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Optimal monitoring in dynamic procurement contracts

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Abstract

What kind of information should government institutions monitor in order to manage public contracts efficiently? I analyze this question in a principal-agent model of procurement with moral hazard and dynamic private information. A principal can buy a good from an agent. The agent can make a cost-reducing investment at a privately known cost before observing a shock to his production costs. The principal can monitor the investment, the shock or both at a cost. I show that it is never optimal to monitor both. Monitoring the investment is always at least as effective as monitoring the shock. The two instruments are equivalent if the investment decision is close to the efficient investment decision.

Keywords: procurement, monitoring, dynamic contracts

JEL Classification: D82, D86, H57

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1 Introduction

Public procurement constitutes a sizable part of economic activity in advanced economies. In 2013 the average OECD country spent 29% of total government expenditure on public procurement. This represents, on average, 12.1% of GDP (OECD, 2015a). The potential cost savings through more efficient public procurement procedures are large.\(^1\) In the aftermath of the Great Recession – as many governments have been forced to reduce their debt – international organizations such as the OECD and national audit offices have urged public institutions to adopt better procurement procedures (NAO/OGC, 2008; GAO, 2013; OECD, 2015b). Among other measures for the effective management of public procurement, these organisations stress the importance of monitoring. In particular, they recommend monitoring the cost developments and other performance measures of private suppliers. However, monitoring is costly and not all sources of additional information help government institutions to manage public contracts effectively.\(^2\)

This poses the question of what kind of information a government institution should monitor in order to optimally manage public contracts. I analyze this question in a principal-agent model of procurement where the agent can produce a good for the principal. Prior to production, the agent can make a cost-reducing investment at a privately known cost. Production costs are determined by the investment and a cost shock. This shock represents the agent’s uncertainty of production costs at the time of making the investment. The principal can monitor the agent’s investment decision, the cost shock or both at a cost.

I show that it is never optimal for the principal to monitor both. Furthermore, the principal achieves at least the same payoff – gross monitoring costs – if she monitors investment as though she was monitoring the shock. Both instruments perform equally well if the investment decision is close to being efficient.

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\(^1\) Cost savings of 1% of public procurement expenditure represents 43 billion EUR across OECD countries (OECD, 2015b).

\(^2\) For example, a large number of performance indicators (NAO, 2014) did not prevent the overbilling of the companies G4S and Serco in the UK.
The model captures two important aspects of public procurement projects. First, cost-reducing investments and the sequential, private learning of production costs are realistic features of many procurement contexts. Procurement contracts frequently run for several years. This is the case for construction projects of airports and high-speed internet networks as well as the management and maintenance of public facilities such as hospitals and highways. Furthermore, suppliers can often make cost-reducing investments at an early stage of the project, e.g. at the planning phase. These investments influence the total costs of the project completion. Additionally, production costs are influenced by exogenous cost factors that a supplier observes after the investment decision. Examples of such factors include ground conditions and input prices for construction projects, and weather conditions for maintenance contracts. The model reflects both the dynamic aspect of learning during procurement contracts and the opportunity to invest in cost reductions.

Secondly, monitoring plays a central role in the management of public procurement projects. Suppliers’ investment decisions can be monitored using key performance indicators such as quality milestones (Garvin et al., 2011). Exogenous cost factors can be monitored by third-party experts. In practice, government agencies monitor a wide variety of outcomes. Typically, these outcomes can be classified as either endogenous or exogenous cost factors. In this paper, I focus on the monitoring of investments and cost shocks as relevant examples for each type.

I solve the principal’s problem by analyzing optimal contracts under the different monitoring regimes. If the agent’s investment decision is monitored, the principal first has to elicit the investment cost and then the value of the shock. The principal’s optimal contract induces underinvestment and efficient production of the good independently of the investment decision. The two sources of asymmetric information can be treated separately. The agent knows his investment cost before a contract is signed. Thus, the

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3 In the UK, public-private partnerships under the scheme of the Private Finance Initiative usually run for 20 to 30 years (HMT, 2012).
4 For instance, the Canadian government has employed KPMG as a third-party expert for a national shipbuilding procurement project (PWGSC, 2013).
principal has to give an information rent to the agent and therefore distorts the investment decision. In contrast, the cost shock is realized after the parties have signed a contract. The principal does not have to give any rent to the agent for this information. She can induce efficient production and extract the surplus with a fixed fee. This implies that the principal has nothing to gain by monitoring both the cost shock and the investment decision.

If only the shock is monitored, the principal has to elicit the investment cost and induce the appropriate investment decision. Moral hazard concerning the investment decision is irrelevant if the distortion in the investment decision is not too large. The previously optimal contract is then still implementable. In contrast, if the principal wishes to implement a large distortion in investment, moral hazard becomes relevant. In this case, the optimal contract features distortions in the investment and the production decision.

Finally, I analyze the optimal contract without monitoring. The principal achieves a strictly smaller payoff than in the case where the shock is monitored. If the principal does not monitor, the agent can play *double deviations* by combining a deviation in the investment decision with a false report on the cost shock. Such deviations are not possible if the shock is monitored and they decrease the principal’s payoff. This contrasts with the case where investment is monitored. Here, the agent cannot benefit from false reports about the cost shock. The unobserved investment decision ‘connects’ the two sources of private information. The principal cannot separate them as in the case of investment monitoring. The optimal contract without monitoring induces two types of inefficiencies: underinvestment in cost reduction and underproduction.

These results have the following implications for the optimal monitoring policy. First, the principal monitors either the shock or the investment decision. Second, monitoring investment is always at least as effective as monitoring the shock. Third, the principal can sometimes achieve the same payoff (gross monitoring costs) under the two monitoring regimes. This is the case if the distortion in the investment decision is not too high.
Following the seminal work by Laffont and Tirole (1986), an important part of the literature on procurement contracting assumes that the principal observes the production costs of the agent.\(^5\) This paper contributes to the literature on monitoring in principal-agent models, in which the principal’s information is determined endogenously. Maskin and Riley (1985) and Khalil and Lawarrée (1995) analyze the question of input-vs-output monitoring, where a principal can monitor either an agent’s effort or his realized return. Dewatripont and Maskin (1995) show that a principal may optimally restrict what he can monitor in order to avoid renegotiation. Khalil (1997) analyzes a principal-agent model where the principal cannot commit to the monitoring policy. Strausz (1997) looks at the delegation of monitoring to a third party under limited commitment. This paper is the first to provide an analysis of the monitoring of post-contractual private information and moral hazard under full commitment.\(^6\)

This paper is also related to the literature on R&D and optimal procurement mechanisms which was initiated by Tan (1992), Piccione and Tan (1996), and Bag (1997). They analyze competitive procurement mechanisms when suppliers can invest in cost-reducing R&D. As the investment decision in this paper can be interpreted as an investment in R&D, the contribution to this literature lies in analyzing the effect of unobserved and cost-reducing R&D on dynamic information rents. Cisternas and Figueroa (2015) analyze the optimal procurement mechanism when a buyer procures two projects sequentially and the winner of the first round can invest in cost reduction before the second round. Liu and Lu (2015) analyze optimal contracts in a model of procurement with an unobservable R&D effort in cost reduction under dynamic adverse selection.

A further related literature studies optimal contracts for \textit{public-private partnerships} (PPP). Iossa and Martimort (2012, 2014) analyze when a public buyer should delegate the design and the implementation of a project to the same firm – which corresponds to a PPP – or different firms – such as under traditional forms of procurement. Hoppe and

\(^5\)See also Laffont and Tirole (1993).

\(^6\)In contrast to the literature on costly state verification initiated by Townsend (1979), the principal here decides ex-ante which information of the agent to observe.
Schmitz (2013) analyze the implication of this decision for innovative procurement. Engel et al. (2013) study optimal contracts for PPPs from a public finance perspective. Closely related to the current paper, Iossa and Martimort (2016) study optimal contracts for a PPP where a single supplier has private information and a moral hazard decision after contracting. In contrast to the current paper, they consider a risk-averse supplier who has no private information at the outset of the contractual relationship. Furthermore, they study the benefit of complete contracts over incomplete contracts, where only complete contracts condition on communication with the supplier. Public-private partnerships are frequently used for the realization of mid to long-term procurement projects where learning of new information and cost-reducing investments by suppliers play an important role. This paper contributes to the literature by providing an explicit analysis of the optimal monitoring policies for a government authority that is entering into a public-private partnership with a private supplier.

This paper is furthermore related to the literature on information rents in dynamic principal-agent models. In models of dynamic adverse selection, Baron and Besanko (1984), Eső and Szentes (2007a,b), and Pavan et al. (2014) show that the principal does not have to give rents for the private information that agents learn after contracting. Krähmer and Strausz (2015) qualify this insight by showing that an agent receives post-contractual information rents if the set of signal realizations learned before contracting is discrete. Baron and Besanko (1984) and Eső and Szentes (2015) argue that privacy of post-contractual information may also be irrelevant in the presence of dynamic adverse selection and moral hazard. I show that their result does not extend to the setting in the current paper. I elaborate on this point in the discussion section.

In the following section I introduce the model. Section 3 presents the optimal contract when the principal monitors the investment decision. Section 4 considers the case where the principal monitors the shock. Section 5 analyzes the optimal contracts without monitoring. Section 6 presents and analyzes the optimal monitoring policy. Section 7 discusses the role of post-contractual information rents and the robustness of the results.
Section 8 concludes.

2 The model

A government institution (the principal) can procure a good from a supplier (the agent). The principal values the good by \( v \). Prior to production, the agent can make a cost-reducing investment decision \( x \in \{0, 1\} \). The investment decision leads to investment costs of \( \kappa \cdot x \) to the agent. \( \kappa \) is the private information of the agent and is drawn from an interval \([\kappa, \overline{\kappa}] \subset \mathbb{R}_+\) according to the distribution function \( F \). \( F \) has a log-concave density function \( f \), so that \( F(\kappa)/f(\kappa) \) is weakly increasing and \((1 - F(\kappa))/f(\kappa) \) is weakly decreasing (Bagnoli and Bergstrom, 2005). The agent’s production cost is determined by the investment decision \( x \) and a shock \( \varepsilon \) that is realized after the investment is made.

