BDPEMS
Berlin Doctoral Program in Economics and Management Science

Working Paper Series
2015-04

The Maximum Punishment Principle and Precision of Audits under Limited Commitment

Martin Pollrich, Humboldt-Universität zu Berlin

Sponsored by

EINSTEIN Foundation.de
Einstein Stiftung Berlin
Einstein Foundation Berlin

BDPEMS
Spandauer Straße 1
10099 Berlin
Tel.: +49 (0)30 2093 5780
mail: bdpems@hu-berlin.de
http://www.bdpems.de

The Maximum Punishment Principle and Precision of Audits under Limited Commitment

Martin Pollrich*

March 23, 2015

Abstract

For optimal audit contracts the principle of maximum deterrence applies: penalties imposed by the contract are either zero or at their maximal level. Additionally, an imperfect audit technology which reveals the agent's type only with an error makes the principal worse off. In this paper I show that both statements are no longer true when the principal cannot commit to an audit strategy. Both intermediate penalties and imperfect audits facilitate the creation of incentives for the principal to carry out an audit.

JEL classification: D82, D86, C72

Keywords: Auditing, limited commitment, contract theory

*Humboldt-Universität zu Berlin, *martin.pollrich@hu-berlin.de*
The author is grateful to Roland Strausz for helpful advice and suggestions. Financial support by Deutsche Forschungsgemeinschaft through SFB/TR 15 and RTG 1659 is gratefully acknowledged.

1 Introduction

Two generally recognized facts with regard to the optimal use of auditing in contract theory are: the maximum punishment principle and that imperfect audit technologies make the principal worse off. This paper shows that neither of the two holds when commitment to the audit policy is not possible. Without commitment the contract has to provide incentives for the principal to carry out an audit. When the principal decides whether to audit, all that matters is the expected revenue from doing so compared to the audit cost. Intermediate penalties allow to fine-tune the revenue from carrying out an audit and therefore relax the problem of inducing an audit in the first place. Also imperfect audits have a similar beneficial effect.

The maximum punishment principle, established by Baron and Besanko (1984), states that if the optimal contract under full commitment calls for a penalty, this penalty is either zero or at the maximal feasible level. Audits are introduced to deter the agent from misreporting and maximal deterrence is achieved with maximal penalties. Rather than reducing the penalty, the principal reduces the probability of an audit which saves audit costs.

Without commitment to an audit strategy, penalties serve another purpose: providing incentives for the principal to actually carry out an audit. Facing this decision, the principal compares the cost associated with an audit to the expected revenue from doing so. Hence, intermediate penalties considerably increase the leeway in fine-tuning the principal's incentives. For example, setting all penalties *above* the audit cost ensures that the principal's expected revenue from an audit exceeds the cost from carrying it out, and therefore renders the audit profitable irrespective of the principal's belief on the agent's type. Building on this intuition, I show that with intermediate penalties the principal benefits from implementing a positive level of audits, even though this was undesirable when complying to the principle of maximum deterrence.

The second part of this paper deals with imperfect audits. Again, under full commitment the principals benefits from increasing the precision of the audit technology, as shown for example by Kofman and Lawarrée (1993) and Laffont and Tirole (1992). There are two reason for this. First, imperfect audits lower the deterrent effect because a misreport may remain undetected. Second, mistakenly penalizing the agent requires compensation via a larger transfer. The latter feature can have also a positive impact under limited commitment. The error in the audit technology increases the expected revenue from carrying out an audit, which relaxes the problem of incentivizing the principal. For this reason the principal benefits from auditing with an imperfect technology, as compared to a perfect audit technology because with the former it is cheaper to create incentives for himself. Chen and Liu (2008) make a similar point, but in their model the principal is restricted to offering menus of contracts. I allow for general mechanisms that use mediators and show that nevertheless imperfect audits can be beneficial.

