Optimal monitoring in dynamic procurement contracts

Andreas Asseyer, Humboldt- Universität zu Berlin
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Andreas Asseyer∗
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Abstract
Which information should government authorities monitor when they procure goods from private suppliers? I analyze this question in a principal-agent model of procurement with moral hazard concerning cost-reducing investments and dynamic adverse selection about investment cost and a production cost shock. The principal can monitor the investment, the shock, or both at a cost. I show that it is never optimal to monitor investment and shock. Monitoring investment is always at least as effective as monitoring the shock. The two instruments are equivalent if the level of investment cost is high. Monitoring may decrease efficiency in the optimal contract.

Keywords: procurement, monitoring, dynamic contracts

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∗Humboldt-Universität zu Berlin, Wirtschaftswissenschaftliche Fakultät, Spandauer Str. 1, 10099 Berlin; email: andreas.asseyer@hu-berlin.de. I would like to thank Helmut Bester and Roland Strausz for advice, and Jacques Crémer, Daniel Garrett, Holger Herbst, Doh-Shin Jeon, Johannes Johnen, Thomas Mariotti, Patrick Rey, Stefan Terstiege and participants of the Micro Colloquium at Free University and Humboldt University in Berlin and the Applied Theory Workshop at Toulouse School of Economics for helpful comments. I am grateful for hospitality at Toulouse School of Economics where parts of this paper were written.

1 Introduction

Public procurement constitutes a sizable fraction of economic activity in advanced economies: In 2011, the expenditure for public procurement in OECD-countries represents on average 29% of total government spending, or 13% of GDP (OECD, 2013). In many of these countries however, the outcomes of public procurement projects are often perceived as unsatisfactory by the general public.\(^1\) Moreover, government controlling institutions such as national audit offices criticize the current practice of public procurement (GAO, 2013; NAO, 2014). In order to improve the performance of public contracts, audit offices advise government authorities to extend the use of monitoring in order to gather more information about the performance of suppliers and the development of costs (GAO, 2013; NAO, 2014; OFPP, 2014). At the same time, it is recognized that not all sources of additional information help the government authority equally well to achieve a better outcome.\(^2\) It therefore arises the question what kind of information a public buyer should try to observe in order to improve the performance of a public contract in the most effective way.

I analyze this question in a principal-agent model of procurement where the agent can produce a good for the principal. Before production, the agent can make a cost-reducing investment at a privately known cost that determines together with a shock the costs of production. The principal decides whether to monitor the agent’s investment decision, the cost shock, or both at a monitoring cost, and designs the contract between the two parties accordingly.

In this model, I show that the principal optimally monitors either the investment or the cost shock. Furthermore, monitoring of investment allows the principal to achieve at least the same payoff – gross of monitoring costs – as monitoring of the shock. However, both instruments are equivalent if the level of the fixed cost of investment is high.

The model captures three important aspects of public procurement projects. First, cost-reducing investments and sequential, asymmetric learning of production costs are realistic features of many procurement projects. Projects such as the construction of airports, the extension of high-speed internet networks, or the introduction of highway toll systems take a considerable amount of time to be realized. Furthermore, the supplier can in these examples often make

\(^1\)Examples include the construction of the new airport for the city of Berlin (Cottrell, 2014) and the overbilling of government authorities in the UK by the outsourcing companies G4S and Serco (Travis, 2013).

\(^2\)For example, the excessive prices paid under public contracts to the companies Serco and G4S in the UK were not prevented by a large number of performance indicators (NAO, 2014).
investments at an early stage of the project, e.g. during the planning phase. These investments influence the total costs of project completion together with exogenous factors such as input prices or ground conditions that a supplier observes after the investment decision.

Second, monitoring plays a central role in the management of public procurement projects. In practice, a wide array of outcomes are monitored by government authorities. In this paper, I consider the monitoring of investments, as a cost factor which is endogenous to suppliers, and the monitoring of a cost shock, such as input prices or ground conditions, which are exogenous cost factors to suppliers. In practice, investment decisions are monitored using key performance indicators such as quality milestones (Garvin et al., 2011) and input prices or ground conditions can be monitored by employing third-party experts\(^3\).

Third, cost-reducing investments are likely to lead to investment costs that have a fixed cost component. For example, a supplier can reduce production costs by adopting a new technology prior to production. Åstebro (2004) provides empirical evidence for fixed costs of learning that arise when firms adopt new technology. Due to fixed costs of investment, the supplier displays ’lumpy’ investment behavior.\(^4\)

I solve the principal’s problem by analyzing optimal contracts under the different monitoring policies. If the agent’s investment decision is monitored, the principal has to elicit first the investment cost and then the value of the shock from the agent. I show that the principal’s optimal contract induces underinvestment and efficient production of the good independently of the investment decision. The two sources of asymmetric information can be treated separately: As the agent knows the fixed cost of investment before contracting, the principal has to give information rent to the agent and therefore distorts the investment decision. In contrast, the cost shock only realizes after the parties have signed a contract. The principal does not have to give rent to the agent for this information as it is possible to induce efficient production and to extract the surplus with a fixed fee. Thus, the principal cannot gain by additionally monitoring the cost shock.

Next, I consider the case where the principal monitors only the shock and has to elicit the fixed cost of investment while inducing the appropriate investment decision. Moral hazard

\(^3\)For instance, the government of Canada employs KPMG as a third party expert for a national shipbuilding procurement project (PWGSC, 2013).

\(^4\)Lumpy investment behavior is empirically widely documented (Doms and Dunne, 1998) and has implications in many different fields of economics. See e.g. Caplin and Leahy (2010) on the history of applications of the \((S, s)\)-model.
concerning the investment decision may be irrelevant, such that the previously optimal contract is still implementable. This is the case if the investment decision in the optimal contract is not too much distorted away from efficient investment, because this distortion creates an incentive for the agent to deviate in order to appropriate the whole additional surplus generated through the investment. If the distortion of the investment decision in the previously optimal contract is large, moral hazard is relevant. In order to discourage the agent from deviating, the principal optimally induces a smaller distortion in the investment decision and introduces an inefficiency in production for the agent who announces not to invest.

Finally, I analyze the case without monitoring. I show that the principal achieves a strictly worse outcome compared to the case where the shock is monitored. Independently of previous reports, the agent has an incentive to truthfully report production costs. If the agent deviates from the equilibrium strategy by taking a different investment decision, then it is optimal to misreport the cost shock such that the principal again holds the correct belief about production costs. This implies that optimal deviation strategies of the agent include false reports about the shock. As the agent makes use of the possibility to misreport the shock, the set of feasible mechanisms from which the principal can choose is smaller than in the case where the shock is monitored. This restriction arises due to private information which the agent learns after contracting. The unobserved investment decision ‘connects’ the two sources of asymmetric information, such that the principal cannot deal with these separately as it was the case under investment monitoring. The optimal contract induces two types of inefficiencies: underinvestment in cost-reduction and underproduction if the supplier does not invest.

These results have the following implications for the optimal monitoring policy. First, the principal monitors either the shock or the investment decision. This follows from the fact that the principal does not have to conceit information rents to the agent for private information about the cost shock if investment is monitored. Second, monitoring of investment is always at least as effective as monitoring of the shock. However the principal may achieve the same payoff (gross of monitoring costs) under the two monitoring options. This turns out to be the case if the level of fixed costs of investment is high.

Different monitoring decisions have mostly ambiguous effects on the efficiency of the interaction between principal and agent. Comparing the optimal contracts under monitoring of
investment and no monitoring, production is more efficient under the first monitoring regime whereas investment is more efficient under the second monitoring regime. However one can show that the optimal contract under monitoring of the shock includes greater distortions in both investment and production than the optimal contract under no monitoring, if the principal’s value for the good is high. Privacy of information that arises after contracting may therefore increase efficiency.


This paper is also related to the literature on R&D and optimal procurement mechanisms which was initiated by Tan (1992), Piccione and Tan (1996), and Bag (1997). As the investment decision in this paper can be interpreted as an investment in R&D, the contribution to this literature lies in analyzing the effect of fixed costs of R&D on dynamic information rents. Cisternas and Figueroa (2014) analyze the optimal procurement mechanism when a buyer procures two projects sequentially and the winner of the first round can invest in cost reduction before the second round. Liu and Lu (2015) analyze optimal contracts in a model of procurement with an unobservable R&D effort in cost-reduction under dynamic adverse selection.

A further related literature analyzes optimal contracts for public-private partnerships (see Iossa and Martimort (2012, 2014); Engel et al. (2013); Hoppe and Schmitz (2013)). Public-private partnerships are frequently used for the realization of mid to long-term procurement projects where learning of new information and cost-reducing investments by suppliers play an important role. This paper contributes to the literature by providing an explicit analysis of optimal monitoring policies for a government authority that enters into a public-private partnership with a supplier.