The production cost is given by \( c_x(\varepsilon) \). For both \( x \in \{0, 1\} \), the function \( c_x(\cdot) \) has the image \([c, \overline{c}] \subset \mathbb{R} \), is strictly increasing and twice continuously differentiable. Without loss of generality I assume that the shock \( \varepsilon \) is uniformly distributed on the unit interval.\(^7\) The investment is cost-reducing in the sense that \( c_1(\varepsilon) < c_0(\varepsilon) \) for all \( \varepsilon \in (0, 1) \). Furthermore, let \( v \in (c, \overline{c}) \). There are two simple monitoring technologies. The principal can perfectly observe the investment decision \( x \) at a monitoring cost \( C_i > 0 \), and the shock \( \varepsilon \) at a monitoring cost \( C_s > 0 \). I assume that the principal cannot monitor probabilistically.\(^8\) Both parties are risk-neutral and have outside options associated with a payoff of zero. Let \( q \) be the probability of production and \( t \) be a transfer. The agent’s payoff is \( t - c_x(\varepsilon)q - \kappa x \) and the principal’s payoff gross monitoring costs is \( vq - t \). The timing of the game is as follows:

i) The agent learns \( \kappa \).

\(^7\)If the shock \( \varepsilon \) leading to production costs \( \hat{c}_x(\varepsilon) \) is distributed according to a continuous and strictly increasing distribution function \( H \) on some interval, then the random variable \( \varepsilon \equiv H(\varepsilon) \) is uniformly distributed on \([0, 1]\). The cost functions can be redefined as \( c_x(\varepsilon) = c_x(H(\varepsilon)) \equiv \hat{c}_x(\varepsilon) \). An assumption on \( c_x(\cdot) \) would then translate into a joint assumption on \( \hat{c}_x(\cdot) \) and \( H \).

\(^8\)An alternative assumption is that the principal has to spend \( C_i^* \) or \( C_s^* \) to install the monitoring technology independently of whether it is used later on and the agent observes whenever a monitoring technology is installed.
ii) The principal decides what to monitor and offers a contract. The agent observes the principal’s decision and accepts or rejects the contract. If the agent chooses reject, the game ends and both parties receive zero payoffs. Otherwise, the game continues.

iii) The agent makes the investment decision \( x \).

iv) The shock \( \varepsilon \) is realized.

v) The agent can produce the good.

**Complete information benchmark**

Suppose the investment cost \( \kappa \), the investment decision \( x \), and the shock \( \varepsilon \) are publicly observed. In this case, the principal can extract the whole social surplus. For any \( \kappa \), she chooses the investment decision and the probability of production, which maximize social surplus:

\[
\max_{x,q_x(\cdot)} \int_0^1 (v - c_x(\varepsilon))q_x(\varepsilon)d\varepsilon - \kappa \cdot x
\]

Independently of the investment decision, the principal procures the good from the agent if her valuation lies above the production costs: \( v \geq c_x(\varepsilon) \) for \( x \in \{0, 1\} \). I denote the thresholds of the shock by \( \varepsilon_x^* \) which satisfy \( c_x(\varepsilon_x^*) = v \) with \( x \in \{0, 1\} \). The principal induces the cost-reducing investment if the investment cost lies below the additional social value created by the investment, i.e., if \( \kappa \leq \kappa^* \) where

\[
\kappa^* \equiv \int_{\varepsilon_0}^{\varepsilon_1} (v - c_1(\varepsilon))d\varepsilon - \int_0^{\varepsilon_0} (v - c_0(\varepsilon))d\varepsilon. \tag{1}
\]

\( \kappa^* \) is the *efficient investment cutoff* and it equals the additional social surplus that is generated through the investment. I assume \( \kappa^* \in (\underline{\kappa}, \overline{\kappa}) \). Graphically, \( \kappa^* \) can be represented as the areas \( A \) and \( B \) in Figure 1. In this figure, the production costs of the agent are depicted as functions of the cost shock for both investment decisions \( x = 0 \) and
Figure 1: Efficient production and investment decisions

\[ c_0(\varepsilon) \]
\[ c_1(\varepsilon) \]

\[ x = 1. \] Area \( A \) represents the expected cost savings of the investment for values of the cost shock under which the good is produced independently of the investment decision. Area \( B \) represents the additional social value of production for values of the cost shock where the good is only produced if the agent invests.

3 Monitoring investment

In this section, I analyze the principal’s optimal contract when she monitors the agent’s investment decision. For now, I assume that the principal monitors the investment and the cost shock. In that case, she only needs to elicit the investment cost from the agent. I solve for the optimal allocation. I then show that this allocation can be implemented with the same expected transfers even if the principal monitors only the investment and not the cost shock. This shows that the principal has nothing to gain from monitoring the cost shock when she is also monitoring the investment decision.

The principal offers the agent a menu of two contracts. The first contract requires the agent to invest and fixes a probability of production \( q_1(\varepsilon) \) and an expected transfer \( t_1(\varepsilon) \) as functions of the shock. The second contract requires the agent not to invest and fixes the probability of production \( q_0(\varepsilon) \) and the expected transfer \( t_0(\varepsilon) \). I denote the
expected payoff of the contract \((q_x(\varepsilon), t_x(\varepsilon))\) – gross investment costs – by

\[
U_x = \int_0^1 (t_x(\varepsilon) - c_x(\varepsilon)q_x(\varepsilon)) d\varepsilon,
\]

for \(x \in \{0, 1\}\). An agent with investment cost \(\kappa\) chooses the first contract if and only if \(U_1 - \kappa \geq U_0\). I denote by \(\hat{\kappa}\) the threshold at which the agent is indifferent between the two contracts. The threshold thus satisfies \(\hat{\kappa} = U_1 - U_0\). The agent participates if at least one of the contracts gives him a higher payoff than his outside option. The principal’s expected payoff from the menu of contracts is

\[
\Pi \equiv F(\hat{\kappa}) \int_0^1 (vq_1(\varepsilon) - t_1(\varepsilon)) d\varepsilon + (1 - F(\hat{\kappa})) \int_0^1 (vq_0(\varepsilon) - t_0(\varepsilon)) d\varepsilon.
\]

The first term represents the expected payoff if the agent invests, the second term is the expected payoff if the agent does not invest. Using equation (2), the principal’s payoff can be expressed as a function of the probability of production, the investment threshold, and the expected payoffs of the agent from the two contracts. Using furthermore the relationship between expected payoffs and the investment threshold, the principal’s expected payoff can be written as

\[
\tilde{\Pi} \equiv F(\hat{\kappa}) \left( \int_0^1 (v - c_1(\varepsilon))q_1(\varepsilon) d\varepsilon - \hat{\kappa} \right) + (1 - F(\hat{\kappa})) \left( \int_0^1 (v - c_0(\varepsilon))q_0(\varepsilon) d\varepsilon \right) - U_0.
\]

The principal chooses the probabilities of production \(q_1(\cdot)\) and \(q_0(\cdot)\) and the investment threshold \(\hat{\kappa}\) in order to maximize her expected payoff – subject to the participation constraint \(U_0 \geq 0\). It is optimal for the principal to make the participation constraint of the non-investing agent binding. Furthermore, the principal procures the good from the agent if and only if it is efficient and sets \(q^*_x(\varepsilon) \equiv 1(\varepsilon \leq \varepsilon^*_x)\) for \(x \in \{0, 1\}\). The optimal investment threshold denoted by \(\kappa^i\) satisfies

\[
\kappa^* = \int_0^{\varepsilon^*_1} (v - c_1(\varepsilon)) d\varepsilon - \int_0^{\varepsilon^*_0} (v - c_0(\varepsilon)) d\varepsilon = \kappa^i + \frac{F(\kappa^i)}{f(\kappa^i)}.
\]
and $\kappa^i < \kappa^*$. The principal induces investment if the social surplus generated by investment exceeds the virtual investment costs. I denote by $\Pi^i$ the payoff that the principal achieves. In contrast to the case with complete information, the principal needs to give an information rent to the agent for the private information on the investment cost. The principal faces an efficiency-rent extraction trade-off and distorts the investment decision. Whereas if the principal chooses efficient production conditional on the investment, the induced investment decision leads to underinvestment.

The optimal allocation can be implemented at the same revenue by a menu of contracts under which the production decision is delegated to the agent. The first contract prescribes investment, the second contract prohibits investment. In both contracts the agent decides whether to produce the good at price $v$. The first contract demands an initial payment of $T_1 = \int_0^{\epsilon^*} (v - c_1(\epsilon)) d\epsilon - \kappa^i$. The initial payment of the second contract is $T_0 = \int_0^{\epsilon^*} (v - c_0(\epsilon)) d\epsilon$. These contracts implement the principal’s optimal investment decisions and allocations. As the principal can delegate the production decision to the agent, the contracts do not require the principal to observe the shock. I state this first result as a proposition.

**Proposition 1.** If investment is monitored, the principal has nothing to gain from monitoring the shock.

Because the shock is statistically independent of the investment cost, the principal can extract the private information on the shock from the agent without giving up any rent. Therefore, the principal cannot increase her payoff by monitoring the shock in addition to the investment. This reflects the result by Baron and Besanko (1984). In the following sections I show that the situation is different if investment is unobservable.

## 4 Monitoring the shock

In the following I suppose that the principal monitors only the shock. Under this monitoring policy, the optimal contract needs to give the agent incentives to reveal his costs.
of investment and to take the right investment decision. I show that moral hazard concerning the investment decision may be irrelevant. In this case, the principal can achieve the same payoff – gross monitoring costs – as in the case where she monitors investment. If moral hazard is relevant, the principal optimally chooses to introduce underproduction by non-investing agents in order to reduce rent payments to investing agents. In this case, the optimal contract features investment and production distortions.

