On Informational Asymmetries in Merger Policy

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Abstract

This paper presents a model of merger policy with privately informed firms and an uninformed antitrust authority that may investigate the proposed merger at a cost. I distinguish between strongly opposed preferences and weakly opposed preferences over mergers. The former case may for example arise when the merging firms in a horizontal merger know about the substitutability of their products or whether other firms can produce a similar product, while the latter may arise when the merging firms know about efficiency gains due to the merger. In equilibrium mergers which are not investigated are always cleared and welfare-enhancing mergers are always implemented. Under strongly opposed preferences the worst mergers, in terms of social welfare are proposed by the firms, while moderately welfare-reducing mergers are not. Under weakly opposed preferences all mergers which have a sufficiently large though possibly negative welfare effect are proposed. When filing fees are available the first best can be implemented under weakly opposed preferences but not under strongly opposed preferences. Filing fees have a beneficial deterrence effect. As in previous models a total welfare standard is not optimal for the antitrust authority. I argue that the case of strongly opposed preferences is of practical relevance.

Keywords: mergers; competition policy; antitrust; asymmetric information

JEL Classification Numbers: L40, L41, D82, K21

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1 Introduction

Mergers are an important area of concern for policy-makers. While mergers have the potential to improve welfare by reducing costs of production through synergies, policy-makers are often concerned with the adverse consequences on competition resulting from an increase in the concentration of production in some industries. Balancing these two concerns is the focus of merger policy as practiced by antitrust authorities. This paper considers antitrust policy in the case in which the firms are privately informed about the consequences of a merger, while the antitrust authority is at an informational disadvantage.

In the model I study it is assumed that a merger typically leads to some increase in market power, but also leads to some efficiency gains. The focus lies on what the implications of asymmetric information are for the conduct of merger policy. We distinguish two cases. In the first case, which I refer to as the case of strongly opposed preferences, the firms are informed about the degree to which post-merger they will be able to exploit their market power, while there is relatively little private knowledge regarding the level of efficiency gains to be achieved. The ability to exploit market power may be seen as capturing the strength of competition between the merging firms’ products, the cost competitiveness of non-merging competitors or the degree of substitutability of non-merging firms products. In the second case, which I refer to as the case of weakly opposed preferences, the firms are better informed than the antitrust authority about the extent to which synergies will lead to cost savings post-merger.

In the model the antitrust authority may carry out a costly investigation of the effects of the merger. After an investigation the authority will only allow mergers that raise welfare to proceed. Other mergers are blocked. It is also shown that in equilibrium the antitrust authority will clear a merger if there was no investigation. Whenever the cost of investigating a merger is sufficiently low, the antitrust authority carries out an investigation. All firms that expect to make a profit from merging, taking account of a small cost of proposing to merge, will propose a merger to the antitrust authority. This includes all welfare-increasing mergers. Relatively unprofitable welfare-reducing mergers are deterred by the threat of an investigation, while relatively profitable welfare-reducing mergers are proposed but may be blocked following an investigation. By allowing the antitrust authority to carry out costly investigation, the severe incentive problems arising in the case of strongly opposed preferences may be partially overcome.

The case of private information concerning efficiencies (i.e. weakly opposed preferences) has been analyzed by some previous papers, e.g. Heidhues and Lagerlöf (2005), Ottaviani and Wickelgren (2011) and in particular Besanko and Spulber (1993). The case of strongly opposed preferences has received relatively little attention in the previous literature. However in some cases the policy implications differ markedly across the two cases. In particular it can be shown that filing fees, independent of the outcome of any investigation, may be used in the case of weakly opposed preferences to implement the first best, without the use of investigations. An earlier analysis of this
case by Besanko and Spulber (1993) only considered marginal changes of filing fees that did not change the type of equilibrium. Even in the Besanko and Spulber (1993) model a filing fee may be found that implements the first best.

In contrast, when preferences are strongly opposed filing fees are not sufficient to implement the first best. To see this consider a simple example in which two firms may either compete in prices or be active in completely unrelated markets. If the firms are active in independent markets, then the merger will have no effect beyond efficiency gains, while if firms compete before the merger then the merger will lead to an increase in prices. In any case, the gain from a merger is larger to the merging firms when they compete strongly before the merger. If the authority simply offered to allow any merger, subject to a filing fee being paid, then the merger to monopoly would most likely be able to be profitable after fees being paid. Hence such a mechanism would lead to the worst mergers, from a social point of view, being allowed. This is why the use of filing fees is limited when preferences are strongly opposed.

Using our model it can be shown that, like in the model of Besanko and Spulber (1993), it is not optimal for the antitrust authority to be endowed with a total welfare standard. The government would prefer to have the authority be tougher against mergers. One way to achieve this is by letting the antitrust authority use a welfare standard which puts less weight on producers surplus than on consumer surplus.