This paper furthermore makes a point related to the literature on information rents in dynamic principal-agent models. In models of dynamic adverse selection, Baron and Besanko 5In contrast to the literature on costly state verification initiated by Townsend (1979), the principal here decides ex-ante which information of the agent to observe.
(1984), Eső and Szentes (2007a,b) and Pavan et al. (2014) show that the principal does not have to give rent for private information which the agent learns after contracting. Krähmer and Strausz (2015) qualify this insight by showing that an agent receives post-contractual information rents if the set of signal realizations learned before contracting is discrete. Baron and Besanko (1984) and Eső and Szentes (2013) argue that privacy of post-contractual information may also be irrelevant in the presence of dynamic adverse selection and moral hazard. In this paper I show that their result does not extend to my setting due to the presence of fixed costs of investment. I elaborate this point in the discussion section of the paper.

In the next section I introduce the model. I then solve for the optimal contracts under the different monitoring policies: Section 3 presents the optimal contract when the principal monitors the investment decision. Section 4 considers the case where the principal monitors the shock. Section 5 analyzes the optimal contracts with no monitoring. Section 6 presents the implications for the optimal monitoring policy. Section 7 discusses the effect of monitoring on the efficiency of the interaction between the principal and the supplier, the robustness of the results, and the role of post-contractual information rents. Section 8 concludes.

2 The model

A government authority (the principal) can procure a good from a supplier (the agent). The principal values the good by \( v \). Prior to production, the agent can make a cost-reducing investment decision \( x \in \{0, 1\} \). The investment decision leads to investment costs of \( x \cdot \kappa \) to the agent where \( \kappa \) is a fixed cost of investment.\(^6\) \( \kappa \) is private information to the agent and drawn from an interval \([\kappa, \bar{\kappa}] \subset \mathbb{R}_+\) according to the distribution function \( F \). \( F \) has a log-concave density function \( f \), so that \( F(\kappa)/f(\kappa) \) is weakly increasing and \((1 - F(\kappa))/f(\kappa) \) is weakly decreasing (Bagnoli and Bergstrom, 2005). The agent’s production cost is determined by the investment decision \( x \) and a shock \( \varepsilon \) that realizes after the investment is made. The production cost is given by \( c_x(\varepsilon) \). For both \( x \in \{0, 1\} \), the function \( c_x(\cdot) \) has the image \([c_x, \bar{c}_x] \subset \mathbb{R} \), is strictly increasing and twice continuously differentiable. Without loss of generality I can assume that the shock is uniformly distributed on the unit interval.\(^7\) The investment is cost-reducing in the sense that

\(^6\)None of the results depend on the binary investment decision. See also the discussion in section 7.

\(^7\)If the shock \( \varepsilon \) leading to production costs \( \tilde{c}_x(\varepsilon) \) is distributed according to a continuous and strictly increasing distribution function \( H \) on some interval, then the random variable \( \varepsilon \equiv H(\varepsilon) \) is uniformly distributed on \([0, 1]\). The cost functions can be redefined as \( c_x(\varepsilon) = c_x(H(\varepsilon)) \equiv \tilde{c}_x(\varepsilon) \). An assumption on \( c_x(\cdot) \) would then translate
\(c_1(\varepsilon) < c_0(\varepsilon)\) for all \(\varepsilon \in (0, 1)\). Furthermore, let \(v \in (\underline{v}, \overline{v})\). There are two simple monitoring technologies. The principal can perfectly observe the investment decision \(x\) at a monitoring cost \(C^i > 0\), and the shock \(\varepsilon\) at a monitoring cost \(C^s > 0\). I assume that the principal cannot monitor probabilistically.\(^8\) Both parties are risk-neutral and have outside options associated with a payoff of zero. Let \(q\) be the probability of production and \(t\) be a transfer. The agent’s payoff is \(t - c_x(\varepsilon)q - x\kappa\) and the principal’s payoff gross of monitoring costs is \(vq - t\). The timing of the game is as follows

i) The agent learns \(\kappa\).

ii) The principal decides what to monitor and offers a contract. The agent observes the principal’s decision and accepts or rejects the contract. If the agent rejects, the game ends and both parties receive zero payoffs. Otherwise, the game continues.

iii) The agent makes the investment decision.

iv) The shock \(\varepsilon\) realizes.

v) The agent can produce the good.

**Complete information benchmark**

Suppose the investment cost \(\kappa\), the shock \(\varepsilon\), and the investment decision \(x\) are publicly observed. In this case the principal can extract the whole social surplus. For any \(\kappa\), she chooses the investment decision and the probability of production such that social surplus is maximized:

\[
\max_{x, q, \kappa} \int_0^1 (v - c_x(\varepsilon))q(\varepsilon) - \kappa \cdot x
\]

The principal procures the good from the agent if \(v \geq c_x(\varepsilon)\) for a given investment decision \(x\). There exists a threshold \(\varepsilon_x^*\) which satisfies \(c_x(\varepsilon_x^*) = v\), for both \(x \in \{0, 1\}\). The principal induces the cost-reducing investment if \(\kappa \leq \kappa^*\) where

\[
\kappa^* \equiv \int_{\varepsilon_x^*}^{\varepsilon_1} (v - c_1(\varepsilon))d\varepsilon - \int_{\varepsilon_0}^{\varepsilon_x^*} (v - c_0(\varepsilon))d\varepsilon.
\tag{1}
\]

\(^8\)An alternative assumption is that the principal has to spend \(C^i\) or \(C^s\) to install the monitoring technology independently of whether it is used later on and the agent observes whenever a monitoring technology is installed.
3 Monitoring investment

In this section, I assume at first that the principal monitors the investment and the cost shock and elicits the investment cost from the agent. I solve for the optimal allocation in this case and show that it can be implemented with the same expected transfers even if the principal only monitors investment. This shows that the principal can not gain from monitoring the cost shock when she monitors investment.

The principal offers the agent a menu of two contracts. The first contract requires the agent to invest and fixes a probability of production \(q_1(\varepsilon)\) and an expected transfer \(t_1(\varepsilon)\), both as functions of the shock. The second contract prescribes the agent not to invest. The probability of production and the expected transfer are stipulated as \(q_0(\varepsilon)\) and \(t_0(\varepsilon)\). I denote the expected payoff gross of investment costs of the two contracts for \(x \in \{0, 1\}\) by

\[
U_x \equiv \int_0^1 (t_x(\varepsilon) - c_x(\varepsilon) q_x(\varepsilon)) \, d\varepsilon. \tag{2}
\]

An agent with investment cost \(\kappa\) chooses the first contract if and only if \(U_1 - \kappa \geq U_0\). I denote by \(\hat{\kappa}\) the threshold at which the agent is indifferent between the two contracts. The agent participates if at least one of the contracts gives a positive expected payoff. The principal’s
expected payoff from the menu of contracts is

$$\Pi \equiv F(\hat{\kappa}) \int_0^1 (vq_1(\epsilon) - t_1(\epsilon))\,d\epsilon + (1 - F(\hat{\kappa})) \int_0^1 (vq_0(\epsilon) - t_0(\epsilon))\,d\epsilon.$$  \hspace{1cm} (3)

The principal’s payoff can be expressed as a function of the probability of production, the investment threshold, and the expected payoffs of the agent from the two contracts. Using furthermore the relationship between expected payoffs and the investment threshold, the principal’s expected payoff is

$$\hat{\Pi} \equiv F(\hat{\kappa}) \left( \int_0^1 (v - c_1(\epsilon))q_1(\epsilon)d\epsilon - \hat{\kappa} \right) + (1 - F(\hat{\kappa})) \left( \int_0^1 (v - c_0(\epsilon))q_0(\epsilon)d\epsilon \right) - U_0.$$  \hspace{1cm} (4)

The principal maximizes the payoff subject to $U_0 \geq 0$, choosing the probabilities of production $q_1(\cdot)$ and $q_0(\cdot)$ and the investment threshold $\hat{\kappa}$. It is optimal to make the participation constraint of the non-investing agent binding. Furthermore the principal procure the good from the agent if and only if it is efficient to do so and sets $q^*_x(\epsilon) \equiv 1(\epsilon \leq \epsilon^*_x)$ for $x \in \{0, 1\}$. The optimal investment threshold denoted by $\kappa^i$ satisfies

$$\int_0^{\epsilon^*_1} (v - c_1(\epsilon))d\epsilon - \int_0^{\epsilon^*_0} (v - c_0(\epsilon))d\epsilon = \kappa^i + \frac{F(\kappa^i)}{f(\kappa^i)}.$$  \hspace{1cm} (5)

and $\kappa^i < \kappa^*$. The principal induces investment if the social surplus generated by the investment exceeds the virtual investment costs. I denote by $\Pi^i$ the payoff that the principal achieves. Whereas the principal chooses efficient production conditional on the investment, the induced investment decision leads to underinvestment.

The optimal direct contracts can be implemented by a menu of two indirect contracts. The first contract prescribes investment, the second contract prohibits investment. In both contracts the agent decides whether to produce the good at price $v$. The first contract demands an initial payment of $T_1 = \int_0^{\epsilon^*_1} (v - c_1(\epsilon))d\epsilon - \kappa^i$. The initial payment of the second contract is $T_0 = \int_0^{\epsilon^*_0} (v - c_0(\epsilon))d\epsilon$. These contracts implement the principal’s optimal investment decisions and allocations. As the principal can delegate the production decision to the agent, the contracts do not require the principal to observe the shock. The first result follows from this observation.