2 Model

Consider a principal who wants to delegate the production of q units of a good to an agent. The principal's value of q units is $V(q)$, where $V(\cdot)$ is strictly increasing, strictly concave and $V(0) = 0$. The production costs θq of the agent are unobservable to the principal, but it is common knowledge that θ belongs to the set $\Theta = \{\theta_l, \theta_h\}$. The agent can have low costs (θ_l) or high costs (θ_h) with the respective probabilities ϕ and $1 - \phi$. Denote $\Delta\theta = \theta_h - \theta_l > 0$ the spread of uncertainty on the agent's marginal cost. The agent is informed about his type θ when taking the production decision. Both the principal and the agent are risk-neutral and I assume that it is always optimal for the principal to employ either type of the agent.[1].

The principal purchases q units and compensates the agent with a transfer t. In addition the principal possesses an audit technology, that provides the principal with a possibly imperfect signal about the agent's type. Using the audit result, a contract can call for the application of a penalty $\widetilde{P}$. The penalty is independent of the output-based transfer from the principal to the agent. Furthermore, the transfer cannot be made contingent on the principal's audit decision. An audit costs $c > 0$.

Concerning the details of the audit technology and feasible penalties, I consider the following three regimes:

$\mathcal{MP}$ *Maximum Punishment and Perfect Audits.* An audit reveals the agent's true type with certainty and $\widetilde{P} \in \{0, P\}$ for some $P > 0$.

$\mathcal{FP}$ *Flexible Punishment and Perfect Audits.* An audit reveals the agent's true type with certainty and $\widetilde{P} \in [0, P]$ for some $P > 0$.

$\mathcal{MI}$ *Maximum Punishment and Imperfect Audits.* The penalty is $\widetilde{P} \in \{0, P\}$ for some $P > 0$. The probability that an audit revelas type θ_i when the agent's true type is θ_j is given by $\pi_{ij} \in (0, 1)$.[2]

The principal can contractually commit to a transfer, a quantity and a punishment scheme, but not to an audit policy. A punishment scheme determines a penalty depending on the finding of an audit.

3 Maximum Punishment and Perfect Audits

Pollrich (2015) studies optimal mechanisms in the model as described above with regime $\mathcal{MP}$. A

[1]The Inada conditions $\lim_{q \to 0} V'(q) = +\infty$ and $\lim_{q \to 0} qV'(q) = 0$ are sufficient to rule out shutdown (see Laffont and Martimort, 2009, chap. 2.6).

[2]Of course, $\pi_{li} + \pi_{hi} = 1$ for all $i = l, h$.

crucial result that I will use in the remainder is the following

Lemma 1. *There exists a unique value $P^* \in (c/\phi, \infty)$, such that the optimal mechanism uses audits if and only if $P > P^*$.*

Following Lemma 1, whenever $P < P^*$ the optimal mechanism does not use audits. Then the revelation principle implies the principal cannot do better than offering an incentive compatible menu $\Gamma = \{(t_l, q_l), (t_h, q_h)\}$. Denote Γ^{na} the optimal menu offer and $\mathcal{V}^{na}$ the principal's profit from offering it. Standard computations show that

$$\mathcal{V}^{na} = \phi\big(V(q_l^{na}) - \theta_l q_l^{na} - \Delta\theta q_h^{na}\big) + (1-\phi)\big(V(q_h^{na}) - \theta_h q_h^{na}\big) \tag{1}$$

An implication of Lemma 1 is, that the principal's profit cannot exceed $\mathcal{V}^{na}$ whenever $P < P^*$. This holds in particular for all P such that $c/\phi < P < P^*$.

4 Failure of the Maximum Punishment Principle

In this section I consider regime $\mathcal{FP}$, i.e., I allow for penalties different from zero or P. The following Proposition shows that the no-audit contract is never optimal when $\phi P > c$.

Proposition 1. *For any $P > c/\phi$ the no-audit contract is not optimal.*

Proof. Consider the following menu offer

$$\Gamma = \{\gamma_l, \gamma_h\} = \{(t_l, q_l, P_l(\cdot)), (t_h, q_h, P_h(\cdot))\} \tag{2}$$

with $q_l = q_l^{na}, q_h = q_h^{na}, t_h = \theta_h q_h^{na} + \alpha c$ and $t_l = \theta_l q_l^{na} + \Delta\theta q_h^{na} - \alpha(P - c)$. Penalty schemes are $P_l(\cdot) \equiv 0$, and $P_h(\theta_h) = c$ as well as $P_h(\theta_l) = P$.

