{n} \neq n$ and $f_{\hat{n}}^{\text{naive}}$ this requires $\prod_{j \neq \hat{n}}(1 - F_{j}^{\text{new}}(f_{\hat{n}}^{\text{naive}})) = \prod_{j \neq n}(1 - F_{j}^{\text{new}}(f_{n}^{\text{naive}}))$, which implies $F_{n}^{\text{new}}(f_{\text{naive}}) = F_{\hat{n}}^{\text{new}}(f_{\text{naive}}) = F_{\text{new}}(f_{\text{naive}})$. Using this symmetry in the above equation leads to the expression of (1) on $(c - \alpha \bar{a}, c)$.

Note that the probability mass below $c$ is not equal to one. In fact, we only know from Lemma 2 that $\min\{f_{\text{soph}}^{\text{soph}}, (f_{\hat{n}}^{\text{new}}, \hat{n} \neq n)\} \leq c \forall n$ with probability one. New-customer prices can be strictly larger than $c$ with positive probability, but these prices are never payed by customers and are therefore inconsequential for consumer welfare and firms’ profits. Thus, either new-customer prices have a mass point at $c$ and sophisticated customer prices can be strictly larger than $c$ or the other way around. I report the strategy with the mass point on $c$ to ease the exposition of results. This leads to the distribution as in (1).

**Mixed strategies for naive-customer prices.** Take a firm $n$ that sets $f_{n}^{\text{new}}$ to all consumers that are not in $n$’s customer base. In order to win firm $j$’s customers and break even, it has to offer a new-customer price $f_{n}^{\text{new}}$ such that (i) $f_{n}^{\text{new}} < f_{\hat{n}}^{\text{new}} \forall \hat{n} \neq j$ and (ii) $f_{n}^{\text{new}} < f_{j}^{\text{naive}}$. If $f_{n}^{\text{new}}$ is such that (i) is satisfied, but $j$’s naive-customer price is still smaller, than $n$ attracts only the sophisticated consumers of $j$, since $f_{j}^{\text{soph}} \geq c$. Hence, the expected profit of attracting $j$’s customers is

$$((1 - F_{j}^{\text{new}}(f_{n}^{\text{new}})))^{N-2}[(1 - F_{j}^{\text{naive}}(f_{n}^{\text{new}}))s_{j}(f_{n}^{\text{new}} + \alpha \bar{a} - c) + F_{j}^{\text{naive}}(f_{n}^{\text{new}})s_{j}(1 - \alpha)(f_{n}^{\text{new}} - c)]$$

(10)

Summing over all $j \neq n$ leads to $n$’s expected profits from new-customer prices:

$$(1 - F_{n}^{\text{new}}(f_{n}^{\text{new}}))^{N-2}(f_{n}^{\text{new}} + \alpha \bar{a} - c) \sum_{j \neq n}(1 - F_{j}^{\text{naive}}(f_{n}^{\text{new}}))s_{j} + (1 - \alpha)(f_{n}^{\text{new}} - c) \sum_{j \neq n}F_{j}^{\text{naive}}(f_{n}^{\text{new}})s_{j} = \text{const.}$$

(11)

Lemma 2 established that all naive-customer prices on $(c - \alpha \bar{a}, c)$ occur with positive probability.
and that $F_{\text{new}}(c - \alpha \tilde{a}) = F_{j, \text{new}}(c - \alpha \tilde{a}) = 0$. I have shown above that expected profits from new-customer prices are zero. Now consider $f_{n, \text{naive}} \in (c - \alpha \tilde{a}, c)$. Rewriting the equation gives

$$\sum_{j \neq n} F_{j, \text{naive}}(f_{n, \text{new}}) s_j = (1 - s_n) \frac{(f_{n, \text{new}} + \alpha \tilde{a} - c)}{\alpha(f_{n, \text{new}} + \tilde{a} - c)}, \quad \forall n$$