**Proposition 1.** If investment is monitored, the principal cannot gain from monitoring the
The principal can costlessly extract the private information about the shock from the agent, as the shock is statistically independent from the investment cost. Therefore the principal cannot increase her payoff by monitoring the shock. This reflects the result by Baron and Besanko (1984). In the next sections I show that the situation is different if investment is unobservable.

4 Monitoring the shock

In the following I suppose that the principal monitors only the shock. Under this monitoring policy, the optimal contract needs to give the agent incentives to reveal his costs of investment and to take the right investment decision. I show that moral hazard concerning the investment decision may be irrelevant. In this case, the principal can achieve the same payoff gross of monitoring costs as under monitoring of investment. If moral hazard is relevant, the principal optimally chooses to introduce underproduction by non-investing agents in order to reduce rent payments to investing agents.

The principal offers a menu of two contracts. The first contract is supposed to be chosen by the agent if he decides to invest, whereas the second contract is targeted at the agent if he does not invest. Both contracts fix a probability of production and an expected transfer \( q_x(\varepsilon) \) and \( t_x(\varepsilon) \) for \( x \in \{0, 1\} \).\(^9\) Denote by \( K_x \) the values of investment costs for which the agent takes the investment decision \( x \) in equilibrium. A menu of contracts is incentive compatible if an agent with investment costs in the set \( K_x \) finds it optimal to choose the contract \( (q_x(\cdot), t_x(\cdot)) \) and the action \( x \). A menu of contracts is individual rational if the agent always prefers one of the contracts over rejecting the principal’s offer. Formally, incentive compatibility and individual rationality under monitoring of the shock require

\[
\int_0^1 (t_x(\varepsilon) - c_x(\varepsilon) q_x(\varepsilon)) \, d\varepsilon - \kappa x \geq \max \left\{ \int_0^1 (t_{x'}(\varepsilon) - c_{x''}(\varepsilon) q_{x'}(\varepsilon)) \, d\varepsilon - \kappa x'', 0 \right\} \tag{6}
\]

for \( \kappa \in K_x \) and \( x, x', x'' \in \{0, 1\} \).

\(^9\)The revelation principle due to Myerson (1986) allows to focus on truthful direct mechanisms with random recommendations concerning the investment decision. The contracts studied here are mechanisms with deterministic recommendations, i.e. investment decisions. Under the assumptions on \( F \) and Assumptions 1 and 2, this can be shown to be without loss of optimality.
Let $U_x$ be the agent’s expected payoff from the contract $(q_x(\cdot), t_x(\cdot))$ gross of investment cost. The joint condition of incentive compatibility and individual rationality can then be expressed as

$$U_x - \kappa x \geq \max \left\{ U_{x'} + \int_0^1 (c_{x'}(\varepsilon) - c_{x''}(\varepsilon)) q_{x'}(\varepsilon) d\varepsilon - \kappa x'', 0 \right\}$$

(7)

for $\kappa \in K_x$ and $x, x', x'' \in \{0, 1\}$. One can characterize this condition as follows.

**Lemma 1.** A menu of contracts is incentive compatible and individual rational under monitoring of the shock if and only if for some $\hat{\kappa} \in [\kappa, \bar{\kappa}]$

1. $K_1 = [\kappa, \hat{\kappa}]$, $K_0 = (\hat{\kappa}, \bar{\kappa}]$, and $U_1 - U_0 = \hat{\kappa}$;
2. $U_0 \geq 0$;
3. $\int_0^1 (c_0(\varepsilon) - c_1(\varepsilon)) q_0(\varepsilon) d\varepsilon \leq \hat{\kappa}$;
4. $\int_0^1 (c_0(\varepsilon) - c_1(\varepsilon)) q_1(\varepsilon) d\varepsilon \geq \hat{\kappa}$.

If the agent optimally invests at some level of investment cost, then it is still optimal to invest if the investment costs are lower. This implies condition 1. Condition 2 guarantees individual rationality: Independently of the level of investment cost, the agent can always achieve a net benefit of $U_0$. If $U_0$ is better than the outside option, the agent always accepts one of the contract offers. Under condition 3, the agent has no incentive to choose the contract aimed at non-investing agents and to invest nevertheless. Conversely, under condition 4, it is unprofitable to pick the contract for investing agents and to abstain from investment.

By condition 1 in Lemma 1, the expected payoff of the principal can be expressed as in equation (4). The optimal menu of contracts for the principal is therefore the solution of the following problem:

$$\max_{U_0, \hat{\kappa}, (q_x(\cdot))_{x \in \{0, 1\}}} \bar{\Pi} \quad \text{s.t. conditions 2 to 4 in Lemma 1}$$

(8)

Note that this problem is equivalent to the principal’s problem under monitoring of investment and shock with the additional constraints 3 and 4. The principal can therefore not achieve a higher payoff (gross of monitoring costs) than under monitoring of investment. However she
achieves the same payoff if conditions 3 and 4 are not binding. The optimal investment and trading rules from the benchmark with observable investment always satisfy condition 4, and satisfy condition 3 if the distortion away from the first best investment decision is not too large:

**Lemma 2.** Under monitoring of the shock, the principal can implement the investment decision characterized by the threshold $\kappa^i$ and efficient production with the same expected transfers as under monitoring of investment if and only if

$$\kappa^* - \kappa^i \leq \int_{\varepsilon_0^*}^{\varepsilon_1^*} (v - c_1(\varepsilon))d\varepsilon. \quad (9)$$

If this condition is violated, moral hazard is relevant. In this case, the unobservability of the investment decision adds agency costs to the principal’s problem. This is the case, if the best deviation of an agent with low investment costs – i.e. $\kappa \in K_1$ – is to choose the contract $(q_0(\cdot), t_0(\cdot))$ and to invest nevertheless. Under efficient production, the benefit of this deviation (gross of investment cost) is given by the area $A$ in Figure 1. $A$ represents the expected cost savings that the agent can keep for himself when deviating. If this area is smaller than the rent $\kappa^i$ that the agent receives in the optimal contract under monitoring of investment, then moral hazard is irrelevant. In equation (9), this inequality is reformulated – using that the sum of the areas $A$ and $B$ in Figure 1 equal $\kappa^*$. The reformulation shows that moral hazard is irrelevant if underinvestment in the optimal contract under monitoring of investment is not too large.

If moral hazard is relevant, the principal could still implement efficient production. In this
case, the principal would have to give the investing agent a rent equal to the area $A$ in Figure 1. This would imply an investment threshold equal to the area $A$ and therefore higher than the optimal threshold under monitoring of investment. The principal can increase her payoff by reducing the probability of production for the agent who does not invest. As illustrated in Figure 2, this allows the principal to profitably reduce the rent of the investing agent to the area $A'$. The investment cost threshold is also affected because the agent optimally invests as long as the investment costs are smaller than the area $A'$. Note that it is never beneficial for the principal to push the threshold down to $\kappa^i$, the optimal threshold under monitoring of investment. At this threshold, the principal is indifferent between trading efficiently with an investing agent or with a non-investing agent. However, production with non-investing agents has to be inefficient under the threshold $\kappa^i$ if moral hazard is relevant. The principal therefore prefers a strictly higher threshold.

The optimal threshold is also strictly smaller than the efficient investment threshold $\kappa^*$. This follows from the observation that the area $A'$ in Figure 2 is smaller than the area $A$ in Figure 1, whereas the efficient investment threshold equals the sum of the areas $A$ and $B$ in Figure 1. It follows that the optimal contract under shock monitoring and relevant moral hazard includes inefficient production by non-investing agents and a smaller distortion in the investment decision than in the optimal contract with monitoring of investment.

In order to characterize the optimal contract and state the result formally, it is helpful to define – for any given incentive compatible and individual rational menu of contracts that implements an investment threshold $\tilde{\kappa}$ – the principal’s gain from investment by the agent with investment cost $\kappa \leq \tilde{\kappa}$ as

$$G(q_1(\cdot), q_0(\cdot), \tilde{\kappa}) \equiv \int_0^1 (v - c_1(\varepsilon))q_1(\varepsilon)d\varepsilon - \int_0^1 (v - c_0(\varepsilon))q_0(\varepsilon)d\varepsilon. \quad (10)$$

If moral hazard is relevant, condition 3 of Lemma 1 is not satisfied with efficient production whereas condition 4 is satisfied. In the optimal contract, condition 3 is therefore satisfied with equality and the investing agent receives a rent (gross of investment cost) given by

$$U_1^*(q_0(\cdot)) \equiv \int_0^1 (c_0(\varepsilon) - c_1(\varepsilon))q_0(\varepsilon)d\varepsilon. \quad (11)$$
This rent is equal to the investment threshold by condition 1 in Lemma 1. With slight abuse of notation, I denote by $U_1^s(\varepsilon_0)$ the agent’s rent for $q_0(\varepsilon) = 1(\varepsilon \leq \varepsilon_0)$, and by

$$u_1^s(\varepsilon_0) = c_0(\varepsilon_0) - c_1(\varepsilon_0)$$

(12)

the first derivative. $u_1^s(\varepsilon_0)$ is the marginal change in the agent’s rent if the good is produced for a shock of size $\varepsilon_0$. The principal maximizes the virtual surplus which is the difference between the social surplus and the rent of the agent. Under the following assumption the virtual surplus is decreasing in the cost shock.