The principal offers a menu of two contracts. One contract should be chosen by the agent if he invests, and the other contract is for the agent if he does not invest. Both contracts fix a probability of production \( q_x(\varepsilon) \) and an expected transfer \( t_x(\varepsilon) \) for \( x \in \{0, 1\} \). Denote by \( K_x \) the set of all values of the investment cost for which the agent takes the investment decision \( x \) in equilibrium. A menu of contracts is incentive compatible if an agent with investment costs in \( K_x \) finds it optimal to choose the contract \((q_x(\cdot), t_x(\cdot))\) and the action \( x \). A menu of contracts is individual rational if the agent always prefers one of the contracts over rejecting the principal’s offer. Formally, if the principal monitors the shock, incentive compatibility and individual rationality require

\[
\int_0^1 (t_x(\varepsilon) - c_x(\varepsilon)q_x(\varepsilon)) d\varepsilon - \kappa x \geq \max \left\{ \int_0^1 (t_{x'}(\varepsilon) - c_{x'}(\varepsilon)q_{x'}(\varepsilon)) d\varepsilon - \kappa x'', 0 \right\}
\]

for \( \kappa \in K_x \) and \( x, x', x'' \in \{0, 1\} \).

Let \( U_x \) be the agent’s expected payoff from the contract \((q_x(\cdot), t_x(\cdot))\) gross investment cost. The joint condition of incentive compatibility and individual rationality can then be expressed as

\[
U_x - \kappa x \geq \max \left\{ U_{x'} + \int_0^1 (c_{x'}(\varepsilon) - c_{x''}(\varepsilon))q_{x'}(\varepsilon)d\varepsilon - \kappa x'', 0 \right\}
\]

for \( \kappa \in K_x \) and \( x, x', x'' \in \{0, 1\} \). Incentive compatibility and individual rationality can

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\(^9\)The revelation principle due to Myerson (1986) allows us to focus on truthful direct mechanisms with random recommendations concerning the investment decision. The contracts studied here are mechanisms with deterministic recommendations, i.e., investment decisions. Under the assumptions on \( F \) and assumptions 1 and 2, this can be shown to be without loss of optimality.
be characterized as follows.

**Lemma 1.** If the principal monitors the shock, a menu of contracts is incentive compatible and individual rational if and only if for some $\kappa \in [\underline{\kappa}, \bar{\kappa}]$

1. $K_1 = [\underline{\kappa}, \kappa]$, $K_0 = (\kappa, \bar{\kappa}]$, and $U_1 - U_0 = \hat{\kappa}$;
2. $U_0 \geq 0$;
3. $\int_0^1 (c_0(\varepsilon) - c_1(\varepsilon)) q_0(\varepsilon) d\varepsilon \leq \hat{\kappa}$;
4. $\int_0^1 (c_0(\varepsilon) - c_1(\varepsilon)) q_1(\varepsilon) d\varepsilon \geq \hat{\kappa}$.

If the agent optimally invests at some level of investment cost, then it is still optimal to invest if the investment costs are lower. This implies condition 1. Condition 2 guarantees individual rationality: Independently of the level of investment cost, the agent can always achieve a net benefit of $U_0$. If $U_0$ is better than the outside option, the agent always accepts one of the contract offers. Under condition 3, agents with low investment costs have no incentive to choose the contract aimed at non-investing agents and to invest nevertheless. With this deviation, the agent forgoes the rent provided under the contract $(q_1(\cdot), t_1(\cdot))$ but can keep any reduction in production costs. The rent therefore needs to exceed the reduced costs. Conversely, under condition 4, it is unprofitable for agents with high investment costs to pick the contract for investing agents and to abstain from investment. In this deviation, the agent picks the contract $(q_1(\cdot), t_1(\cdot))$ and receives the rent payment associated with it. However, he incurs a loss as he produces with the less-efficient production technology. The allocation is incentive compatible if this loss exceeds the rent.

By condition 1 in Lemma 1, the expected payoff of the principal can be expressed as in equation (4). The optimal menu of contracts for the principal is therefore the solution of the following problem:

$$\max_{U_0, \kappa, (q_1(\cdot))_{\epsilon \in \{0, 1\}}} \hat{\Pi} \quad \text{s.t. conditions 2 to 4 in Lemma 1}$$ (8)
This problem is equivalent to the principal’s problem under the monitoring of investment and shock with the additional constraints 3 and 4. The principal cannot achieve a higher payoff (gross monitoring costs) than under investment monitoring. However, she achieves the same payoff if conditions 3 and 4 are not binding at the optimum. The optimal investment and production decisions from the optimal contract with investment monitoring always satisfy condition 4. Furthermore, condition 3 is satisfied if the distortion away from the first best investment decision is not too large:

**Proposition 2.** If the shock is monitored, the principal can implement the investment decision characterized by the threshold \( \kappa^i \) and efficient production with the same expected transfers as under investment monitoring if and only if

\[
\kappa^i \geq \kappa^* - \int_{\epsilon_0^*}^{\epsilon_1^*} (v - c_1(\epsilon)) d\epsilon. \tag{9}
\]

If this condition is violated, moral hazard is relevant. In this case, the unobservability of the investment decision adds agency costs to the principal’s problem. This is the case if the best deviation of an agent with low investment costs – i.e., \( \kappa \in K_1 \) – is to choose the contract \((q_0(\cdot), t_0(\cdot))\) and invest nevertheless. Under efficient production, the benefit of this deviation (gross investment cost) is given by the area \( A \) in Figure 1. \( A \) represents the expected cost savings that the agent can keep for himself when deviating. If this area is smaller than the rent \( \kappa^i \) that the agent receives in the optimal contract under investment monitoring, then moral hazard is irrelevant. Equation (9) expresses this comparison – using the fact that the sum of areas \( A \) and \( B \) in Figure 1 equal \( \kappa^* \). The reformulation shows that moral hazard is irrelevant if the principal wants to implement a large investment threshold which is close to the efficient investment threshold. In contrast, if the principal aims to implement a small investment threshold, then moral hazard becomes relevant.

If moral hazard is relevant, the principal could still implement efficient production. In this case, the principal would have to give the investing agent a rent equal to the area \( A \) in
Figure 2: Optimal contract under shock monitoring

Figure 1. This would imply an investment threshold equal to the area $A$. This threshold would be higher than the optimal threshold under monitoring of the investment decision.

The principal can increase her payoff by reducing the probability of production for the agent who does not invest. As illustrated in Figure 2, this allows the principal to profitably reduce the rent of the investing agent to the area $A'$. The investment cost threshold is also affected because the agent’s investment is optimal as long as the investment costs are smaller than the area $A'$. Note that it is never beneficial for the principal to push the threshold down to $\kappa_i^*$, the optimal threshold under monitoring of investment. At this threshold, the principal is indifferent between trading efficiently with an investing agent or with a non-investing agent. However, production with non-investing agents is inefficient under the threshold $\kappa_i^*$ if moral hazard is relevant. The principal therefore prefers a strictly higher threshold. The optimal threshold is also strictly smaller than the efficient investment threshold $\kappa^*$. This follows from the observation that the area $A'$ in Figure 2 is smaller than the area $A$ in Figure 1, whereas the efficient investment threshold equals the sum of the areas $A$ and $B$ in Figure 1. It follows that the optimal contract under shock monitoring and the relevant moral hazard includes inefficient production by non-investing agents and a smaller distortion in the investment decision than in the optimal contract with investment monitoring.
In order to characterize the optimal contract, it is helpful to define – for any given incentive compatible and individual rational menu of contracts that implements an investment threshold \( \hat{\kappa} \) – the principal’s gain from investment by the agent with investment cost \( \kappa \leq \hat{\kappa} \):

\[
G(q_1(\cdot), q_0(\cdot), \hat{\kappa}) \equiv \int_0^1 (v - c_1(\varepsilon))q_1(\varepsilon)d\varepsilon - \hat{\kappa} - \int_0^1 (v - c_0(\varepsilon))q_0(\varepsilon)d\varepsilon.
\] (10)

If an agent with investment cost \( \kappa \) invests, the principal can extract the expected social surplus of \( \int_0^1 (v - c_1(\varepsilon))q_1(\varepsilon)d\varepsilon - \kappa \), reduced by the rent \( \hat{\kappa} - \kappa \). If the same agent does not invest, the principal does not give an information rent and receives the social surplus of \( \int_0^1 (v - c_0(\varepsilon))q_0(\varepsilon)d\varepsilon \). \( G(\cdot, \cdot, \cdot) \) is therefore the part of the social value of the investment which the principal can appropriate.

A potential deviation of an agent with a low investment cost is the following: Select contract \((q_0(\cdot), t_0(\cdot))\) instead of contract \((q_1(\cdot), t_1(\cdot))\) and choose the investment decision \( x = 1 \). The agent can then keep the reduction in production costs due to the investment for himself. If moral hazard is relevant then this deviation is attractive and the agent has to be given an information rent equal to the reduction in production costs. Formally, condition 3 of Lemma 1 is binding in the optimal contract. The investing agent therefore receives a rent (gross investment cost) of

\[
U^*_{1s}(q_0(\cdot)) \equiv \int_0^1 (c_0(\varepsilon) - c_1(\varepsilon))q_0(\varepsilon)d\varepsilon.
\] (11)

This rent is equal to the investment threshold by condition 1 in Lemma 1. With slight abuse of notation, I denote by \( U^*_1(\varepsilon_0) \) the agent’s rent for \( q_0(\varepsilon) = 1(\varepsilon \leq \varepsilon_0) \), and by

\[
u^*_1(\varepsilon_0) = c_0(\varepsilon_0) - c_1(\varepsilon_0)
\] (12)

the first derivative. \( u^*_1(\varepsilon_0) \) is the marginal change in the agent’s rent if the good is produced for a shock of size \( \varepsilon_0 \). The principal maximizes the virtual surplus—which is
the difference between the social surplus and the rent of the agent. Under the following assumption, the virtual surplus is decreasing in the cost shock.