The results in this paper are to some extent foreshadowed by earlier work by Rey (2002) which, in terms of the message of the paper also appears close to this paper. That paper highlights the informational problem faced by the antitrust authority, who without knowing the motivation for a merger, about which the firms have better information, needs to make a decision. Rey provides a few examples of the motivations that the firm may privately be aware of: efficiency gains, increased market power, enhanced scope for collusion. Rey also alludes to the way in which the nature of information may affect the effectiveness of competition policy. In the case of private information regarding efficiencies it is suggested that firms could be "asked to 'pay' in one form or another for any negative external effect of the merger, so as to ensure that only socially desirable mergers are proposed". Like the present paper, Rey sees a problem of enforcement whenever firms’ private information does not cover efficiency gains. In that case, he argues that the "authority would prefer mergers that generate only a small increase in market power, but it is precisely the firms with the highest ability to exert market power that will be the most eager to get their merger accepted". Rey does not elaborate on his verbal discussion and continues to suggest that in the latter case a pooling equilibrium may be obtained. The present paper formalizes many of the ideas in Rey (2002). The basic intuition of Rey is shown to be correct in that mergers when firms’ private information mainly covers non-efficiency factors are indeed problematic. The way that this paper tries to circumvent the problem of non-responsiveness as alluded to by Rey, is to introduce the ability for the antitrust authority to investigate mergers. This is then shown to lead to a tractable model with which to analyze informational aspects in merger policy in the spirit of Rey (2002).
The model allows to discuss two principal effects through which antitrust authority may be beneficial. First, there is the beneficial screening effect. This effect captures the fact that a proposed merger, if investigated by an antitrust authority, will be prohibited if this merger is found to reduce welfare. Second, there is the beneficial deterrence effect. Mergers which would reduce welfare are not proposed by the firms, who expect the merger to be investigated and subsequently blocked. This effect tends to be less visible than the first since data on merger opportunities that were foregone due to expected antitrust problems is harder to collect than information on mergers which were blocked by antitrust authorities. Two possible negative effects of merger policy are ignored in the benchmark case, but may appear in some of the applications. These negative effects of antitrust policy may also be present in practice. First, by assuming that the investigations reveal all information to the antitrust authority, there is no possibility that the antitrust authority, after an investigation, incorrectly clears a welfare-decreasing merger or incorrectly blocks a welfare-improving merger. This is the adverse screening effect. Mistakes in screening mergers can also change the deterrence effect of antitrust authority. For example, if welfare-improving mergers may be blocked, then this may lead some firms to stop proposing welfare-improving mergers. In this case there would be over-deterrence of some mergers. This effect is the adverse deterrence effect. In the applications some of these negative effects of merger policy will be considered. The adverse screening effect appears only under strongly opposed preferences.

The paper proceeds as follows. Section 1.1 reviews related literature. Section 2 introduces the model and provides some examples of economic environments to which this model may be applicable. Section 3 studies the equilibrium of the game. Section 4 considers some extensions to the benchmark model. The use of filing fees, an imperfect investigation technology, commitment and different welfare standards are considered. Section 5 discusses the policy relevance and implications of the model. Section 6 concludes.

1.1 Literature Overview

This paper fits into the general literature on antitrust policy, in particular with regard to mergers. While there is some discussion of the benefits and costs of merger policy (see e.g. Crandall and Winston (2003), Baker (2003), Baker and Shapiro (2008)), this discussion is framed without the use of a formal model. This paper presents such a model which allows to discuss issues such as the deterrence effect of merger policy. Furthermore, a formal model may have the additional gain in clarifying how merger policy may be improved, rather than debating whether an authority erred in allowing or blocking a particular merger. Most of the literature focuses on assessing the likely consequences of a merger and whether antitrust authorities do so correctly. Carlton (2009) highlights that in assessing antitrust policy of mergers one needs information both on the price change induced by the merger as well as the antitrust authority’s prediction of the price change, taking account of selection effects, which are the focus of this paper. There are now a fair number
of empirical studies, that attempt to estimate the post-merger price effects of mergers to assess the quality of merger policy. For examples, see Ashenfelter and Hosken (2010), Ashenfelter, Hosken and Weinberg (2013), Aguzzoni et al. (2015) and Aguzzoni et al. (2014).

A number of other studies are concerned with assessing the decisions of antitrust authorities empirically. Studies of this kind, following Eckbo (1983) and Eckbo (1992), typically rely on stock market information to assess the likely consequences of mergers. For example, Duso, Neven and Röller (2007) analyze decisions of the European Commission between 1990 and 2001 using stock market information to infer the likely consequences of a merger. They find that institutional and political factors affected merger control decisions, in addition to the consumer surplus effect of the merger. In a subsequent paper Duso, Gugler and Szücs (2013) assess the changes to European merger control in 2004 using a sample covering 368 mergers between 1990 and 2007. This paper also makes use of stock market data to infer the likely consequences of a merger.