**Assumption 1.** $v - c_0(\varepsilon) - (F(\kappa^s)/(1 - F(\kappa^s))) \cdot u_1^s(\varepsilon)$ is decreasing in $\varepsilon$.\(^{10}\)

The principal then chooses the following menu of contracts under monitoring of the shock.

**Proposition 2.** If the shock is monitored and Assumption 1 is satisfied, the principal achieves an optimal payoff $\Pi^s$ through the menu of contracts $\{(q_1^s(\cdot), t_1^s(\cdot)), (q_0^s(\cdot), t_0^s(\cdot))\}$ and the investment threshold $\kappa^s$:

1. If moral hazard is irrelevant, then production is efficient and the investment threshold is the same as with monitoring of investment: $q_1^s(\cdot) = q_1^s(\cdot)$ for $x \in \{0, 1\}$, and $\kappa^s = \kappa^i$.

2. If moral hazard is relevant, then production is efficient if the agent invests. If the agent does not invest, there is underproduction. The investment threshold is lower than efficient and higher than with monitoring of investment:

$$q_1^s(\cdot) = q_1^s(\cdot) \quad and \quad q_0^s(\varepsilon) = 1(\varepsilon \leq \varepsilon_0^s) \quad with \quad \varepsilon_0^s < \varepsilon_0^s;$$

(13)

$$\kappa^s = \int_{0}^{\varepsilon_0^s} (c_0(\varepsilon) - c_1(\varepsilon))d\varepsilon \in (\kappa^i, \kappa^*);$$

(14)

$$(1 - F(\kappa^s))(v - c_0(\varepsilon_0^s)) + f(\kappa^s)G(q_1^s(\cdot), q_0^s(\cdot), \kappa^s)u_1^s(\varepsilon_0^s) = F(\kappa^s)u_1^s(\varepsilon_0^s).$$

(15)

In both cases, optimal transfers satisfy $\int_0^1 t_1^s(\varepsilon)d\varepsilon = \int_0^1 (v - c_x(\varepsilon))q_2^s(\varepsilon)d\varepsilon - x \cdot \kappa^s$ for $x \in \{0, 1\}$.

The principal’s trade-off can be seen from equation (15): The left hand side captures the marginal beneficial effects on the principal’s payoff when the good is procured for the shock

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\(^{10}\)This assumption is similar to standard assumptions made in the literature on sequential screening (Courty and Li, 2000). It is satisfied if $\kappa^s$ is small enough.
and the right hand side represents the marginal adverse effects. The first term on the left
hand side is the marginal increase in social surplus. The term on the right hand side is the
marginal increase in the agent’s rent. The marginal effect on gross rents increases the fraction
of agents who invest. This effect is beneficial for the principal and is captured by the second
term on the left hand side. In contrast to a standard adverse selection problem with an efficient
and an inefficient type, the fraction of the efficient, i.e. investing, agents is endogenous to the
mechanism.

5 No monitoring

In this section, I suppose that the principal monitors neither the investment nor the shock.
Thus, the principal wants to elicit information about investment costs and about the shock
from the agent. At the same time, the optimal contract needs to provide incentives to take
the ‘right’ investment decision. I show that the principal achieves a strictly lower payoff than
under monitoring of investment or under monitoring of the shock. Both, moral hazard concern-
ing the investment decision and post-contractual adverse selection concerning the cost shock
are therefore always relevant in this case. The optimal contract induces underinvestment and
underproduction of non-investing agents.

The principal offers a menu of two contracts. The first contract targets the agent who
makes the investment, the second contract is to be chosen by the agent who does not invest.
Both contracts specify a probability of production \( q_x(\varepsilon') \) and an expected transfer \( t_x(\varepsilon') \) as
functions of a report \( \varepsilon' \) about the shock for both investment decisions \( x \in \{0, 1\} \).

\[ K_x \] be the
set of investment cost values for which the agent takes the decision \( x \) in equilibrium. Incentive
compatibility regarding the shock requires that a agent who has made the investment decision
\( x \) and has chosen the appropriate contract reports \( \varepsilon \) truthfully:

\[
t_x(\varepsilon) - c_x(\varepsilon)q_x(\varepsilon) \geq t_x(\varepsilon') - c_x(\varepsilon)q_x(\varepsilon') \tag{16}
\]

for all \( \varepsilon, \varepsilon' \in [0, 1] \) and \( x \in \{0, 1\} \). Incentive compatibility regarding the whole menu of contracts
requires that the agent with investment cost in \( K_x \) chooses the investment decision \( x \), the

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\[ ^{11} \] The same comment as in footnote 9 applies.
contract \((q_x(\cdot), t_x(\cdot))\), and reports \(\varepsilon\) truthfully. The menu of contracts is individually rational if the agent always prefers one of the contracts to his outside option. The joint condition of incentive compatibility and individual rationality under no monitoring can be expressed as

\[
\int_0^1 (t_x(\varepsilon) - c_x(\varepsilon)q_x(\varepsilon)) d\varepsilon - \kappa x \geq \max \left\{ \int_0^1 \max_{\varepsilon' \in [0,1]} (t_{x'}(\varepsilon') - c_{x'}(\varepsilon)q_{x'}(\varepsilon')) d\varepsilon - \kappa x', 0 \right\}
\]

(17)

for all \(\kappa \in K_x, x, x', x'' \in \{0, 1\}\). For the characterization of this constraint, it is helpful to make the following observation. An agent who has chosen the contract for the investment decision \(x\) but has made the decision \(x'\) makes a false report about the shock: The false report is optimally chosen such that the principal has a correct belief about production costs.

**Lemma 3.** Incentive compatibility regarding the shock implies

\[
c_x^{-1}(c_{x'}(\varepsilon)) \in \arg \max_{\varepsilon' \in [0,1]} t_x(\varepsilon') - c_{x'}(\varepsilon)q_x(\varepsilon')
\]

for all \(x, x' \in \{0, 1\}\) and all \(\varepsilon \in [0, 1]\).

Using this result, incentive compatibility and individual rationality can be characterized.

**Lemma 4.** A menu of contracts is incentive compatible and individual rational under no monitoring if and only if for some \(\hat{\kappa} \in [\kappa, \overline{\kappa}]\)

1. \(K_1 = [\kappa, \overline{\kappa}], K_0 = (\hat{\kappa}, \overline{\kappa}],\) and \(U_1 - U_0 = \hat{\kappa};\)
2. \(U_0 \geq 0;\)
3. \(\int_0^1 c_0'(\varepsilon)q_0(\varepsilon)(c_0^{-1}(c_0(\varepsilon)) - \varepsilon) d\varepsilon \leq \hat{\kappa};\)
4. \(\int_0^1 c_1'(\varepsilon)q_1(\varepsilon)(\varepsilon - c_0^{-1}(c_1(\varepsilon))) d\varepsilon \geq \hat{\kappa};\)
5. \(q_x(\varepsilon)\) is decreasing in \(\varepsilon\) and

\[
t_x(\varepsilon) = c_x(\varepsilon)q_x(\varepsilon) + t_x(1) - c_x(1)q_x(1) + \int_\varepsilon^1 c_x'(z)q_x(z) dz.
\]

Conditions 3, 4, and 5 differ from Lemma 1. Conditions 3 and 4 reflect that the agent optimally lies about the shock on a deviation path where the agent takes a different investment decision than in equilibrium. Condition 5 follows from standard monotonicity and revenue
equivalence requirements that are necessary and sufficient for incentive compatibility regarding
the report of the shock.

Due to condition 1, the principal’s payoff from an incentive compatible menu of contracts
can be expressed as in equation (4). The principal’s problem is then to

\[
\max_{U_0, \hat{\kappa}, (q_x(\cdot))_{x \in \{0, 1\}}} \Pi \quad \text{s.t. conditions 2 to 5 in Lemma 4.} \tag{18}
\]

It now arises the question whether the principal can still implement the investment and pro-
duction decisions from the optimal contracts under monitoring of investment and monitoring
of the shock. This turns out to be impossible.

**Lemma 5.** Under no monitoring, efficient production \(q_x^*(\cdot)\) for \(x \in \{0, 1\}\) is incentive compatible
only if the investment threshold is efficient: \(\hat{\kappa} = \kappa^*\). Furthermore the optimal investment and
production decisions under monitoring of the shock \(\{q_1^*(\cdot), q_0^*(\cdot), \kappa^*\}\) do not satisfy the joint
condition of incentive compatibility and individual rationality under no monitoring.