Then there exists a PBE where an agent of type θ_i picks contract γ_i with certainty and the principal audits with probability α whenever the agent picks contract γ_h, and does not audit otherwise.

To see this, consider first the agent's decision. Given the auditing policy of the principal, the low-cost type earns $\Delta\theta q_h^{na} - \alpha(P - c)$, no matter which contract she picks. For small enough α, this payoff is non-negative which guarantees participation. The high-cost type earns an expected rent of zero from picking contract γ_h, because with probability α she is audited and pays penalty c. Contract γ_l yields the high-cost type $-\Delta\theta(q_l^{na} - q_h^{na}) - \alpha(P - c) < 0$, because $q_l^{na} > q_h^{na}$ and $P > c$. This confirms the agent's strategy.

The principal trivially has no incentive to audit when the agent picks contract γ_l. Auditing when the agent picked contract γ_h yields a sure penalty payment of c, because it is certain that the audit reveals type θ_h. Hence, any strategy $\widetilde{\alpha} \in [0,1]$ is optimal, because an audit costs c. This confirms the principal's strategy and thus the equilibrium.

The principal's payoff from offering menu Γ with the associated PBE as outlined above is

$$\phi\big(V(q_l^{na}) - \theta_l q_l^{na} - \Delta\theta q_h^{na} + \alpha(P-c)\big) + (1-\phi)\big(V(q_h^{na}) - \theta_h q_h^{na} - \alpha c\big) = \mathcal{V}^{na} + \alpha(\phi P - c) \quad (3)$$

Provided $P > c/\phi$ the payoff exceeds $\mathcal{V}^{na}$ and the no-audit contract cannot be optimal. □

Under limited commitment, the penalty serves a dual purpose. First, as in the case of full commitment, it is used to deter the low-cost type from misreporting. This deterrence is maximal, when the respective penalty is set at its maximal level - which is the maximum punishment principle. On the other hand, the penalty can be used to incentivize the principal to carry out an audit in the first place. To this end, a penalty to the high-cost type is specified, which just compensates the principal for its audit cost. In particular this penalty is far from maximal.

Proposition 1 together with Lemma 1 implies failure of the maximum punishment principle. For any $P \in (c/\phi, P^*)$ Proposition 1 implies that the no audit contract is not optimal. On the other hand, by Lemma 1 the principal does not want to use audits when penalties are restricted to zero and P. Consequently, the optimal mechanism features auditing *and* intermediate penalties.

Corollary 1. *For $P \in (c/\phi, P^*)$ the principal is strictly better off with a mechanism that specifies penalties other than 0 and P, in other words the maximum punishment principle fails.*

5 Imperfect Audits

This section assume regime $\mathcal{MI}$, i.e., maximal punishments but audits can be imperfect.[3] I make a further simplifying assumption on the audit technology:

$$\pi_{ll} = 1, \quad \text{and} \quad \pi_{hh} = \pi = \frac{c}{P} \in (0,1). \quad (4)$$

This implies that an audit can perfectly identify a low-cost agent, but high-cost types are only identified with probability $\pi < 1$. Because probabilities add up to one, we have $\pi_{hl} = 0$ and $\pi_{lh} = 1 - \pi > 0$.

Under full commitment, the principal is clearly worse off with the imperfect audit technology, because he has to compensate the high-cost type for the expected punishment imposed on her. This is no longer the case under limited commitment. Consider first the following Proposition.

Proposition 2. *If $P \in (c/\phi, P^*)$ the no-audit contract is not optimal.*

[3] The interplay of imperfect audits and general penalties is left for future research.