(12)

$$\Leftrightarrow \sum_{j=1}^{N} F_{j, \text{naive}}(f_{n, \text{new}}) s_j = (1 - s_n) \frac{(f_{n, \text{new}} + \alpha \tilde{a} - c)}{\alpha(f_{n, \text{new}} + \tilde{a} - c)} + s_n F_{n, \text{naive}}(f_{n, \text{new}}), \quad \forall n$$

(13)

$$\Leftrightarrow g(f_{n, \text{new}}) = (1 - s_n) \Omega(f_{n, \text{new}}) + s_n F_{n, \text{naive}}(f_{n, \text{new}}), \quad \forall n$$

(14)

For each $n$, the condition implies $F_{n, \text{naive}}(f_{n, \text{new}}) = g(f_{n, \text{new}}) = \frac{\eta \alpha}{s_n} - \frac{1 - s_n}{s_n} \Omega(f_{n, \text{new}})$. Plugging this into (11) pins down $g(f) = \Omega(f)$ for all $f$ and therefore $F_{n, \text{naive}}(f_{n, \text{new}}) = \Omega(f)$. Hence, in all second-period shrouding equilibria, naive customer prices are mixed symmetrically according to (2).

(ii) I show now that after histories in which shrouding occurs and at least one firm has no customer base and another has one, firms always educate about hidden fees if the product is socially desirable and $\eta > 0$. Firms make no profit and consumers pay marginal costs.

Given shrouding occurs with positive probability, the same reasoning as in (i) implies that firms can earn $\tilde{s}_n \alpha (1 - \alpha) \tilde{a}$ conditional on shrouding from their old naive customers while firms earn zero expected profits from new-customer prices and old sophisticates.\(^{44}\) But then firms without a customer base earn zero total profit since they have no customer base to exploit and their shrouding condition reduces to $0 \geq \eta \alpha \min \{(1 - \alpha) \tilde{a}, v - c\}$. As long as $v > c$ and $\eta > 0$, they have a strict incentive to educate customers about hidden fees. When $\eta$ is equal to zero, profits are zero after unshrouding. Firms without customer base are indifferent between shrouding and unshrouding and there are potentially multiple equilibria.

(iii) After all other histories, hidden fees are unshrouded in period 1. Thus, standard Bertrand arguments imply that each consumer pays marginal costs.

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\(^{44}\tilde{s}_n (\geq s_n)\) is the market share a firm gets when not all firms sell to consumers but $n$ does.
B.4 Proof of Proposition 4

The results of Proposition 3 pin down the continuation payoffs after period 1 and can be used to study equilibrium behavior in period 1.

Lemma 3 establishes that when \( v > c \) and \( \eta > 0 \), firms can achieve positive continuation profits if and only if each firm has a positive customer base, i.e. when prices in the first period are identical with positive probability.

First, I study equilibria in which firms always set the same transparent price \( f_1 \) in the first period. Given the reduced-game profits starting from \( t = 1 \) specified in (3), the only possible profitable deviations are either (i) shrouding and undercutting competitors or (ii) unshrouding hidden fees and attracting the remaining profitable customers.

(i) is unprofitable if \( s_n(f_1 + \alpha \bar{a} - c) + s_n \alpha (1 - \alpha) \bar{a} \geq f_1 + \alpha \bar{a} - c \), which is equivalent to \( f_{n1} \leq c - \alpha \bar{a} + \frac{s_n}{1 - s_n} \alpha (1 - \alpha) \bar{a} \).