**Assumption 1.** \( v - c_0(\varepsilon) - (F(\kappa^*)/(1 - F(\kappa^*))) \cdot u^*_1(\varepsilon) \) is decreasing in \( \varepsilon \).

The principal then chooses the following menu of contracts under shock monitoring.

**Proposition 3.** If the shock is monitored and Assumption 1 is satisfied, the principal achieves an optimal payoff \( \Pi^s \) through the menu of contracts \( \{ (q^s_1(\cdot), t^s_1(\cdot)), (q^s_0(\cdot), t^s_0(\cdot)) \} \) and the investment threshold \( \kappa^s \):

1. If moral hazard is irrelevant, then production is efficient and the investment threshold is the same as with monitoring of the investment: \( q^s_x(\cdot) = q^*_x(\cdot) \) for \( x \in \{0, 1\} \), and \( \kappa^s = \kappa^i \).

2. If moral hazard is relevant, then production is efficient if the agent invests. If the agent does not invest, there is underproduction. The investment threshold is lower than efficient and higher than with monitoring of the investment:

\[
q^s_1(\cdot) = q^*_1(\cdot) \quad \text{and} \quad q^s_0(\varepsilon) = 1(\varepsilon \leq \varepsilon^*_0) \quad \text{with} \quad \varepsilon^*_0 < \varepsilon^*_0; \tag{13}
\]

\[
\kappa^s = \int_{0}^{\varepsilon^*_0} (c_0(\varepsilon) - c_1(\varepsilon))d\varepsilon \in (\kappa^i, \kappa^*); \tag{14}
\]

\[
(1 - F(\kappa^s))(v - c_0(\varepsilon^*_0)) + f(\kappa^s)G(q^s_1(\cdot), q^s_0(\cdot), \kappa^s)u^*_1(\varepsilon^*_0) = F(\kappa^s)u^*_1(\varepsilon^*_0). \tag{15}
\]

In both cases, optimal transfers satisfy \( \int_0^1 t^s_x(\varepsilon)d\varepsilon = \int_0^1 (v - c_x(\varepsilon))q^s_x(\varepsilon)d\varepsilon - x \cdot \kappa^s \) for \( x \in \{0, 1\} \).

Moral hazard connects the production decision of non-investing agents and the investment cutoff. As illustrated in Figure 2, the principal can reduce the production probability of non-investing agent below the efficient cutoff. This reduces the information rent of the agent, but it also reduces the efficiency of production and the probability that the agent invests.

10This assumption is similar to standard assumptions made in the literature on sequential screening (Courty and Li, 2000). It is satisfied if \( \kappa^* \) is small enough.
Formally, the principal’s trade-off can be seen from equation (15): The left-hand side captures the marginal beneficial effects on the principal’s payoff when the good is procured for the shock $\varepsilon$ and the right-hand side represents the marginal adverse effects. The first term on the left-hand side is the marginal increase in social surplus. The term on the right-hand side is the marginal increase in the agent’s rent. The marginal effect on gross rents increases the fraction of agents who invest. This effect is beneficial for the principal and is captured by the second term on the left-hand side. In contrast to a standard adverse selection problem with an efficient and an inefficient type, the fraction of the efficient, i.e., investing, agents is endogenous to the mechanism.

5 No monitoring

In this section, I analyze the principal’s optimal contract when there is no monitoring. In this case, the principal needs to elicit information on investment costs and the shock from the agent. At the same time, the optimal contract needs to provide incentives to the agent to take the right investment decision.

I show that without monitoring, the principal achieves a strictly lower payoff – gross monitoring costs – than by using either monitoring technology. Both sources of agency costs, moral hazard concerning the investment decision, and adverse selection concerning the cost shock, are therefore always relevant. The optimal contract induces underinvestment and underproduction of non-investing agents.

The principal offers a menu of two contracts. The first contract targets the agent who is making the investment, the second contract is designed for the agent who does not invest. Both contracts specify a probability of production $q_x(\varepsilon')$ and an expected transfer $t_x(\varepsilon')$ as functions of a report $\varepsilon'$ on the shock for both investment decisions $x \in \{0, 1\}$.\textsuperscript{11} $K_x$ be the set of investment cost values for which the agent takes the decision $x$ in equilibrium.

\textsuperscript{11}See comment in footnote 9.
Incentive compatibility regarding the shock requires the following: A truthful report on the cost shock has to be optimal for an agent who has selected the correct contract and the appropriate investment decision. Formally, this implies

\[ t_x(\varepsilon) - c_x(\varepsilon)q_x(\varepsilon) \geq t_x(\varepsilon') - c_x(\varepsilon)q_x(\varepsilon') \]  

for all \( \varepsilon, \varepsilon' \in [0,1] \) and \( x \in \{0,1\} \).

Incentive compatibility regarding the whole menu of contracts requires that the agent with investment cost in \( K_x \) chooses the investment decision \( x \), the contract \((q_x(\cdot), t_x(\cdot))\), and reports \( \varepsilon \) truthfully. The menu of contracts is individually rational if the agent always prefers one of the contracts to his outside option. The joint condition of incentive compatibility and individual rationality under no monitoring can be expressed as

\[ \int_0^1 (t_x(\varepsilon) - c_x(\varepsilon)q_x(\varepsilon)) \, d\varepsilon - \kappa x \geq \max \left\{ \int_0^1 \max_{\varepsilon' \in [0,1]} (t_{x'}(\varepsilon') - c_{x'}(\varepsilon)q_{x'}(\varepsilon')) \, d\varepsilon - \kappa x'', 0 \right\} \]  

for all \( \kappa \in K_x, x, x', x'' \in \{0,1\} \). For the characterization of this constraint, it is helpful to make the following observation: An agent who has chosen the contract for the investment decision \( x \) but has made the decision \( x' \) falsely reports on the shock. The false report is optimally chosen such that the principal has a correct belief about production costs.

**Lemma 2.** Incentive compatibility regarding the shock implies

\[ c_x^{-1}(c_{x'}(\varepsilon)) \in \arg \max_{\varepsilon' \in [0,1]} t_x(\varepsilon') - c_{x'}(\varepsilon)q_{x'}(\varepsilon') \]  

for all \( x, x' \in \{0,1\} \) and all \( \varepsilon \in [0,1] \).

After a deviation, the agent optimally corrects his previous lie. Suppose the agent picks the contract \((q_x(\cdot), t_x(\cdot))\) and deviates to the investment decision \( x' \). For a report \( \varepsilon' \), the principal believes that the agent has the production costs \( c_x(\varepsilon') \), whereas the agent has the true production costs \( c_{x'}(\varepsilon) \). Lemma 2 implies that the agent optimally chooses
\[ \varepsilon' \text{ such that the principal holds the correct belief about the agent’s production cost, i.e., } c_x(\varepsilon') = c_{x'}(\varepsilon). \] This result is most intuitive for the case where each contract stipulates a strike price \( r_x \) which the agent receives if the good is produced. On the equilibrium path, the agent makes a report that induces production if \( c_x(\varepsilon) \leq r_x \). On a deviation path, the agent makes such a report if \( c_{x'}(\varepsilon) \leq r_x \). It is then straightforward to see that the report \( \varepsilon' \) such that \( c_x(\varepsilon') = c_{x'}(\varepsilon) \) is optimal for the agent on the deviation path.

Using the result from Lemma 2, incentive compatibility and individual rationality can be characterized.

**Lemma 3.** If the principal does not monitor, a menu of contracts is incentive compatible and individual rational if and only if for some \( \hat{\kappa} \in [\underline{\kappa}, \overline{\kappa}] \)

1. \( K_1 = [\underline{\kappa}, \hat{\kappa}], \quad K_0 = (\hat{\kappa}, \overline{\kappa}], \quad \text{and } U_1 - U_0 = \hat{\kappa}; \)
2. \( U_0 \geq 0; \)
3. \( \int_0^1 c_0'(\varepsilon)q_0(\varepsilon)(c_0^{-1}(c_0(\varepsilon)) - \varepsilon)d\varepsilon \leq \hat{\kappa}; \)
4. \( \int_0^1 c_1'(\varepsilon)q_1(\varepsilon)(\varepsilon - c_0^{-1}(c_1(\varepsilon)))d\varepsilon \geq \hat{\kappa}; \)
5. \( q_x(\varepsilon) \) is decreasing in \( \varepsilon \) and

\[ t_x(\varepsilon) = c_x(\varepsilon)q_x(\varepsilon) + t_x(1) - c_x(1)q_x(1) + \int_{\varepsilon}^1 c_x'(z)q_x(z)dz. \]

Conditions 3, 4, and 5 differ from Lemma 1. Conditions 3 and 4 reflect that the agent optimally lies about the shock on a deviation path where the agent takes a different investment decision than in equilibrium. Condition 3 ensures that an agent with a low investment cost has no incentive to select contract \((q_0(\cdot), t_0(\cdot))\), to invest nevertheless, and to combine this deviation with a false report on \( \varepsilon \). The left-hand side of condition 3 corresponds to the areas \( A' \) and \( B' \) in Figure 3, when the probability of production is \( q_0(\varepsilon_0) = 1(\varepsilon \leq \varepsilon_0) \). The area \( A' \) represents the cost reduction which the agent realizes for values of the cost shock where the good is produced under both contracts. The area \( B' \) is
an additional gain, since the probability of production is higher on the equilibrium path due to the opportunity to misreport the cost shock. Similarly, under condition 4 the agent with high investment costs does not gain from choosing the contract \((q_1(\cdot), t_1(\cdot))\), making the investment decision \(x = 0\), and misreporting the cost shock. Condition 5 follows from standard monotonicity and revenue equivalence requirements that are necessary and sufficient for incentive compatibility regarding the shock.