If the principal offers a contract that stipulates efficient production, incentive compatibility
requires that the agent is willing to produce after both investment decisions as long as \(c_x(\varepsilon) \leq v\).
This implies that an agent who deviates by choosing the menu \((q_0(\cdot), t_0(\cdot))\) and the investment
decision \(x = 1\), finds it optimal to make a report \(\varepsilon' \leq \varepsilon_0^*\) and to induce production as long as
\(c_1(\varepsilon) \leq v\), i.e. as long as production is efficient. On this deviation path, the agent receives
the whole social surplus generated through the investment. This corresponds to the sum of the
areas A and B in Figure 1. The principal could still implement efficient production but has to
leave a rent equal to \(A + B\) to an investing agent. Therefore the agent would invest as long as his
investment cost lies below the efficient investment cost threshold \(\kappa^*\). Suppose now the principal
were to implement the production decisions from the optimal contract under monitoring of the
shock. Under no monitoring, this would result in a strictly higher investment threshold than
under monitoring of the shock. Figure 3 illustrates this result: Given the production threshold
\(\varepsilon_0\), investing agents receive a rent (gross of investment costs) equal to the area \(A'\) if the principal
monitors the shock. If the principal does not monitor, the rent increases to the areas \(A'\) and
\(B'\). The optimal investment threshold under monitoring of the shock is therefore not feasible
anymore.
In the optimal contract under no monitoring, the principal introduces underproduction for non-investing agents. As illustrated in Figure 3, this reduces the information rent of an investing agent to the sum of the areas $A'$ and $B'$. The investment cost threshold equals the sum of these areas and therefore lies below the efficient threshold $\kappa^*$. However, it is never optimal for the principal to push the investment threshold below the optimal level with monitoring of investment $\kappa^i$, as production by non-investing agents is inefficient.

It follows that condition 3 in Lemma 4 is satisfied with equality in the optimal menu of contracts. The agent’s rent gross of investment cost is therefore given by

$$U^n_1(q_0(\cdot)) = \int_0^1 c_0'(\varepsilon) q_0(\varepsilon) (c_1^{-1}(c_0(\varepsilon)) - \varepsilon) d\varepsilon.$$  \hspace{1cm} (19)

I denote by $U^n_1(\varepsilon_0)$ the agent’s rent for $q_0(\varepsilon) = 1(\varepsilon \leq \varepsilon_0)$, and by

$$u^n_1(\varepsilon_0) = c_0'(\varepsilon) (c_1^{-1}(c_0(\varepsilon)) - \varepsilon)$$  \hspace{1cm} (20)

the first derivative. I make an assumption that ensures that the principal benefits from a lower shock in the optimal contract. Technically speaking, this assumption ensures that the virtual surplus is decreasing in the shock $\varepsilon$.

**Assumption 2.** \( v - c_0(\varepsilon) - (F(\kappa^*)/(1 - F(\kappa^*))) \cdot u^n_1(\varepsilon) \) is decreasing in $\varepsilon$.\textsuperscript{12}

\textsuperscript{12}Like Assumption 1, this is satisfied if $\kappa^*$ is small enough.
The principal offers the following menu of contracts.

**Proposition 3.** If there is no monitoring and Assumption 2 is satisfied, the principal achieves an optimal payoff $\Pi^n$ through the menu of contracts $\{(q^n_i(\cdot), t^n_i(\cdot)), (q^n_0(\cdot), t^n_0(\cdot))\}$ and the investment threshold $\kappa^n$: If the agent invests, production is efficient. If the agent does not invest, there is underproduction. The investments threshold is lower than efficient and higher than with observable investment. For $x \in \{0, 1\}$

\[
q^n_i(\cdot) = q^n_1(\cdot) \quad \text{and} \quad q^n_0(\varepsilon) = 1(\varepsilon \leq \varepsilon_0^n) \quad \text{with} \quad \varepsilon_0^n < \varepsilon_0^*; \tag{21}
\]

\[
\kappa^n = \int_0^{\varepsilon_0^n} c_0(\varepsilon)(c^{-1}_1(\varepsilon_0(\varepsilon)) - \varepsilon) d\varepsilon \in (\kappa^i, \kappa^s); \tag{22}
\]

\[
(1 - F(\kappa^n))(v - c_0(\varepsilon_0^n)) + f(\kappa^n)G(q^n_i(\cdot), q^n_0(\cdot), \kappa^n)u^n_i(\varepsilon_0^n) = F(\kappa^n)u^n_1(\varepsilon_0^n); \tag{23}
\]

\[
t^n_x(\varepsilon) = c_x(\varepsilon)q^n_x(\varepsilon) + \int_\varepsilon^1 c'_x(z)q^n_x(z)dz - \int_0^1 zc'_x(z)q^n_x(z)dz + \kappa^n \cdot x. \tag{24}
\]

Equation (23) reflects a very similar trade-off as equation (15) for the optimal contract under monitoring of the shock. On the left hand side are the marginal beneficial effects of production by non-investing types at shock $\varepsilon$: a marginal increase in efficiency and the marginal gain from the increase in the fraction of investing agent types. The right hand side reflects the marginal increase in rent payments that have to be given to the agent. In the next section I analyze the implications for the principal’s optimal monitoring policy. Assumptions 1 and 2 are maintained throughout the section.

### 6 The optimal monitoring policy

It is straightforward to derive the implications for the optimal choice of a monitoring policy.

**Proposition 4.** The principal’s optimal monitoring policy is given by:

- **no monitoring** if $C^i \geq \Pi^i - \Pi^n$ and $C^s > \Pi^s - \Pi^n$;

- **monitoring of the shock** if $C^s \leq \min \{C^i + \Pi^s - \Pi^i, \Pi^s - \Pi^n\}$;

- **monitoring of the investment** if $C^i < \min \{C^s + \Pi^i - \Pi^s, \Pi^i - \Pi^n\}$.

There exist monitoring costs $C^s > 0$ and $C^s > 0$ such that any of these cases can occur.
In order to analyze the effect of the level of fixed cost of investment on the optimal monitoring policy, I introduce the family of distribution functions \( \{ F_z(\kappa) \}_{z \in (z, Z)} \) of the investment cost which satisfies \( F_z(\kappa) = F(\kappa - z) \) for all \( \kappa \) and \( z \). Furthermore \( F_z(\kappa^*) = 1 \) and \( F_z(\kappa^*) = 0 \). These distributions functions are therefore generated by moving the support of the distribution \( F \). A higher \( z \) corresponds to a higher level of investment cost.

**Proposition 5.** If the level of fixed costs of investment is high, the principal can achieve the same payoff gross of monitoring costs under monitoring of the shock and monitoring of the investment decision. For a low level of fixed costs of investment, the principal achieves a higher gross payoff under monitoring of the investment, i.e. \( \exists z' \in (z, Z) \), such that \( \Pi^i = \Pi^s \) for \( z \geq z' \) and \( \Pi^i > \Pi^s \) for \( z < z' \).

Figure 4 illustrates the results from the two propositions.

The principal optimally monitors either the investment or the shock. This builds on the one hand on Proposition 1: If the principal can control the investment decision through monitoring, there is no additional gain from observing the shock. On the other hand, the result hinges on the fact that the principal can achieve a strictly higher payoff under monitoring of the shock compared to no monitoring. Under no monitoring, the principal has to give a positive rent to the agent for the information about the cost shock, even though the agent learns the shock only after contracting. This result arises due to the fixed cost of investment: If the principal
does not monitor, the best deviation of the agent combines the choice of the contract aimed at
a non-investing agent with a positive investment and a lie about the cost shock. As Lemma
3 shows, this lie is strict, due to the fact that the investment on the deviation path is strictly
different from zero. The fixed costs of investment therefore create the motive to monitor the
shock. In the next section, I relate this result to the discussion on post-contractual information
rent in the literature on dynamic mechanism design.

It then arises the question on the relative performance of monitoring of the investment
decision relative to monitoring of the shock. It turns out that – gross of monitoring costs – the
principal achieves never a higher payoff under monitoring of the shock relative to monitoring
of investment. However, if fixed costs of investment play an important role, monitoring of the
shock is as effective as monitoring of investment. The principal therefore simply chooses the
monitoring technology with lower costs in that case. As the results in section 4 show, both
monitoring technologies are equally effective, if moral hazard is irrelevant. If the level of fixed
costs of investment is high and investment is monitored, the principal chooses an investment
threshold close to the efficient threshold as the agent invests only with a small probability. If
the principal monitors the cost shock instead of the investment, an agent with low investment
costs cannot profitably deviate by choosing the contract for non-investing agents and investing
nevertheless. On this deviation, the agent would forego the large information rent that ensures
that agents with low investment cost choose to invest. Moral hazard is therefore irrelevant and
monitoring of the shock is as effective as monitoring of investment if the level of fixed costs of
investment is high.