To check for (ii), I need to establish the optimal deviation under unshrouding. Given all other firms shroud and play \( f_1 \), a firm \( n \) can make sure to attract only all profitable customers after unshrouding, i.e. only all non-avoiding naives and neither educated avoiding naives nor sophisticates, by setting \( \tilde{f}_1 = \max\{c, f_1\} \) and \( \tilde{\alpha}_1 < \min\{f_1 + \bar{a}, v\} - \tilde{f}_1 \). The resulting deviation profits are bounded by \( \eta \alpha \min\{f_1 + \bar{a} - c, v - c\} \). Note that I do not have to consider the case where \( f_1 \geq c \) since the resulting deviation profits of \( \eta \alpha (f_1 + \bar{a} - c) \leq f_1 + \alpha \bar{a} - c \) for all \( f_1 \geq c \), and therefore deviation (i) is always preferred. Thus, consider \( f_1 < c \), in which case only non-avoiding naives are profitable after unshrouding. Hence, the optimal deviation profits with unshrouding are \( \eta \alpha \min\{f_1 + \bar{a} - c, v - c\} \). Deviating in this way is unprofitable if \( s_n(f_1 + \alpha \bar{a} - c) + s_n \alpha (1 - \alpha) \bar{a} \geq \eta \alpha \min\{f_1 + \bar{a} - c, v - c\} \forall n \). Thus I have to consider three cases. First, if \( f_1 + \bar{a} < v \) and \( s_n < \eta \alpha \forall n \), we get \( f_1 \leq c - \alpha \bar{a} + \frac{\eta \alpha + s_n(1 - \alpha)}{\eta \alpha - s_n} \alpha \bar{a} \forall n \). This is always larger than \( c - \alpha \bar{a} + \frac{s_n}{1 - s_n} \alpha (1 - \alpha) \bar{a} \), the upper bound from (ii), which is why (i) does not need to be considered in this case. Second, if \( f_1 + \bar{a} < v \) and \( s_{\max} > \eta \alpha \), we get a lower bound for prices of \( f_1 \geq c - \alpha \bar{a} - \frac{\eta \alpha + s_n(1 - \alpha)}{\eta \alpha - s_n} \alpha \bar{a} \forall n \). Since the latter is increasing in \( s_n \), the lower bound is most restrictive for the firm with the largest market share \( s_{\max} \). Comparing this lower bound with the lowest price that induces zero profits \( c - \alpha \bar{a} - \alpha (1 - \alpha) \bar{a} \) shows that the latter is always larger. Therefore, deviation (ii) is not binding in this case. Third, if \( f_1 + \bar{a} \geq v \), I get another lower bound at \( f_1 \geq c - \alpha \bar{a} + \frac{\eta \alpha}{s_n} (v - c) - \alpha (1 - \alpha) \bar{a} \). This is most restrictive for the firm with the smallest market share \( s_{\min} \). Thus, the latter case imposes a lower bound on prices and thereby imposes minimal positive shrouding profits of \( \eta \alpha (v - c) \) in the last two cases respectively.
Thus, deviation (i) induces an upper bound on prices and (ii) can induce a lower bound if \( f_1 + \bar{a} \geq v \). In the latter case, shrouding equilibrium profits are always strictly positive and above \( \eta \alpha (v - c) \).

Note that there can be no equilibrium in which firms play mixed strategies with a continuous distribution function. When firms mix on some interval with a continuous distribution function, conditional on prices of this interval occurring, the probability of having the same prices is zero and continuation profits are zero as well. Thus, standard Bertrand arguments such as those in the proof of Proposition 2 establish the usual contradiction.

There can, however, be shrouding equilibria in which firms mix over a finite number of prices, each price being played by each firm with positive probability. These prices must be within the range derived above, for otherwise (i) or (ii) above is a profitable deviation. Since continuation profits cannot be larger as when all firms coordinate on the same price with probability one, and the largest such price is given by \( f_1 = c - \alpha \bar{a} + \frac{s_{min}}{1 - s_{min}} \alpha (1 - \alpha) \bar{a} \), profits must be below \( s_n \frac{s_{min}}{1 - s_{min}} \alpha (1 - \alpha) \bar{a} + s_n \alpha (1 - \alpha) \bar{a} \) for all firms. At the same time, shrouding profits must be at least \( \eta \alpha (v - c) > 0 \) if total prices for naives are larger than \( v \) and zero otherwise for each firm.