Due to condition 1, the principal’s payoff from an incentive compatible menu of contracts can be expressed as in equation (4). The principal’s problem is then

\[
\max_{U_0, \tilde{\kappa}, (q_x(\cdot))_{x \in \{0, 1\}}} \tilde{\Pi} \quad \text{s.t.} \quad \text{conditions 2 to 5 in Lemma 3.} 
\]

(18)

Can the principal still implement the investment and production decisions from the optimal contracts with monitoring? This turns out to be impossible.

**Proposition 4.** Without monitoring, efficient production \(q^*_x(\cdot)\) for \(x \in \{0, 1\}\) is incentive compatible only if the investment threshold is efficient: \(\tilde{\kappa} = \kappa^*\). Furthermore, the optimal investment and production decisions under monitoring the shock \(\{q^*_1(\cdot), q^*_0(\cdot), \kappa^*\}\) do not satisfy the joint condition of incentive compatibility and individual rationality without monitoring.

If the principal offers a contract that stipulates efficient production, incentive compatibility requires the agent to be willing to produce after either investment decision and to make a report that induces production as long as \(c_x(\varepsilon) \leq v\). This implies that an agent who deviates by choosing the menu \((q_0(\cdot), t_0(\cdot))\) and the investment decision \(x = 1\), finds it optimal to make a report \(\varepsilon' \leq \varepsilon_0^*\) and to induce production as long as \(c_1(\varepsilon) \leq v\), i.e., as long as production is efficient. On this deviation path, the agent receives the whole social surplus generated through the investment. This corresponds to the sum of the areas \(A\) and \(B\) in Figure 1. The principal could still implement efficient production but has to leave a rent equal to \(A + B\) to an investing agent. Therefore, the agent will invest as long as his investment cost lies below the efficient investment cost threshold \(\kappa^*\).
Figure 3: Optimal contract under no monitoring

Suppose now that the principal implements the production decisions from the optimal contract under monitoring the shock. Without monitoring, this results in a strictly higher investment threshold than under shock monitoring. Figure 3 illustrates this result: Given the production threshold $\varepsilon_0$, investing agents receive a rent (gross investment costs) equal to area $A'$ if the principal monitors the shock. If the principal does not monitor, the rent increases to areas $A'$ and $B'$. The optimal investment threshold under monitoring the shock is therefore no longer feasible.

In the optimal contract without monitoring, the principal introduces underproduction for non-investing agents. As illustrated in Figure 3, this reduces the information rent of an investing agent to the sum of areas $A'$ and $B'$. The investment cost threshold equals the sum of these areas and therefore lies below the efficient threshold $\kappa^*$. However, it is never optimal for the principal to push the investment threshold below the optimal level with monitoring of investment $\kappa^i$, as production by non-investing agents is inefficient.

It follows that the agent’s most attractive deviation strategy is to select the contract for non-investing agents, invest nevertheless, and misreport the cost shock such that the principal holds the correct belief about production costs. Formally, this implies that condition 3 in Lemma 3 is satisfied with equality in the optimal menu of contracts. The investing agent’s rent, gross investment cost, is therefore determined by the non-investing
agents’ probability of production and given by

\[ U^n_1(q_0(\cdot)) = \int_0^1 c_0'(\varepsilon)q_0(\varepsilon)(c_1^{-1}(c_0(\varepsilon)) - \varepsilon) d\varepsilon. \]  

(19)

I denote by \( U^n_1(\varepsilon_0) \) the agent’s rent for \( q_0(\varepsilon) = 1(\varepsilon \leq \varepsilon_0) \), and by

\[ u^n_1(\varepsilon_0) = c_0'(\varepsilon)(c_1^{-1}(c_0(\varepsilon)) - \varepsilon) \]  

(20)

the first derivative. I make an assumption that ensures that the principal benefits from a lower shock in the optimal contract. Technically speaking, this assumption ensures that the virtual surplus is decreasing in the shock \( \varepsilon \).

**Assumption 2.** \( v - c_0(\varepsilon) - (F(\kappa^*)/(1 - F(\kappa^*))) \cdot u^n_1(\varepsilon) \) is decreasing in \( \varepsilon \).\(^{12}\)

Without monitoring, the principal offers the following menu of contracts.

**Proposition 5.** If there is no monitoring and Assumption 2 is satisfied, the principal achieves an optimal payoff \( \Pi^n \) through the menu of contracts \( \{(q^n_1(\cdot), t^n_1(\cdot)), (q^n_0(\cdot), t^n_0(\cdot))\}\) and the investment threshold \( \kappa^n \): If the agent invests, production is efficient. If the agent does not invest, there is underproduction. The investments threshold is lower than efficient and higher than with investment monitoring. For \( x \in \{0, 1\} \)

\[
q^n_1(\cdot) = q^*_1(\cdot) \quad \text{and} \quad q^n_0(\varepsilon) = 1(\varepsilon \leq \varepsilon^n_0) \quad \text{with} \quad \varepsilon^n_0 < \varepsilon^*_0; 
\]

\[
\kappa^n = \int_0^{\varepsilon^n_0} c_0'(\varepsilon)(c_1^{-1}(c_0(\varepsilon)) - \varepsilon) d\varepsilon \in (\kappa^*, \kappa^*); 
\]

\[
(1 - F(\kappa^n))(v - c_0(\varepsilon^n_0)) + f(\kappa^n)G(q^n_1(\cdot), q^n_0(\cdot), \kappa^n)u^n_1(\varepsilon^n_0) = F(\kappa^n)u^n_1(\varepsilon^n_0); 
\]

\[
t^n_1(\varepsilon) = c_x(\varepsilon)q^n_x(\varepsilon) + \int_{\varepsilon}^1 c_x'(z)q^n_x(z) dz - \int_0^1 zc_x'(z)q^n_x(z) dz + \kappa^n \cdot x. 
\]

(24)

The principal faces again an efficiency-rent extraction trade-off. If the principal reduces non-investing agents’ probability of production below the efficient level then the

\(^{12}\)Like Assumption 1, this is satisfied if \( \kappa^* \) is small enough.
rent payment to investing agents can be reduced. This is illustrated in Figure 3. However, this reduces the efficiency of the production decision for non-investing agents and decreases the fraction of investing agents. Formally, the principal’s trade-off is reflected in equation (23). On the left-hand side are the marginal beneficial effects of production by non-investing types at shock $\varepsilon$: a marginal increase in efficiency and the marginal gain from the increase in the fraction of investing agent types. The right-hand side reflects the marginal increase in rent payments that have to be given to the agent. In the next section, I analyze the implications for the principal’s optimal monitoring policy. Assumptions 1 and 2 are maintained throughout the section.

6 The optimal monitoring policy

It is straightforward to derive the implications for the optimal choice of a monitoring policy.

Proposition 6. The principal’s optimal monitoring policy is given by:

- no monitoring if $C_i \geq \Pi^i - \Pi^a$ and $C^a > \Pi^a - \Pi^i$;
- monitoring the shock if $C^a \leq \min \{C_i + \Pi^a - \Pi^i, \Pi^a - \Pi^i\}$;
- monitoring the investment if $C^a < \min \{C^a + \Pi^i - \Pi^a, \Pi^i - \Pi^a\}$.

There exist monitoring costs $C_i > 0$ and $C^a > 0$ such that each monitoring regime can be optimal.

The principal finds her optimal monitoring policy by comparing the payoffs net of monitoring costs under the different monitoring regimes. This gives the conditions presented in the proposition. Apart from the policy where the principal monitors the investment and the shock, each monitoring regime is optimal for some values of the monitoring costs $C_i$ and $C^a$.

The principal optimally monitors either the investment or the shock as long as monitoring costs are low enough. On one hand, this result builds on Proposition 1: if the
principal can control the investment decision through monitoring, there is no additional gain from observing the shock. On the other hand, the result is implied by the fact that the principal achieves a strictly higher payoff under monitoring of the shock than without monitoring. If the principal does not monitor then she has to give a positive rent to the agent for private information of the cost shock. Without monitoring, the best deviation of the agent with low investment cost is to select the contract for non-investing agents, make the investment, and falsely report the cost shock. As Lemma 2 shows, the agent’s false report is strict due to the fact that the investment on the deviation path is strictly different from zero. The agent can therefore secure a positive information rent for the private knowledge he acquires after signing a contract. In the discussion section, I relate this result to the insights on post-contractual information rents from the literature on dynamic mechanism design.

The next result shows that both monitoring technologies can be perfect substitutes. This is the case if moral hazard is irrelevant. In contrast, if moral hazard is relevant, then monitoring of the investment is the more effective instrument.

**Proposition 7.** If moral hazard is irrelevant, both monitoring technologies give the same payoff gross monitoring costs. If moral hazard is relevant, the payoff gross monitoring costs is higher under investment monitoring; i.e., if equation (9) is satisfied, then $\Pi^i = \Pi^*$, otherwise $\Pi^i > \Pi^*$.

Without monitoring, the agent’s optimal deviation exploits moral hazard regarding the investment and private information about the shock. The principal can directly eliminate moral hazard by monitoring the investment decision. As argued above, this also resolves the problem of private information. If moral hazard is irrelevant under shock monitoring then the principal can also eliminate both sources of agency cost by observing the shock. If moral hazard is irrelevant, then the agent’s best deviation under shock monitoring does not make use of a deviation in the investment decision. In this case, both monitoring technologies are equally effective. The principal then simply chooses the monitoring technology with lower monitoring costs. If moral hazard is relevant, then
monitoring the shock is not enough to eliminate the agency costs resulting from moral hazard regarding the investment. In this case, shock monitoring the shock is not as effective as investment monitoring. With shock monitoring, the agent’s best deviation is to pick the contract for non-investing agents even if investment costs are low, and to make the investment nevertheless. Thus, the principal only monitors the shock if this is sufficiently cheaper than investment monitoring.