7 Discussion

Efficiency

The optimal contracts under the different monitoring policies can often not be ranked unam-
biguously in terms of efficiency. If a principal moves from no monitoring to monitoring of
investment, efficiency of production increases whereas efficiency of investment decreases. The
optimal contracts under monitoring of investment and monitoring of the shock lead either to
identical investment and production decisions, or imply lower efficiency of investment and higher
efficiency of production under monitoring of investment.
Figure 5: The beneficial effect of privacy of information on efficiency

However, a comparison in terms of efficiency is possible for the optimal contracts under monitoring of the shock and no monitoring in some cases.

**Proposition 6.** If the principal’s value for the good is high, then investment and production are more efficient under no monitoring than under monitoring of the shock: There exists \( \hat{v} \in (\underline{v}, \overline{v}) \) such that \( \varepsilon^s_0 < \varepsilon^n_0 \) and \( \kappa^s < \kappa^n \) for \( v > \hat{v} \).

Perhaps surprisingly, private learning of the shock increases total efficiency, if the value of the good is high. Privacy of post-contractual information may therefore increase the efficiency in optimal contracts.

In the optimal contracts under both monitoring of the shock and no monitoring, the principal reduces information rent of the investing agent by trading less frequently with the non-investing agent. The effect of a small reduction in the probability of production on the information rent differs between the two cases. Proposition 6 uses that the marginal effect on rents is smaller under no monitoring for high realizations of the shock. This is illustrated in Figure 5: When reducing the production threshold for the non-investing agent from the efficient level \( \varepsilon^s_0 \) to the smaller level \( \varepsilon_0 \), the agent’s rent decreases from \( A \) to \( A' \) under monitoring of the shock. If the principal does not monitor, the rent decreases from \( A + B \) to \( A' + B' \). Note that for high values of \( v \), \( B' \) is larger than \( B \). The reduction of the production threshold is therefore less effective in reducing information rents under no monitoring than under monitoring of the shock. Thus, the principal finds it optimal to induce a smaller production distortion under no monitoring. The
agent therefore receives a higher total value of information rent under no monitoring, which implies a higher investment threshold. It follows that the optimal contract is more efficient under no monitoring.

Robustness

All results were presented using a simple model with a binary investment decision. The results do not hinge on this assumption and can easily be extend to a more realistic setup. Åstebro (2004) presents empirical evidence for fixed costs of learning that have to be borne by firms that want to adopt a new technology, independently of the extent to which these technologies are applied in the firms’ production process later on. The model presented in this paper could be extended to capture the extent of technology adoption:

Let the investment $x$ be chosen from the interval $[0, 1]$ to represent the extent to which a new technology is adopted. This results in production costs $c_x(\varepsilon)$ with $c_x(\varepsilon) < c_{x'}(\varepsilon)$ for all $\varepsilon \in (0, 1)$ whenever $x > x'$, and $c_x(0) = \underline{c}$ and $c_x(1) = \overline{c}$ for all $x$. For any positive $x$, the agent has to bear a privately observable fixed cost of learning $\kappa$ and variable implementation costs $k(x)$, which is continuous, increasing, and satisfies $k(0) = 0$. I make the same assumptions on the distribution of $\kappa$ as before. For $k(\cdot) = 0$, this model is equivalent to the binary investment decision studied in the main text. In the optimal contracts under all monitoring policies, the agent produces with the old technology ($x = 0$) if the costs of learning are above a threshold, and adopts the new production technology to a strictly positive extent $x^* > 0$, if the fixed costs of learning are below a threshold. If the principal monitors the shock and moral hazard is relevant, the optimal deviation strategy of the agent with low investment costs consists in choosing the contract for the non-investing agent and adopting the production technology to a positive extent $\hat{x} > 0$. Under no monitoring, the agent optimally deviates by choosing the contract for the non-investing agent, adopting the production technology to a positive extent $\tilde{x} > 0$, followed by a strict lie about the cost shock. In this extension of the model, all implications for the optimal monitoring policy remain the same as in the original model.
Relevance of private post-contractual information

In the literature on dynamic mechanism design, a central question concerns the relevance of privacy of post-contractual information. Baron and Besanko (1984), Eső and Szentes (2007a,b) and Pavan et al. (2014) show that in many setups of dynamic adverse selection, the principal can costlessly elicit private information which the agent learns after contracting. Baron and Besanko (1984) and Eső and Szentes (2013) argue that this insight also holds in models of dynamic adverse selection and moral hazard.

In contrast, Krähmer and Strausz (2015) show in a model of pure adverse selection, that privacy of post-contractual information matters if the agent’s private information learned before contracting (ex-ante type) is drawn from a discrete set. In this paper, privacy of post-contractual information turns out to be relevant in a model of dynamic adverse selection and moral hazard. Whereas the agent’s private information, i.e. the fixed cost of investment and the cost shock, are drawn from continuous distributions, the agent chooses an unobservable action from a discrete set. As in Krähmer and Strausz (2015), the agent’s best deviation strategy under no monitoring includes a strict lie about post-contractual information. As such a deviation is not feasible if post-contractual information is publicly observed, privacy of post-contractual information is relevant. However, there are a two notable differences to the result in Krähmer and Strausz (2015). First, the envelope theorem (Theorem 2 in Milgrom and Segal (2002)) could be applied in my setup in order to determine the agent’s expected utility as a function of his ex-ante type. This is not feasible in Krähmer and Strausz (2015) due to the discreteness of the agent’s ex-ante type. Second, Krähmer and Strausz (2015) show that the principal’s optimal allocation under observable post-contractual information can still be implemented if ex-post information is unobservable, however at a lower revenue. In this paper, and as shown in Lemma 5, the principal cannot implement her optimal allocation under monitoring of the shock – consisting of the investment and production decisions – if she does not monitor. Privacy of post-contractual information restricts the set of implementable allocations for the principal, but does not change the principal’s ability to extract surplus from a given allocation as in Krähmer and Strausz (2015). This also explains the relation to Eső and Szentes (2013). They show in a general model of dynamic adverse selection and moral hazard that the principal can achieve the same revenue under public and private post-contractual information, if the optimal allocation and
unobservable actions under public post-contractual information are still implementable if post-contractual information is private. In the model presented in this paper, the principal cannot implement the optimal production and investment decisions with monitoring of the shock, if there is no monitoring. The result of Eső and Szentes (2013) does therefore not apply in this case.

As I argue above, these results does not hinge on the assumption of a discrete set of investment options, but also arise under the natural assumption of fixed costs of investment and a continuous set of investment options. Fixed costs of investment lead to discrete jumps of the agent’s investment decision in equilibrium and make privacy of post-contractual information relevant.

8 Conclusion

What kind of information should government authorities try to observe when they procure goods and services from private suppliers? This paper provides an analysis of this question in a principal-agent model where the principal can decide whether to monitor the agent’s investment and/or a cost shock which the agent learns after contracting. I show that the principal optimally monitors either the investment decision or the cost shock. Fixed costs of investment create the motive to monitor the cost shock. Under monitoring of investment, the principal achieves at least the same payoff – gross of monitoring costs – as under monitoring of the shock. If the level of fixed costs of investment is high, both monitoring technologies are equally effective. Monitoring can have negative effects on the efficiency of the interaction between principal and agent. If the value of the good is high, investment and production decisions are more efficient if the principal does not monitor compared to the case of shock monitoring.
9 Appendix

Proof of Proposition 1 The proof follows from the discussion in the main text.

Proof of Lemma 1 Condition 1 is equivalent to (7) for \( x = 1, x' = x'' = 0 \), and for \( x = 0, x' = x'' = 1 \). Condition 2 is equivalent to (7) for \( x = x' = x'' = 0 \). Conditions 3 and 4 are implied by condition 1, (7) for \( x = x'' = 1, x' = 0 \), and (7) for \( x = x'' = 0, x' = 1 \). Conversely, conditions 1, 3, and 4 imply (7) for \( x = x' = x'' = 1 \). It remains to prove that conditions 1 to 4 imply (7) for \( x = x' = 1, x'' = 0 \), and for \( x = x' = 0, x'' = 1 \). The first constraint can be rewritten as

\[
\int_0^1 (c_0(\varepsilon) - c_1(\varepsilon)) q_1(\varepsilon) d\varepsilon \geq \kappa
\]

for \( \kappa \in K_1 \). By condition 1, \( \kappa \leq \hat{\kappa} \) for \( \kappa \in K_1 \), and the constraint is implied by condition 4. The second constraint can be written as

\[
\int_0^1 (c_0(\varepsilon) - c_1(\varepsilon)) q_0(\varepsilon) d\varepsilon \leq \kappa
\]

for \( \kappa \in K_0 \). By condition 1, \( \kappa \geq \hat{\kappa} \) for \( \kappa \in K_0 \). The constraint is therefore implied by condition 3.