### B.5 Proof of Proposition 5

**Step 1: Period 2** In the following Lemma I summarize results on continuation equilibria. Afterwards, I study the first period.

**Lemma 4** (Period 2 with Disclosure Policy). An Equilibrium with shrouding in period 2 exists if and only if shrouding occurs in period 1. Shrouding occurs in period 2 either with probability one or with probability zero. When shrouding occurs, both customer types pay a total price of \( c \) and naives a hidden fee \( \bar{a} \). Profits are zero in any continuation equilibrium.

**Proof of Lemma 4.** First, I analyze continuation equilibria given shrouding occurs in period 1. By the exact same argument as in the proof of Proposition 3, continuation equilibrium profits are zero whenever some firm unshrouded in period 1.

Suppose prices were shrouded in period 1. Then continuation equilibrium profits must be zero conditional on shrouding and unshrouding. Suppose otherwise. Note that whether shrouding or unshrouding occurs, firms have symmetric information on customers and can charge those that were naive and sophisticated in period 1 separately in period 2. The markets for consumers who were naive or sophisticated in period 1 can therefore be treated as separate markets in period 2. For
consumers that were sophisticated in period 1, the market is a standard Bertrand market and the results follow immediately. Recall that sophisticates are unaffected by shrouding. For the market for consumers that were naive in period 1, the argument is similar to the one used in the proof on Lemma 3(i) Step (II). Take the firm that earns the largest strictly positive profits conditional on either unshrouding or shrouding. If these profits occur conditional on shrouding, take the supremum for which these profits occur and denote it by $\bar{f}$. For positive profits to occur, each competitor must set larger prices with positive probability. I.e., competitors set prices in $[\bar{f}, \bar{f} + \epsilon)$ with positive probability for each $\epsilon > 0$, or $\bar{f}$ would be shifted upwards. But then competitors can increase their profits conditional on shrouding discretely by shifting probability mass from $[\bar{f}, \bar{f} + \epsilon)$ slightly below $\bar{f}$. Since losses conditional on unshrouding are below $\epsilon$, this deviation is strictly profitable for some $\epsilon$ small enough, a contradiction. If the largest profits occur conditional on unshrouding the same argument applied to total prices applies. Thus, expected profits are zero for all customers conditional on shrouding and unshrouding. In particular when firms shroud with probability one, a firm’s demand is independent of $\bar{a}$ and hence any firm sets $a_{n2} = \bar{a}$, and standard Bertrand arguments applied to each market imply that $f_{n2}^{soph} = c$ and $f_{n2}^{naive} = c - \bar{a}$. When shrouding occurs with probability zero, all consumers pay $f_{n2}^{soph} = f_{n2}^{naive} = c$ since all are aware of hidden fees, whether they can avoid them or not.

I study unshrouding incentives next. When firms shroud with probability one, all consumers pay a total price equal to marginal costs. Unshrouding and undercutting total prices for competitors’ non-avoiding naive customers reduces total prices below marginal costs and can therefore not profitably attract these customers. I now establish that shrouding either occurs with probability one or with probability zero. Suppose otherwise. Recall that firms earn zero profits in expectation whether shrouding or unshrouding occurs. When shrouding occurs, customers that were naive in period 1 must pay a transparent price below marginal cost and a hidden fee of $\bar{a}$. If this was not so, a firm could earn strictly positive profits by setting prices for customers that were naive in $t = 1$ of $c - \epsilon$ and $\bar{a}$ for some $\epsilon > 0$ small enough. This would marginally reduce profits on these customers when unshrouding occurs but discretely increase profits when shrouding occurs. Naives of period 1 therefore purchase at a transparent price below $c$ when shrouding occurs and firms earn zero expected profits from them. But when unshrouding occurs, the share of naive customers in period 2 drops discretely to $\eta\alpha$ and with it the share of naives of period 1 that pay the hidden fee in period 2. Since these customers pay transparent fees below $c$ and profits are zero when shrouding occurs, firms must earn strictly negative profits with these prices when unshrouding occurs. Thus,
these firms are better off by unshrouding with probability one and setting transparent prices to \( c \) and hidden fees to zero, a contradiction.