Figure 4 illustrates the optimal monitoring policy for the case where moral hazard is relevant and for the case where it is not.
Relevance of moral hazard

Moral hazard is relevant if the inequality (9) is satisfied. This condition is not formulated in terms of the fundamentals of the model. The condition can be satisfied under assumptions on the production cost functions $c_0(\cdot)$ and $c_1(\cdot)$ or the distribution function $F$. Here, I want to provide one such condition on the distribution function. Let $\{F_z(\kappa)\}_{z \in [\underline{z}, \overline{z})}$ be a family of distribution functions of the investment cost which satisfies $F_z(\kappa) = F(\kappa - z)$ for all $\kappa$ and $z$. Furthermore, $F_z(\kappa^*) = 1$ and $F_z(\kappa^*) = 0$. These distribution functions are therefore generated by moving the support of distribution $F$. A higher $z$ corresponds to a higher level of investment cost. I can then show the following result.

**Proposition 8.** If the level of investment costs is high then moral hazard is irrelevant under monitoring of the shock, i.e., $\exists z' \in [\underline{z}, \overline{z})$, such that inequality (9) is satisfied if $z \geq z'$.

If the level of investment costs is high, then the probability that the agent will invest is small. The principal has to give an information rent to the agent only in this case. As the probability to pay this rent is small, the principal optimally induces a minor distortion in the investment decision. If this distortion is small, the information rent of investing agents is large. Thus, all deviations for which the agent selects the contract for non-investing agents become relatively less attractive. If the level of investment costs is large enough, moral hazard therefore becomes irrelevant.

Monitoring and efficiency

Monitoring often has ambiguous effects on efficiency. If the principal moves from no monitoring to investment monitoring, production efficiency increases whereas investment efficiency decreases. The optimal contracts under monitoring of investment and monitoring the shock lead either to identical investment and production decisions or imply lower investment efficiency and higher production efficiency under monitoring of investment.

However, shock monitoring can have unambiguously negative effects on efficiency.
Figure 5: The beneficial effect of privacy of information on efficiency

Proposition 9. If the principal’s value for the good is high, then investment and production are more efficient without monitoring than with monitoring the shock: There exists \( \hat{v} \in (\underline{v}, \bar{v}) \) such that \( \varepsilon^* < \varepsilon_0^a \) and \( \kappa^a < \kappa^n \) for \( v > \hat{v} \).

Perhaps surprisingly, private learning of the shock increases total efficiency if the value of the good is high. Privacy of post-contractual information may therefore increase the efficiency in optimal contracts.

In optimal contracts without monitoring and with shock monitoring, the principal reduces the information rent of the investing agent by trading less frequently with the non-investing agent. The effect of a small reduction in the probability of production on the information rent differs between the two cases. Proposition 9 follows from the fact that the marginal effect on rents is smaller without monitoring for high realizations of the shock. This is illustrated in Figure 5: When the production threshold for the non-investing agent is reduced from the efficient level \( \varepsilon^*_0 \) to the smaller level \( \varepsilon_0 \) then the agent’s rent decreases from \( A \) to \( A' \) under monitoring of the shock. If the principal does not monitor, the rent decreases from \( A + B \) to \( A' + B' \). Note that for high values of \( v \), \( B' \) is larger than \( B \). The reduction of the production threshold is therefore less effective in reducing information rents without monitoring than with shock monitoring. Thus, the principal finds it optimal to induce a smaller production distortion without monitoring.
The agent therefore receives a higher total value of information rent without monitoring, which implies a higher investment threshold. It follows that the optimal contract is more efficient without monitoring.

7 Discussion

Relevance of private post-contractual information

In the literature on dynamic mechanism design, a central question concerns the relevance of the privacy of post-contractual information. Baron and Besanko (1984), Eső and Szentes (2007a,b) and Pavan et al. (2014) show that in many setups of dynamic adverse selection, the principal can costlessly elicit private information which the agent learns after contracting. Baron and Besanko (1984) and Eső and Szentes (2015) argue that this insight also holds in models of dynamic adverse selection and moral hazard.

In contrast, Krähmer and Strausz (2015) show in a model of pure adverse selection, that the privacy of post-contractual information matters if the agent’s private information learned before contracting (ex-ante type) is drawn from a discrete set. In this paper, privacy of post-contractual information turns out to be relevant in a model of dynamic adverse selection and moral hazard. Whereas the agent’s private information, i.e., the investment cost and the cost shock, are drawn from continuous distributions, the agent chooses an unobservable action from a discrete set. As in Krähmer and Strausz (2015), the agent’s best deviation strategy under no monitoring includes a strict lie about post-contractual information. As such a deviation is not feasible if post-contractual information is publicly observed, privacy of post-contractual information is relevant. However, there are a two notable differences to the result in Krähmer and Strausz (2015). First, the envelope theorem\(^\text{13}\) can be applied in the setup of the current paper in order to determine the agent’s expected utility as a function of his ex-ante type. This is not feasible in Krähmer and Strausz (2015) due to the discreteness of the agent’s ex-ante type.

\(^{13}\)Theorem 2 in Milgrom and Segal (2002)
Second, Krähmer and Strausz (2015) show that the principal’s optimal allocation under observable post-contractual information can still be implemented if ex-post information is unobservable, though at a lower revenue. In this paper, I show in Proposition 4, that the optimal allocation with shock monitoring – consisting of the investment and production decisions – cannot be implemented without monitoring. Privacy of post-contractual information restricts the set of implementable allocations for the principal, but does not change the principal’s ability to extract surplus from a given allocation, as in Krähmer and Strausz (2015). This also explains the relation to Eső and Szentes (2015). They study a general model of dynamic adverse selection and moral hazard. They show that the principal can achieve the same revenue under public and private post-contractual information, as long as the optimal allocation and the optimal actions under public information remain implementable with private information. In the model of the current paper, the principal cannot implement the optimal production and investment decisions with shock monitoring if she does not monitor (Proposition 4). The precondition of the result by Eső and Szentes (2015) is therefore not satisfied in this model.

**Binary investment decision and fixed costs of investment**

All results were presented using a simple model with a binary investment decision. A binary investment decision reflects non-convexities in the investment decision. In practice, many investment decisions are non-convex. If the investment represents the adoption of a new technology, the supplier is likely to face fixed costs of learning the new technology. Åstebro (2004) empirically documents such fixed costs of learning. More generally, *lumpy* investment behavior is often attributed to fixed costs of investment and such non-convex investment behavior is widely observed among firms.\(^{14}\)

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\(^{14}\)Doms and Dunne (1998) provide empirical evidence and Caplin and Leahy (2010) survey the history of applications of the \((S,s)\)-model, an investment model with fixed costs.
8 Conclusion

What kind of information should government institutions monitor in order to manage public contracts efficiently? This paper provides an analysis of this question in a principal-agent model where the principal can decide whether to monitor an agent’s investment in cost reduction or a shock to production costs that the agent learns after contracting. I show that the principal optimally monitors either the investment decision or the cost shock. Under investment monitoring, the principal achieves at least the same payoff – gross monitoring costs – as under shock monitoring. However, moral hazard may be irrelevant under shock monitoring. In this case, both monitoring technologies are equally effective.

The results suggest that government institutions should monitor endogenous cost factors (such as investments) rather than exogenous cost factors (such as cost shocks) if the costs of monitoring are similar across outcomes. However, if exogenous cost factors are cheaper to monitor then doing so can give a government institution the same control over private suppliers at lower costs.
9 Appendix

Proof of Proposition 1 The proof follows from the discussion in the main text.

Proof of Lemma 1 Condition 1 is equivalent to (7) for \( x = 1, x' = x'' = 0 \), and for \( x = 0, x' = x'' = 1 \). Condition 2 is equivalent to (7) for \( x = x' = x'' = 0 \). Conditions 3 and 4 are implied by condition 1, (7) for \( x = x'' = 1, x' = 0 \), and for \( x = x'' = 0, x' = 1 \). Conversely, conditions 1, 3, and 4 imply (7) for \( x = x' = x'' = 1 \). It remains to prove that conditions 1 to 4 imply (7) for \( x = x' = 1, x'' = 0 \), and for \( x = x' = 0, x'' = 1 \).

The first constraint can be rewritten as

\[
\int_0^1 (c_0(\varepsilon) - c_1(\varepsilon))q_1(\varepsilon)d\varepsilon \geq \kappa
\]

for \( \kappa \in K_1 \). By condition 1, \( \kappa \leq \hat{\kappa} \) for \( \kappa \in K_1 \), and the constraint is implied by condition 4. The second constraint can be written as

\[
\int_0^1 (c_0(\varepsilon) - c_1(\varepsilon))q_0(\varepsilon)d\varepsilon \leq \kappa
\]

for \( \kappa \in K_0 \). By condition 1, \( \kappa \geq \hat{\kappa} \) for \( \kappa \in K_0 \). The constraint is therefore implied by condition 3.

Proof of Proposition 2 Plug \( \hat{\kappa} = \kappa^i \) and efficient production rules in conditions 3 and 4 of Lemma 1. The left-hand side of condition 3 can be rewritten as

\[
\int_0^{\varepsilon^1_0} (c_0(\varepsilon) - c_1(\varepsilon))d\varepsilon = \int_0^{\varepsilon^1_1} (v - c_1(\varepsilon))d\varepsilon - \int_0^{\varepsilon^1_3} (v - c_0(\varepsilon))d\varepsilon - \int_0^{\varepsilon^1_0} (v - c_1(\varepsilon))d\varepsilon
\]

\[= \kappa^* - \int_{\varepsilon^1_0}^{\varepsilon^1_1} (v - c_1(\varepsilon))d\varepsilon
\]

Condition 3 is therefore satisfied if the condition in the lemma is satisfied. The left-hand side of condition 4 can be written as \( \kappa^* - \int_{\varepsilon^1_0}^{\varepsilon^1_1} (v - c_0(\varepsilon))d\varepsilon \) which is
greater than \( \kappa^* \). Condition 4 is therefore always satisfied. Note that conditions 3 and 4 do not restrict the choice of \( U_0 \) and \( U_1 \). One can therefore set \( U_0 \) and \( U_1 \) as in the optimal contract under monitoring of the investment, so that expected transfers are identical in both cases.

\[ \square \]

**Proof of Proposition 3** If moral hazard is irrelevant, the result is immediate. Suppose moral hazard is relevant and consider the principal’s problem (8). Neglect condition 4 of Lemma 1. It is optimal to set \( U_0 = 0 \) and \( q_1(\cdot) = q_1^* (\cdot) \).