Proof of Lemma 2 Plug \( \hat{\kappa} = \kappa^i \) and efficient production rules in conditions 3 and 4 of Lemma 1. The left hand side of condition 3 can be rewritten as

\[
\int_0^{\varepsilon_0^i} (c_0(\varepsilon) - c_1(\varepsilon)) d\varepsilon = \int_0^{\varepsilon_1^i} (v - c_1(\varepsilon)) d\varepsilon - \int_0^{\varepsilon_0^i} (v - c_0(\varepsilon)) d\varepsilon - \int_{\varepsilon_0^i}^{\varepsilon_1^i} (v - c_1(\varepsilon)) d\varepsilon
\]

\[
= \kappa^* - \int_{\varepsilon_0^i}^{\varepsilon_1^i} (v - c_1(\varepsilon)) d\varepsilon
\]

Condition 3 is therefore satisfied if the condition in the lemma is satisfied. The left hand side of condition 4 can by the same steps be written as \( \kappa^* - \int_{\varepsilon_0^i}^{\varepsilon_1^i} (v - c_0(\varepsilon)) d\varepsilon \) which is greater than \( \kappa^* \). Condition 4 is therefore always satisfied. Note that conditions 3 and 4 do not restrict the choice of \( U_0 \) and \( U_1 \). One can therefore set \( U_0 \) and \( U_1 \) as in the optimal contract under monitoring of the investment, so that expected transfers are identical in both cases.
Proof of Proposition 2  If moral hazard is irrelevant, the result is immediate. Suppose moral hazard is relevant and consider the principal’s problem (8). Neglect condition 4 of Lemma 1. It is optimal to set \( U_0 = 0 \) and \( q_1(\cdot) = q_1^*(\cdot) \).

I show now that the optimal threshold \( \kappa^* \) satisfies \( \kappa^* \in (\kappa^i, \kappa^s) \). Suppose \( \kappa^s \leq \kappa^i \). As moral hazard is relevant, \( q_0(\cdot) = q_0^* (\cdot) \) is not feasible. It follows that the marginal gain of an additionally investing agent type exceeds the marginal information rent: \( G(q_1^*, q_0, \kappa^s) - F(\kappa^s)/f(\kappa^s) > G(q_1^*, q_0^*, \kappa^i) - F(\kappa^i)/f(\kappa^i) = 0 \). It is therefore profitable to increase \( \kappa^s \) which also relaxes condition 3. It follows that \( \kappa^s > \kappa^i \). Suppose next \( \kappa^s \geq \kappa^s \). The proof of Lemma 2 implies that \( q_0(\cdot) = q_0^* (\cdot) \) is feasible. The marginal gain of an additionally investing agent type is then lower as the marginal information rent: \( G(q_1^*, q_0, \kappa^s) - F(\kappa^s)/f(\kappa^s) = \kappa^s - \kappa^s - F(\kappa^s)/f(\kappa^s) \leq -F(\kappa^s)/f(\kappa^s) < 0 \). Decreasing \( \kappa^s \) is profitable. It follows \( \kappa^s < \kappa^i \).

Furthermore note that \( G(q_1^*, q_0, \kappa^s) < F(\kappa^s)/f(\kappa^s) \) in any optimum. If this does not hold, it is always profitable to increase \( \kappa^s \) and set \( q_0 \) closer to \( q_0^* \).

Moreover, condition 3 of Lemma 1 is satisfied with equality at the optimum. If condition 3 is satisfied with strict inequality, it is possible to set \( q_0(\cdot) \) closer to \( q_0^*(\cdot) \) and increase the payoff.

One can therefore write the threshold \( \hat{\kappa} \) as a function of \( q_0^*(\cdot) \). Plugging this into (4) and taking the pointwise first order derivative with respect to \( q_0 \) for any \( \varepsilon \) gives

\[
f(\hat{\kappa}(q_0)) \left\{ \frac{1 - F(\hat{\kappa}(q_0))}{f(\hat{\kappa}(q_0))} (v - c_0(\varepsilon)) - \left( \frac{F(\hat{\kappa}(q_0))}{f(\hat{\kappa}(q_0))} - G(q_1^*, q_0, \hat{\kappa}(q_0)) \right) u_1^*(\varepsilon) \right\}.
\]

(25)

For any fixed threshold \( \hat{\kappa} \) that satisfies \( \hat{\kappa} \in (\kappa^i, \kappa^s) \) and \( G(q_1^*, q_0, \hat{\kappa}) < F(\hat{\kappa})/f(\hat{\kappa}) \), this expression is decreasing in \( \varepsilon \) under Assumption 1. If \( u_1^*(\varepsilon) \) is increasing, this follows from \( G(q_1^*, q_0, \hat{\kappa}) < F(\hat{\kappa})/f(\hat{\kappa}) \). If \( u_1^*(\varepsilon) \) is decreasing, it follows from Assumption 1 as \( F(\kappa^s)/(1-F(\kappa^s)) > F(\hat{\kappa})/(1-F(\hat{\kappa})) \cdot G(q_1^*, q_0, \hat{\kappa}) \). Thus, the threshold \( \hat{\kappa} \) is optimally implemented by a step function \( q_0(\varepsilon) = 1(\varepsilon \leq \varepsilon_0) \). Since \( \kappa^s \) satisfies \( \kappa^s \in (\kappa^i, \kappa^s) \) and \( G(q_1^*, q_0, \kappa^s) < F(\kappa^s)/f(\kappa^s) \), there exists a cutoff \( \varepsilon_0^* \) which optimally implements \( \kappa^s \) and this implies (14). There is a unique combination of \( \kappa^s \) and \( \varepsilon_0^* \) which equates the first order derivative to zero and satisfies (15). This follows from \( \kappa^s \) being increasing in \( \varepsilon_0^* \), \( (1-F(\hat{\kappa}))/f(\hat{\kappa}) \) being increasing in \( \kappa^s \), and \( G(q_1^*, q_0^*, \kappa^s) - F(\kappa^s)/f(\kappa^s) \) being increasing in \( \varepsilon_0^* \) and \( \kappa^s \), for \( q_0^*(\varepsilon) = 1(\varepsilon \leq \varepsilon_0^*) \).

It remains to show that \( \varepsilon_0^* < \varepsilon_0^* \). For \( \varepsilon_0^* \geq \varepsilon_0^* \), there is no first-order loss from decreasing \( \varepsilon_0^* \) whereas there is a first order gain from a lower fraction of investing agent types as \( F(\kappa^s)/f(\kappa^s) - \).
$G(q_1^0, q_0^0, \kappa^*) > 0$. Finally, one can easily check that condition 4 of Lemma 1 is satisfied as $\kappa^* < \kappa^*$. Optimal transfers can be derived from the definitions of $U_1$ and $U_0$.

**Proof of Lemma 3** For $\tilde{\epsilon}(\epsilon) = c_x^{-1}(c_{x'}(\epsilon))$, (16) implies

$$t_x(\epsilon') - c_{x'}(\epsilon)q_x(\epsilon) = t_x(\epsilon') - c_x(c_x^{-1}(c_{x'}(\epsilon)))q_x(\epsilon')$$

$$\leq t_x(c_x^{-1}(c_{x'}(\epsilon))) - c_x(c_x^{-1}(c_{x'}(\epsilon)))q_x(c_x^{-1}(c_{x'}(\epsilon)))$$

$$= t_x(\tilde{\epsilon}(\epsilon)) - c_{x'}(\epsilon)q_x(\tilde{\epsilon}(\epsilon))$$

**Proof of Lemma 4** By standard mechanism design arguments, one can show that condition 5 of the lemma is sufficient and necessary for (16). (17) can be rewritten as follows. The left hand side equals $U_x - \kappa x$. Using Lemma 3 and condition 5 one can rewrite the right hand side as follows (where I use the notation $\tilde{\epsilon}(\epsilon) = c_x^{-1}(c_{x'}(\epsilon))$):

$$\int_0^1 \max_{\tilde{\epsilon}(\epsilon)} (t_x(\epsilon') - c_{x'}(\epsilon)q_x(\epsilon')) \, d\tilde{\epsilon} - \kappa x''$$

$$= \int_0^1 (t_x(c_x^{-1}(c_{x'}(\epsilon)))) - c_x(c_x^{-1}(c_{x'}(\epsilon)))q_x(c_x^{-1}(c_{x'}(\epsilon))) \, d\epsilon - \kappa x''$$

$$= \int_0^1 (t_x(1) - c_x(1)q_x(1) + \int_0^1 c_{x'}(z)q_{x'}(z)dz) \, d\epsilon - \kappa x''$$

$$= U_x + \int_0^1 \int_{\tilde{\epsilon}(\epsilon)}^\epsilon c_{x'}(z)q_{x'}(z)dz \, d\epsilon - \kappa x''$$

$$= U_x + \int_0^1 \epsilon c_{x'}(\tilde{\epsilon}(\epsilon))q_{x'}(\tilde{\epsilon}(\epsilon)) \frac{d\tilde{\epsilon}}{d\epsilon} \, d\epsilon - \int_0^1 \epsilon c_{x'}(\epsilon)q_{x'}(\epsilon) \, d\epsilon - \kappa x''$$

$$= U_x + \int_0^1 c_{x'}(\epsilon)q_{x'}(\epsilon)(c_x^{-1}(c_{x'}(\epsilon)) - \epsilon) \, d\epsilon - \kappa x''.$$
for $\kappa \in K_x$ and $x, x', x'' \in \{0, 1\}$. The equivalence of this condition to the conditions 1 to 4 of the lemma can be shown by taking exactly the same steps as in the proof of Lemma 1. 