\( \square \)

**Step 2: Period 1**

By Lemma 4, continuation profits are zero independent of first-period behavior. Hence, the setting is the same as in period 1 of Proposition 2.

\( \square \)

From now on, results are proven for the setting described in the main text with all naives avoiding hidden fees after unshrouding (\( \eta = 0 \))

**B.6 Proof of Proposition 7**

Relative to Propositions 3 and 4, the incentives to unshroud have changed. First, I derive the shrouding condition for shrouding equilibria in period 2. Note that naives can avoid hidden fees after unshrouding so that they cannot be profitably attracted by unshrouding. Given shrouding occurred in period 1 and all firms have a positive customer base, the shrouding-equilibrium prices are the same as in Proposition 3. When unshrouding occurs in \( t=2 \), a share \((1 - \lambda)\) of the old naives remain naive. The situation is the same when these consumers learn about hidden fees, i.e. when naiveté in period 1 is not a perfect predictor of naiveté in period 2. But past naiveté remains an informative signal that competitors do not have. Hence, conditional on unshrouding firms can guarantee themselves only profits of \( s_n \alpha (1 - \lambda)(1 - \alpha)\bar{a} \) by setting naive-customer prices to \( c - (1 - \lambda)\alpha \bar{a} \) and to \( c \) for sophisticates and new customers. Since this is strictly smaller than shrouding profits, the argument in the proof of Lemma 3, part (i) still applies accordingly and firms prefer shrouding over unshrouding when shrouding occurs with positive probability.

Second, I identify the most profitable deviations from a shrouding equilibrium path in period 1. There are three candidates: firms could unshroud without changing prices, firms could unshroud and undercut competitors or they could continue to shroud and undercut competitors. At a given price \( f_1 \) that is charged by all firms, shrouding in \( t = 1 \) is profitable if

\[
s_n(f_1 + \alpha \bar{a} - c) + s_n \alpha (1 - \alpha)\bar{a} \geq s_n(f_1 + (1 - \lambda)\alpha \bar{a} - c) + s_n(1 - \lambda)\alpha (1 - (1 - \lambda)\alpha)\bar{a}, \quad \forall n.
\]

(15)

For positive \( \lambda \), this condition is equivalent to \( \alpha \leq \frac{2}{\lambda - 1} \), which holds for all \( \lambda \). Note that after shrouding in period 1, the second period becomes equivalent to a model with a share of \( \bar{\alpha} = \)
(1 - \lambda)\alpha \) of naive consumers and no option to unshroud. This induces equilibrium profits as in a shrouding equilibrium with a share of naives of \( \tilde{\alpha} \).

Unshrouding and undercutting a price \( f_1 \) is not profitable if

\[
s_n(f_1 + \alpha \bar{a} - c) + s_n\alpha(1 - \alpha)\bar{a} \geq (f_1 + (1 - \lambda)\alpha \bar{a} - c) + (1 - \lambda)\alpha(1 - (1 - \lambda)\alpha)\bar{a}, \ \forall n.
\]

(16)

While simply undercutting is no deviation if

\[
s_n(f_1 + \alpha \bar{a} - c) + s_n\alpha(1 - \alpha)\bar{a} \geq (f_1 + \alpha \bar{a} - c) + (1 - \lambda)\alpha(1 - (1 - \lambda)\alpha)\bar{a}, \ \forall n.
\]

(17)

it can be easily shown that the last condition is more restrictive for all \( \lambda > 0 \). It follows immediately that this condition is equivalent to

\[
f_1 \leq c - \alpha \bar{a} + \frac{s_n}{1 - s_n} \alpha(1 - \alpha)\bar{a} - \frac{1 - \lambda}{1 - s_n} \alpha(1 - \alpha)\bar{a}, \ \forall n.
\]

(18)

Since the r.h.s. is increasing in \( s_n \), the largest price at which no firm has an incentive to undercut is given by the r.h.s evaluated at the smallest market share \( s_{min} \).