I now show that the optimal threshold \( \kappa^* \) satisfies \( \kappa^* \in (\kappa_i, \kappa^*) \). Suppose \( \kappa^* \leq \kappa_i \). As moral hazard is relevant, \( q_0(\cdot) = q_0^* (\cdot) \) is not feasible. It follows that the marginal gain of an additionally investing agent type exceeds the marginal information rent:

\[ G(q_1^*, q_0^*, \kappa^*) - F(\kappa^*)/f(\kappa^*) > G(q_1^*, q_0^*, \kappa^*) - F(\kappa^*)/f(\kappa^*) = 0. \]

It is therefore profitable to increase \( \kappa^* \) which also relaxes condition 3. It follows that \( \kappa^* > \kappa_i \). Suppose next \( \kappa^* \geq \kappa^* \). The proof of Proposition 2 implies that \( q_0(\cdot) = q_0^* (\cdot) \) is feasible. The marginal gain of an additionally investing agent type is then lower as the marginal information rent:

\[ G(q_1^*, q_0^*, \kappa^*) - F(\kappa^*)/f(\kappa^*) \leq - F(\kappa^*)/f(\kappa^*) < 0. \]

Decreasing \( \kappa^* \) is profitable. It follows \( \kappa^* < \kappa^* \).

Furthermore, note that \( G(q_1^*, q_0^*, \kappa^*) \leq F(\kappa^*)/f(\kappa^*) \) in any optimum. If this does not hold it is always profitable to increase \( \kappa^* \) and set \( q_0 \) closer to \( q_0^* \).

Moreover, condition 3 of Lemma 1 is satisfied with equality at the optimum. If condition 3 is satisfied with strict inequality, it is possible to set \( q_0(\cdot) \) closer to \( q_0^* (\cdot) \) and increase the payoff.

One can therefore write the threshold \( \hat{\kappa} \) as a function of \( q_0^* (\cdot) \). Plugging this into (4) and taking the pointwise first-order derivative with respect to \( q_0 \) for any \( \varepsilon \) gives

\[
\begin{align*}
\frac{f(\hat{\kappa}(q_0))}{f(\hat{\kappa}(q_0))} \left\{ 1 - \frac{F(\hat{\kappa}(q_0))}{f(\hat{\kappa}(q_0))}(v - c_0(\varepsilon)) \right. & - \left. \left( \frac{F(\hat{\kappa}(q_0))}{f(\hat{\kappa}(q_0))} - G(q_1^*, q_0, \hat{\kappa}(q_0)) \right) u_1^*(\varepsilon) \right\}.
\end{align*}
\]

(25)

For any fixed threshold \( \hat{\kappa} \) that satisfies \( \hat{\kappa} \in (\kappa_i, \kappa^*) \) and \( G(q_1^*, q_0, \hat{\kappa}) < F(\hat{\kappa})/f(\hat{\kappa}) \), this expression is decreasing in \( \varepsilon \) under Assumption 1. If \( u_1^*(\varepsilon) \) is increasing, this follows
from $G(q^*_1, q_0, \hat{\kappa}) < F(\hat{\kappa})/f(\hat{\kappa})$. If $u_1^*(\varepsilon)$ is decreasing, it follows from Assumption 1 as $F(\kappa^*)/(1 - F(\kappa^*)) > F(\hat{\kappa})/(1 - F(\hat{\kappa}) - f(\hat{\kappa})/(1 - F(\hat{\kappa})) \cdot G(q^*_1, q_0, \hat{\kappa})$. Thus, the threshold $\hat{\kappa}$ is optimally implemented by a step function $q_0(\varepsilon) = 1(\varepsilon \leq \varepsilon^*_0)$. Since $\kappa^s$ satisfies $\kappa^s \in (\kappa^i, \kappa^*)$ and $G(q^*_1, q_0, \kappa^s) < F(\kappa^s)/f(\kappa^s)$, there exists a cutoff $\varepsilon^*_0$ which optimally implements $\kappa^s$ and this implies (14). There is a unique combination of $\kappa^s$ and $\varepsilon^*_0$ which equates the first-order derivative to zero and satisfies (15). This follows from $\kappa^s$ being increasing in $\varepsilon^*_0$, $(1 - F(\hat{\kappa}))/f(\hat{\kappa})$ being increasing in $\kappa^s$, and $G(q^*_1, q^*_0, \kappa^s) - F(\kappa^s)/f(\kappa^s)$ being increasing in $\varepsilon^*_0$ and $\kappa^s$, for $q^*_0(\varepsilon) = 1(\varepsilon \leq \varepsilon^*_0)$.

It remains to show that $\varepsilon^*_0 < \varepsilon^*_0$. For $\varepsilon^*_0 \geq \varepsilon^*_0$, there is no first-order loss from decreasing $\varepsilon^*_0$ whereas there is a first-order gain from a lower fraction of investing agent types as $F(\kappa^s)/f(\kappa^s) - G(q^*_1, q^*_0, \kappa^s) > 0$. Finally, one can easily check that condition 4 of Lemma 1 is satisfied as $\kappa^s < \kappa^*$. Optimal transfers can be derived from the definitions of $U_1$ and $U_0$.

**Proof of Lemma 2** For $\tilde{\varepsilon}(\varepsilon) = c_{x'}^{-1}(c_{x'}(\varepsilon))$, (16) implies

$$t_x(\varepsilon') - c_{x'}(\varepsilon)q_x(\varepsilon') = t_x(\varepsilon') - c_x(c_{x'}^{-1}(c_{x'}(\varepsilon)))q_x(\varepsilon')$$

$$\leq t_x(c_{x'}^{-1}(c_{x'}(\varepsilon))) - c_x(c_{x'}^{-1}(c_{x'}(\varepsilon)))q_x(c_{x'}^{-1}(c_{x'}(\varepsilon)))$$

$$= t_x(\tilde{\varepsilon}(\varepsilon)) - c_{x'}(\varepsilon)q_x(\tilde{\varepsilon}(\varepsilon))$$

**Proof of Lemma 3** By standard mechanism design arguments, one can show that condition 5 of the lemma is sufficient and necessary for (16). (17) can be rewritten as follows. The left-hand side equals $U_x - \kappa x$. Using Lemma 2 and condition 5 one can
rewrite the right-hand side as follows (where I use the notation $\tilde{\varepsilon}(\varepsilon) = c_{x''}^{-1}(c_{x''}(\varepsilon))$):

$$
\int_0^1 \max (t_{x'}'(\varepsilon') - c_{x''}(\varepsilon)q_{x'}(\varepsilon')) \, d\varepsilon - \kappa x''
$$

$$
= \int_0^1 (t_{x'}'(c_{x''}^{-1}(c_{x''}(\varepsilon)))) - c_{x''}(c_{x''}^{-1}(c_{x''}(\varepsilon)))q_{x'}(c_{x''}^{-1}(c_{x''}(\varepsilon)))) \, d\varepsilon - \kappa x''
$$

$$
= \int_0^1 (t_{x'}(1) - c_{x'}(1)q_{x'}(1) + \int_{\tilde{\varepsilon}(\varepsilon)}^1 c_{x'}(z)q_{x'}(z) \, dz) \, d\varepsilon - \kappa x''
$$

$$
= U_x' + \int_{\tilde{\varepsilon}(\varepsilon)}^1 c_{x'}(z)q_{x'}(z) \, dz \, d\varepsilon - \kappa x''
$$

$$
= U_x + \int_{\tilde{\varepsilon}(\varepsilon)}^1 \varepsilon c_{x'}(\tilde{\varepsilon}(\varepsilon))q_{x'}(\tilde{\varepsilon}(\varepsilon)) \frac{d\tilde{\varepsilon}}{d\varepsilon} \, d\varepsilon - \int_0^1 \varepsilon c_{x'}(\varepsilon)q_{x'}(\varepsilon) \, d\varepsilon - \kappa x''
$$

$$
= U_x + \int_{\tilde{\varepsilon}(\varepsilon)}^1 c_{x'}(\varepsilon)q_{x'}(\varepsilon)(c_{x''}^{-1}(c_{x''}(\varepsilon)) - \varepsilon) \, d\varepsilon - \kappa x''.
$$

where the first inequality follows from Lemma 2, the second from condition 5, the fourth from integration by parts, and the fifth from a change of variable from $\varepsilon$ to $\tilde{\varepsilon}$. Under condition 5, (17) is therefore equivalent to

$$
U_x - \kappa x \geq \max \left\{ U_x + \int_{\tilde{\varepsilon}(\varepsilon)}^1 c_{x'}(\varepsilon)q_{x'}(\varepsilon)(c_{x''}^{-1}(c_{x''}(\varepsilon)) - \varepsilon) \, d\varepsilon - \kappa x'', 0 \right\}
$$

for $\kappa \in K_x$ and $x, x', x'' \in \{0, 1\}$. The equivalence of this condition to the conditions 1 to 4 of the lemma can be shown by taking exactly the same steps as in the proof of Lemma 1.