**Proof of Lemma 5** Plug $q_x^*(\varepsilon)$ into the left hand sides of conditions 3 and 4 of Lemma 4.

$$
\int_0^{\varepsilon} c_x'(\varepsilon)(\varepsilon - c_x^{-1}(c_x(\varepsilon)))d\varepsilon = \int_0^{\varepsilon} c_x'(\varepsilon)d\varepsilon - \int_0^{\varepsilon} c_x'(\varepsilon)c_x^{-1}(c_x(\varepsilon))d\varepsilon
$$

where the second equality follows from a change of variable and the third equality follows from integration by parts. $\kappa^*$ is therefore the only threshold that is incentive compatible and individual rational under efficient production.

In order to prove the second part of the Lemma I show that conditions 3 and 4 in Lemma 1 are less restrictive than conditions 3 and 4 in Lemma 4. In order to see this note that for $\bar{\varepsilon}(\varepsilon) = c_x^{-1}(c_{x'}(\varepsilon))$

$$
\int_0^1 c_x'(\varepsilon)q_x(\varepsilon)(c_x^{-1}(c_x(\varepsilon)) - \varepsilon)d\varepsilon = \int_0^1 c_x'(\varepsilon)q_x(\varepsilon)d\varepsilon - \int_0^1 (c_x(\varepsilon) - c_x'(\varepsilon))q_x(\varepsilon)d\varepsilon + \int_0^1 q_x(\varepsilon)d\varepsilon
$$

$$
> (\varepsilon(\varepsilon - c_x'(\varepsilon))q_x(\varepsilon)d\varepsilon \quad \text{for} \quad x = 0, x' = 1 \quad (x = 1, x' = 0)
$$

where the last inequality holds if $q_x(\varepsilon)$ is not constant on $[0, 1]$. This is the case for $q_0^*$ and $q_1^*$. Since condition 3 of Lemma 1 is satisfied with equality for $q_0^*$ and $\kappa^*$, the stricter condition 3 of Lemma 4 cannot be satisfied for $q_0^*$ and $\kappa^*$.

**Proof of Proposition 3** The proof is only sketched as it follows essentially the same steps as the proof of Proposition 2. Consider the principal’s problem defined in (18) and neglect conditions 4 and 5. It is optimal to set $U_0 = 0$ and $q_1(\cdot) = q_1^*(\cdot)$. Using the same arguments as in the proof of Proposition 2, it can be shown that the optimal investment threshold $\kappa^*$ satisfies $\kappa^* \in (\kappa^0, \kappa^*)$. $G(q_1^*, q_0, \kappa^*) < F(\kappa^*)/f(\kappa^*)$, and that condition 3 is satisfied with equality. The last result allows to derive a first order condition analogous to (25) with $u_0^*(\varepsilon)$ instead of $u_1^*(\varepsilon)$. 

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By the same arguments as in the proof of Proposition 2, it can be derived that there is a unique optimal pair \( \kappa^a \) and \( \varepsilon_0^a \) with \( q_0^a(\varepsilon) = 1(\varepsilon \leq \varepsilon_0^a) \) which satisfies (22) and (23), and \( \varepsilon_0^a < \varepsilon_0^s \). Clearly \( q_0^a(\varepsilon) \) is decreasing in \( \varepsilon \), transfers can be chosen to satisfy the requirement of condition 5, and it can be shown that condition 4 is satisfied.

**Proof of Proposition 4**  
By Proposition 1, it is never optimal to monitor the shock and the investment as this would give the payoff \( \Pi^i \) which can also be achieved if only investment is monitored. The conditions for optimality are derived from the payoffs \( \Pi^i - C^i, \Pi^s - C^s, \) and \( \Pi^n \) that can be achieved under monitoring of investment, the shock, and no monitoring. From Proposition 2 it follows that \( \Pi^i \geq \Pi^n \). If \( \Pi^s > \Pi^n \), there exist \( C^i > 0 \) and \( C^s > 0 \) such that any of the three monitoring choices can be optimal. \( \Pi^s > \Pi^n \) follows from the fact that condition 3 of Lemma 4 is more restrictive than condition 3 of Lemma 1, which is implied by (26).

**Proof of Proposition 5**  
By Proposition 2, \( \Pi^i = \Pi^s \) if moral hazard is irrelevant and \( \Pi^i > \Pi^s \) if moral hazard is relevant. It only remains to show that there exists a \( z^* \in [z, \tau] \) such that moral hazard is relevant (i.e. (9) is satisfied) iff \( z \geq z^* \). The optimal investment threshold \( \kappa^i_z \) is a function of \( z \) implicitly defined by

\[
\kappa^i_z + \frac{F_z(\kappa^i_z)}{f_z(\kappa^i_z)} = \kappa^*
\]

From the definition of \( F_z \) it follows that \( F_z(\kappa)/f_z(\kappa) = F(\kappa - z)/f(\kappa - z) \). By log-concavity of \( f, F_z(\kappa)/f_z(\kappa) \) is increasing in \( \kappa \) and decreasing in \( z \). This proves that \( \kappa^i_z \) is increasing in \( z \). Moreover \( \kappa^i_z = \kappa^* \) as \( F_z(\kappa^*) = 0 \). As \( \kappa^* - \kappa^i_z \) is therefore decreasing in \( z \) and zero at \( \tau \), this establishes the existence of \( z^* \).

**Proof of Proposition 6**  
For \( v = \tau \), (15) and (23) are solved by \( \varepsilon_0^s = \varepsilon_0^a = 1 \). As \( u^i_1(1) = u^n_1(1) = 0 \), there is by (14) and (22) no first order effect on \( \kappa^s \) and \( \kappa^n \) for \( v \) smaller but close to \( \tau \). However, \( u^i_1(\varepsilon) > u^n_1(\varepsilon) \) for \( \varepsilon \) close to one. This is implied by the following argument:

Note that \( u^i_1(1) = u^n_1(1) = 0 \) and \( \partial u^i_1(\varepsilon)/\partial \varepsilon = c_0(\varepsilon) - c_1(\varepsilon) \). For \( \varepsilon \) close to one, the following approximation holds

\[
\partial u^i_1(\varepsilon)/\partial \varepsilon \approx c_0(\varepsilon) - c_1(\varepsilon) + c_0(\varepsilon) \left( \frac{c_0(\varepsilon)}{c_1(\varepsilon - 1(c_0(\varepsilon)))} - 1 \right) \approx \frac{(c_0(\varepsilon))^2}{c_1(\varepsilon)} - c_0(\varepsilon)
\]
as $c_1^{-1}(c_0(\varepsilon)) \simeq \varepsilon$ for $\varepsilon$ close to one. It follows for $\varepsilon$ close to one

$$\partial u^n_1(\varepsilon)/\partial \varepsilon - \partial u^s_1(\varepsilon)/\partial \varepsilon_0 \simeq \left( \frac{(c_0'(\varepsilon))^2}{c_1'(\varepsilon)} - c_0'(\varepsilon) \right) - (c_0'(\varepsilon) - c_1'(\varepsilon))$$

$$= \frac{1}{c_1'(\varepsilon)} (c_0'(\varepsilon)^2 - 2c_0'(\varepsilon)c_1'(\varepsilon) + c_1'(\varepsilon)^2) = \frac{1}{c_1'(\varepsilon)} (c_0'(\varepsilon) - c_1'(\varepsilon))^2 > 0$$

It follows by continuity that $u^n_1(\varepsilon) > u^s_1(\varepsilon)$ for $\varepsilon$ close to one. This implies

$$\frac{1 - F(\kappa^n)}{f(\kappa^n)} (v - c_0(\varepsilon^n_0)) - u^n_1(\varepsilon^n_0) \left( \frac{F(\kappa^n)}{f(\kappa^n)} - G(q^*_1, q^*, \kappa^n) \right)$$

$$\simeq \frac{1 - F(\kappa^s)}{f(\kappa^s)} (v - c_0(\varepsilon^s_0)) - u^s_1(\varepsilon^s_0) \left( \frac{F(\kappa^s)}{f(\kappa^s)} - G(q^*_1, q^*, \kappa^s) \right)$$

$$> \frac{1 - F(\kappa^s)}{f(\kappa^s)} (v - c_0(\varepsilon^s_0)) - u^s_1(\varepsilon^s_0) \left( \frac{F(\kappa^s)}{f(\kappa^s)} - G(q^*_1, q^*, \kappa^s) \right) = 0.$$

Thus, $\varepsilon^n_0 > \varepsilon^s_0$ and $\kappa^n > \kappa^s$ for $v$ sufficiently close to $\bar{v}$.

References


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