The lower bound of the interval for equilibrium profits is given by the smallest price that earns firms nonnegative profits when all firms play this price with probability one.

\[ \square \]

B.7 Proof of Proposition 8

The only difference to the proofs of Propositions 3, 4 and Corollary 1 is the upper bound of the interval on which prices \( f_{n2}^{new} \) and \( f_{n2}^{naive} \) are mixed. By choosing \( f_{n2}^{new} \), firms now attract the new arriving customers as well. Despite \( f_{n2}^{soph} = c \), firms earn positive profits from \( f_{n2}^{new} = c - \epsilon \), since marginal losses from sophisticates are offset by positive margins from newly arrived naives.

Thus, competition drives \( f_{n2}^{new} \) down until \( n \) does not benefit from attracting new customers, i.e. until \( ((1 - \gamma) + (1 - s_n)\gamma(1 - \alpha))(f_{n2}^{new} - c) + (1 - \gamma)\alpha \bar{a} \leq 0, \forall n \), which results in \( f_{n2}^{new} \leq c - \frac{(1 - \gamma)}{(1 - \gamma) + (1 - s_n)\gamma(1 - \alpha)}\alpha \bar{a}, \forall n \). For \( N = 2 \), this pins down the interval as stated in the Proposition.

For \( N > 2 \), note that for each \( n \), all \( \hat{n} \neq n \) jointly have to choose \( f_{n2}^{new} \) to make \( n \) indifferent between an interval of naive-customer prices. Since this must hold for all \( n \), they need to mix on the same interval and thus we get \([c - \alpha \bar{a}, c - \frac{(1 - \gamma)}{(1 - \gamma) + (1 - s_{max})\gamma(1 - \alpha)}\alpha \bar{a}] \).

The rest follows as in Propositions 3 and 4. \[ \square \]
B.8 Proof of Proposition 9

The proof is the same as for Propositions 3, 4 and Corollary 1, except for one difference: the smaller share of naive customers in period 2. Here, all new-customer prices $f_{n2}^{\text{new}} < c - \sigma \bar{a}$ are never played in equilibrium with positive probability since they result in negative profits from new customers. This induces mixing of $f_{n2}^{\text{naive}}$ and $f_{n2}^{\text{new}}$ on $[c - \sigma \bar{a}, c]$. Second period profits become $\pi_{n2} = s_n \sigma \alpha (1 - \alpha) \bar{a}$.\hfill \Box

B.9 Proof of Proposition 10

In this proof, I construct an equilibrium for the T-period model in which shrouding occurs in each period.

Lemma 5 (T Periods). Assume the customer bases of all firms is non-empty in $t - 1$. Shrouding can occur in $t$ if and only if shrouding occurred in $t - 1$ and no customer-type switches in $t - 1$.
(Given shrouding in $t - 1$, if customers switch in $t - 1$, unshrouding occurs with probability 1 in $t$ and profits become zero.)

Proof of Lemma 5. Obviously, shrouding in $t$ requires shrouding in all periods before $t$, otherwise consumers are educated about hidden fees in $t$. Let there be shrouding in $t - 1$. Suppose that at least one firm attracted at least one customer type of her competitor in $t - 1$. Then this firm is perfectly informed about her competitor’s customers and the competitor thus earns zero profits in $t$. He is therefore indifferent between unshrouding or not. Due to the same equilibrium selection logic as in Proposition 4, i.e. only unshrouding being robust to the presence of non-avoiding naive customers, unshrouding occurs with probability one. Similarly, if shrouding occurs, we get a contradiction if some customers switched in the past.\hfill \Box

Lemma 5 shows that switching of customers induces unshrouding in the subsequent period and therefore zero profits until period $T$. Hence, firms have a strong incentive not to let customers switch in order to maintain shrouding equilibria with positive profits.