Proof of Proposition 4  Plug $q_{x}^*(\varepsilon)$ into the left-hand sides of conditions 3 and 4 of Lemma 3.

$$
\int_{0}^{\varepsilon_x^*} c_{x'}(\varepsilon - c_{x'}^{-1}(c_{x}(\varepsilon))) \, d\varepsilon = \int_{0}^{\varepsilon_x^*} c_{x'}(\varepsilon) \, d\varepsilon - \int_{0}^{\varepsilon_x^*} c_{x'}(\varepsilon)c_{x'}^{-1}(c_{x}(\varepsilon)) \, d\varepsilon
$$

$$
= \int_{0}^{\varepsilon_x^*} c_{x'}(\varepsilon) \, d\varepsilon - \int_{0}^{\varepsilon_x^*} c_{x'}(\varepsilon) \, d\varepsilon
$$

$$
= \int_{0}^{\varepsilon_x^*} (v - c_{x}(\varepsilon)) \, d\varepsilon - \int_{0}^{\varepsilon_x^*} (v - c_{x'}(\varepsilon)) \, d\varepsilon
$$

$$
= (-1)^{1(x=1)} \kappa^*.
$$
where the second equality follows from a change of variable and the third equality follows from integration by parts. \( \kappa^* \) is therefore the only threshold that is incentive compatible and individual rational under efficient production.

In order to prove the second part of the lemma I show that conditions 3 and 4 in Lemma 1 are less restrictive than conditions 3 and 4 in Lemma 3. In order to see this, note that for \( \tilde{\varepsilon}(\varepsilon) = c_x^{-1}(c_x(\varepsilon)) \)

\[
\int_0^1 c'_x(\varepsilon) q_x(\varepsilon)(c_x^{-1}(c_x(\varepsilon)) - \varepsilon) d\varepsilon = \int_0^1 \int_{\tilde{\varepsilon}(\varepsilon)}^\varepsilon c'_x(z) q_x(z) dz d\varepsilon
\]

\[
= \int_0^1 (c_x(\varepsilon) - c_x'(\varepsilon)) q_x(\varepsilon) d\varepsilon - \int_0^1 (c_x(\varepsilon) - c_x(\tilde{\varepsilon}(\varepsilon))) q_x(\varepsilon) d\varepsilon + \int_0^1 \int_{\tilde{\varepsilon}(\varepsilon)}^\varepsilon c'_x(z) q_x(z) dz d\varepsilon
\]

\[
= \int_0^1 (c_x(\varepsilon) - c_x'(\varepsilon)) q_x(\varepsilon) d\varepsilon + \int_0^1 \int_{\tilde{\varepsilon}(\varepsilon)}^\varepsilon c'_x(z)(q_x(z) - q_x(\varepsilon)) dz d\varepsilon
\]

\[
> (\cdot) \int_0^1 (c_x(\varepsilon) - c_x'(\varepsilon)) q_x(\varepsilon) d\varepsilon \quad \text{for } x = 0, x' = 1 \quad (x = 1, x' = 0)
\]

where the last inequality holds if \( q_x(\varepsilon) \) is not constant on \([0, 1]\). This is the case for \( q_0^a \) and \( q_1^a \). Since condition 3 of Lemma 1 is satisfied with equality for \( q_0^a \) and \( \kappa^a \), the stricter condition 3 of Lemma 3 cannot be satisfied for \( q_0^a \) and \( \kappa^a \).

\[\square\]

**Proof of Proposition 5**  The proof is only sketched as it essentially follows the same steps as the proof of Proposition 3. Consider the principal’s problem defined in (18) and neglect conditions 4 and 5. It is optimal to set \( U_0 = 0 \) and \( q_1(\cdot) = q_1^a(\cdot) \). Using the same arguments as in the proof of Proposition 3, it can be shown that the optimal investment threshold \( \kappa^a \) satisfies \( \kappa^a \in (\kappa^3, \kappa^*) \), \( G(q_1^a, q_0, \kappa^a) < F(\kappa^a)/f(\kappa^a) \), and that condition 3 is satisfied with equality. The last result allows to derive a first-order condition analogous to (25) with \( u^a_1(\varepsilon) \) instead of \( u^a_1(\varepsilon) \). By the same arguments as in the proof of Proposition 3, it can be derived that there is a unique optimal pair \( \kappa^a \) and \( \varepsilon^a_0 \) with \( q_0^a(\varepsilon) = 1(\varepsilon \leq \varepsilon^a_0) \) which satisfies (22) and (23), and \( \varepsilon^a_0 < \varepsilon^*_0 \). Clearly \( q_0^a(\varepsilon) \) is decreasing in \( \varepsilon \), transfers can be chosen to satisfy the requirement of condition 5, and it can be shown that condition 4 is satisfied.  \[\square\]
Proof of Proposition 6  By Proposition 1, it is never optimal to monitor the shock and the investment as this would give the payoff $\Pi^i$ which can also be achieved if only investment is monitored. The conditions for optimality are derived from the payoffs $\Pi^i - C^i$, $\Pi^s - C^s$, and $\Pi^n$ that can be achieved under monitoring of investment, the shock, and no monitoring. From Proposition 3 it follows that $\Pi^i \geq \Pi^n$. If $\Pi^s > \Pi^n$, there exist $C^i > 0$ and $C^s > 0$ such that any of the three monitoring choices can be optimal. $\Pi^s > \Pi^n$ follows from the fact that condition 3 of Lemma 3 is more restrictive than condition 3 of Lemma 1, which is implied by (26).

Proof of Proposition 7  The proof follows from the discussion in the main text.

Proof of Proposition 8  By Proposition 3, $\Pi^i = \Pi^s$ if moral hazard is irrelevant and $\Pi^i > \Pi^s$ is moral hazard is relevant. It only remains to show that there exists $a z' \in [z, \overline{z})$ such that moral hazard is relevant (i.e., (9) is satisfied) iff $z \geq z'$. The optimal investment threshold $\kappa^i_z$ is a function of $z$ implicitly defined by

$$\kappa^i_z + \frac{F_z(\kappa^i_z)}{f_z(\kappa^i_z)} = \kappa^*$$

From the definition of $F_z$ it follows that $F_z(\kappa)/f_z(\kappa) = F(\kappa - z)/f(\kappa - z)$. By log-concavity of $f$, $F_z(\kappa)/f_z(\kappa)$ is increasing in $\kappa$ and decreasing in $z$. This proves that $\kappa^i_z$ is increasing in $z$. Moreover $\kappa^s_z = \kappa^*$ as $F_z(\kappa^*) = 0$. As $\kappa^* - \kappa^i_z$ is therefore decreasing in $z$ and zero at $\overline{z}$, this establishes the existence of $z'$.

Proof of Proposition 9  For $v = \overline{v}$, (15) and (23) are solved by $\varepsilon_0^s = \varepsilon_0^n = 1$. As $u^s_1(1) = u^n_1(1) = 0$, there is by (14) and (22) no first-order effect on $\kappa^s$ and $\kappa^n$ for $v$ smaller but close to $\overline{v}$. However, $u^s_1(\varepsilon) > u^n_1(\varepsilon)$ for $\varepsilon$ close to one. This is implied by the following argument: Note that $u^s_1(1) = u^n_1(1) = 0$ and $\partial u^s_1(\varepsilon)/\partial \varepsilon = c'_0(\varepsilon) - c'_1(\varepsilon)$. For $\varepsilon$
close to one, the following approximation holds

\[
\frac{\partial u^n_1(\varepsilon)}{\partial \varepsilon} = c''_0(\varepsilon) \left( \frac{c'_0(\varepsilon)}{c'_1(c^{-1}_1(c_0(\varepsilon)))} - 1 \right) \simeq \frac{(c'_0(\varepsilon))^2}{c'_1(\varepsilon)} - c'_0(\varepsilon)
\]

as \( c^{-1}_1(c_0(\varepsilon)) \simeq \varepsilon \) for \( \varepsilon \) close to one. It follows for \( \varepsilon \) close to one

\[
\frac{\partial u^n_1(\varepsilon)}{\partial \varepsilon} - \frac{\partial u^n_1(\varepsilon)}{\partial \varepsilon} \simeq \left( \frac{(c'_0(\varepsilon))^2}{c'_1(\varepsilon)} - c'_0(\varepsilon) \right) - (c'_0(\varepsilon) - c'_1(\varepsilon))
\]

\[
= \frac{1}{c'_1(\varepsilon)} (c'_0(\varepsilon)^2 - 2c'_0(\varepsilon)c'_1(\varepsilon) + c'_1(\varepsilon)^2) = \frac{1}{c'_1(\varepsilon)} (c'_0(\varepsilon) - c'_1(\varepsilon))^2 > 0
\]

It follows by continuity that \( u^n_1(\varepsilon) > u^n_0(\varepsilon) \) for \( \varepsilon \) close to one. This implies

\[
1 - \frac{F(\kappa^n)}{f(\kappa^n)} (v - c_0(\varepsilon^n_0)) - u^n_1(\varepsilon^n_0) \left( \frac{F(\kappa^n)}{f(\kappa^n)} - G(q^*_1, q^*, \kappa^n) \right)
\]

\[
\simeq 1 - \frac{F(\kappa^*)}{f(\kappa^*)} (v - c_0(\varepsilon^n_0)) - u^n_1(\varepsilon^n_0) \left( \frac{F(\kappa^*)}{f(\kappa^*)} - G(q^*_1, q^*, \kappa^*) \right)
\]

\[
> 1 - \frac{F(\kappa^*)}{f(\kappa^*)} (v - c_0(\varepsilon^n_0)) - u^n_1(\varepsilon^n_0) \left( \frac{F(\kappa^*)}{f(\kappa^*)} - G(q^*_1, q^*, \kappa^*) \right) = 0.
\]

Thus, \( \varepsilon^n_0 > \varepsilon^n_0 \) and \( \kappa^n > \kappa^* \) for \( v \) sufficiently close to \( \tau \).

References


NAO (2014). Transforming government’s contract management. Report "HC269" by the National Audit Office (NAO), UK.