Proof. Consider the menu offer

$$\Gamma = \{\gamma_l, \gamma_h\} = \{(t_l, q_l, P_l(\cdot)), (t_h, q_h, P_h(\cdot))\} \tag{5}$$

with $q_l = q_l^{na}, q_h = q_h^{na}, t_h = \theta_h q_h^{na} + \pi\alpha P$ and $t_l = \theta_l q_l^{na} + \Delta\theta q_h^{na} - \alpha(P - c)$. Penalty schemes are $P_l(\cdot) \equiv 0$, and $P_h(\theta_h) = 0$ as well as $P_h(\theta_l) = P$ - hence the menu uses only maximal punishments.

Then there exists a PBE where an agent of type θ_i picks contract γ_i with certainty and the principal audits with probability α whenever the agent picks contract γ_h, and does not audit otherwise.

The low-cost type earns $\Delta\theta q_h^{na} - \alpha(P - c)$ from reporting low costs, which is non-negative for α sufficiently small. Reporting high costs yields $\Delta\theta q_h^{na} + \pi\alpha P - \alpha P = \Delta\theta q_h^{na} - \alpha(P - c)$, because $\pi = c/P$. The high-cost type's payoff from contract γ_l is strictly negative. Contract γ_h yields $\theta_h q_h^{na} + \pi\alpha P - \pi\alpha P - \theta_h q_h^{na} = 0$. This confirms the agent's strategy. When the agent picks γ_l the principal has clearly no incentive to audit. Otherwise, auditing yields an expected payoff of $\pi P = c/P \cdot P = c$. Hence, auditing with probability α is a best response.

Offering the menu Γ the principal's payoff in the associated PBE outlined above is

$$\begin{aligned}\mathcal{V}(\Gamma) &= \phi\big(V(q_l^{na}) - \theta_l q_l^{na} - \Delta\theta q_h^{na} + \alpha(P - c)\big) + (1 - \phi)\big(V(q_h^{na}) - \theta_h q_h^{na} - \pi\alpha P\big) \\ &= \mathcal{V}^{na} + \phi\alpha P - \alpha c = \mathcal{V}^{na} + \alpha(\phi P - c)\end{aligned}$$

Thus, for *any* $P > c/\phi$ the no-audit contract cannot be optimal. □

Once more, the intuition behind Proposition 2 lies in the commitment problem. The imperfect audit relaxes the commitment problem, because it does not require any low-cost types to generate positive expected audit revenue. The commitment to penalize an agent whenever the audit reveals low costs is sufficient to incentivize the principal. If the error in the audit technology is sufficiently large, it is possible to generate audit incentives even though the menu separates the agent's types. This eliminates some inefficiencies of the optimal mediated audit mechanism and renders auditing profitable also for comparably low penalties, i.e., for $P < P^*$.

Proposition 2 together with Lemma 1 imply that the principal is strictly better off with imperfect audits for any $P \in (c/\phi, P^*)$. Following Proposition 2 the principal can achieve a strictly larger profit than $\mathcal{V}^{na}$ when auditing is imperfect and $\phi P > c$. On then other hand, with perfect audits the principal's maximal profit is $\mathcal{V}^{na}$ whenever $P < P^*$.

Corollary 2. *If $\pi = c/P$ and $P \in (c/\phi, P^*)$ the principal is better off with imperfect audits than with perfect audits.*

References

Baron, David P. and David Besanko, "Regulation, Asymmetric Information, and Auditing," *RAND Journal of Economics*, 1984, *15* (4), 447–470.

Chen, Hsiao-Chi and Shi-Miin Liu, "Incentive Contracts Under Imperfect Auditing," *Manchester School*, 2008, *76* (2), 131–159.

Kofman, Fred and Jacques Lawarrée, "Collusion in Hierarchical Agency," *Econometrica*, 1993, *61* (3), 629–56.

Laffont, Jean-Jacques and David Martimort, *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press, 2009.

_ **and Jean Tirole**, "Cost Padding, Auditing and Collusion," *Annales d'Économie et de Statistique*, 1992, *25/26.*

Pollrich, Martin, "Mediated Audits," 2015. BDPEMS Working Paper No. 2015-03.