Since the control and state variables are the same as in the two period model, Lemma 3 and Lemma 5 give the profits in period T of shrouding equilibria, namely $\pi_{nT} = s_n \alpha (1 - \alpha) \bar{a}$. Denote by $V_{nt}$ the continuation value of shrouding of firm $n$ at the beginning of period $t$ when the firms continue to shroud until $T$. We know from Lemma 5 that the continuation profit in all other cases is zero. Note that $V_{nT} = \pi_{nT} = s_n \alpha (1 - \alpha) \bar{a}$. 

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Lemma 5 tells us as well that in each shrouding equilibrium path with shrouding in each period, no customer switches the firm until period T and each firm has a positive customer base. Denote one firm by $n$ and the other $\hat{n}$. Then I can find the range of naive customers’ prices $f_{nt}^{naive}$ that prevents $\hat{n}$ from undercutting her competitor with $f_{nt}^{new}$. Undercutting attracts all of $n$’s customers once, earning $s_n(f_{nt}^{naive} + \alpha \bar{a} - c) + 0$, but induces a loss of future shrouding profits $0 + \delta V_{nt+1}$.

Hence, all prices $f_{nt}^{naive} \leq f_{nt}^{naive} + c - \alpha \bar{a} + \frac{\delta}{s_n} \cdot V_{nt+1}$ are not undercut by $\hat{n}$ and we therefore get $f_{nt}^{naive} = \min\{v, f_{nt}^{naive}\}$.

Similarly, I can find the lowest new-customers price $f_{nt}^{new}$ that is undercut by $n$ with $f_{nt}^{soph}$ in order to prevent switching of any customer: $f_{nt}^{new} < f_{nt}^{soph}$ makes $n$ loose her sophisticated customers and gives $s_n \alpha (f_{nt}^{naive} + \bar{a} - c) + 0$ while undercutting and choosing $f_{nt}^{soph} = f_{nt}^{new}$ in order to prevent switching gives $s_n \alpha (f_{nt}^{naive} + \bar{a} - c) + s_n(1 - \alpha)(f_{nt}^{new} - c) + \delta V_{nt}$. Hence, all $f_{nt}^{new} \geq f_{nt}^{new} \equiv c - \frac{\delta}{s_n(1-\alpha)} \cdot V_{nt+1}$.

Using $V_{nt}$, it can be easily shown that $f_{nt}^{new} \leq f_{nt}^{naive}$ for all $T > t > 1$. Therefore, we get prices for these periods of $f_{nt}^{naive} = f_{nt}^{soph} = \min\{v, f_{nt}^{naive}\} = f_{nt}^{new}$. Profits in period $t$ become $\pi_{nt} = \min\{s_n(v + \alpha \bar{a} - c), \delta V_{nt+1}\}$. This allows us to compute the shrouding condition in period $t$

$$V_{nt} \geq \alpha \eta \min\{(1 - \alpha)\bar{a}, v - c\}, \ \forall n \text{ and } t > 1$$ (19)

Using $V_{nt} = \pi_{nt} + \delta V_{nt+1}$ and the definition of $\pi_{nt}$, it is straightforward to show that $V_{nt}$ is decreasing in $t$. Hence, $V_{nT} \geq \alpha \eta \min\{(1 - \alpha)\bar{a}, v - c\}$ implies that all the other shrouding conditions are satisfied.

In period 1 the same argument applies as for Corollary 1 to pin down first period prices. \hfill \Box