The theoretical literature on merger policy can be categorized into two types. First, there are papers which specify how competition between firms in some market works and how this would then be affected by a merger. Prominent examples of this type of paper are Williamson, (1968) and Farrell and Shapiro (1990). Williamson (1968) shows that even a small improvement in efficiency is often sufficient to ensure that a merger to monopoly increases overall welfare. This is because reductions in marginal costs are gained over all units which are produced, while the loss from market power is often only of second-order importance. Furthermore, Williamson showed that for somewhat larger levels of efficiency gains, consumers may also benefit from such mergers to monopoly. Farrell and Shapiro (1990) analyzed the case of an industry in which firms compete in quantities and, upon merging, may realize efficiency gains. By looking at ‘marginal mergers’, Farrell and Shapiro arrive at a condition on the externalities of the merger as an indicator of the welfare effects of mergers.

The second strand of the literature, to which this paper belongs, treats the competitive interactions in the markets affected by a merger in a reduced form way and instead analyzes the implications of an imperfectly informed antitrust authority for optimal mandates given to the authority. One of the first papers in that literature is the one by Besanko and Spulber (1993). In that model, firms are privately informed about the efficiency gains from a merger. The antitrust authority may only clear or block mergers. Their model corresponds to a special case of my model. That paper’s focus is on showing that when an antitrust authority which may not commit to a particular merger policy, may nevertheless be able to implement the commitment outcome by appropriately changing the weight given to producer surplus in the welfare standard that is used to judge mergers. Besanko and Spulber (1993) also consider the use of filing fees as a policy instrument.

The paper closest to the current paper is Sørgard (2009), who investigates the enforcement and deterrence effect of antitrust policy. In that model it is assumed that mergers can be ordered according to their welfare effect and the antitrust authority can conduct investigations, which
are however subject to error. While Sørgard is agnostic about how the relationship between social welfare effect of a merger and its associated profit, his main results focus on cases in which mergers with the largest negative impact on welfare are deterred, which in the model studied here corresponds to the case in which preferences are weakly opposed. The paper focuses on deriving the enforcement effect, that is whether antitrust policy correctly blocks welfare-reducing mergers while allowing welfare-increasing mergers and the deterrence effect. Sørgard (2009), similar Besanko and Spulber (1993) finds welfare gains from commitment to a particular antitrust policy, in this case the likelihood of an investigation. Sørgard (2009) does not assess the use of filing fees as an instrument of merger policy. There is also no discussion of an adverse deterrence effect in Sørgard (2009), which follows from the implicit assumption of weakly opposed preferences in that paper.

The paper by Neven and Röller (2005) considers a political economy model of the merger control process in which merging firms can give ‘bribes’ to the antitrust authority which will then decide on mergers based on the level of bribes and its objective function, which can either be a total welfare or a consumer welfare standard. Another model closely related to that proposed here is by Heidhues and Lagerlöf (2005). Their focus is on the efficiency defense in merger proceedings. While in the model I propose here it is the antitrust authority which needs to spend resources to acquire information, in the model by Heidhues and Lagerlöf it is the merging firms which need to produce verifiable information regarding their efficiencies and thereby resolve the asymmetric information problem. Heidhues and Lagerlöf focus on the question if and under what conditions firms should be allowed to present verifiable evidence regarding efficiencies. Like much of the rest of the literature they also assume that the private information held by the firms pertains only to the level of efficiencies created by a merger.

Ottaviani and Wickelgren (2008) explore the issue of ex ante and ex post merger control. Until the Hart Scott Rodino Act of 1976 mergers in the United States did not have to be notified to antitrust authorities, so that mergers would be assessed only after having taken place. Ottaviani and Wickelgren analyze a model in which both the merging firms and the antitrust authority may learn about the effects of a merger once it has taken place. Having observed the effects of a merger, the antitrust authority can then use its superior information to make a better decision regarding the merger. However, it is costly to unravel previously consummated mergers. They also allow for ex post asymmetric information and allow the firms to signal the effects of the merger by choosing its price appropriately. The model they develop thus differs from the one discussed here by its focus on the timing of merger control, but it shares with the framework developed here a concern with informational issues in antitrust policy. In particular, the authors note that some of their results may depend on whether uncertainty pertains to the cost level post-merger or the ‘increase in market’ power, although this distinction is not developed further in their paper.

Recently the role of asymmetric information in merger control has been analyzed by Giebe and Lee (2015). They analyze to what extent an antitrust authority should use information reported by non-merging firms, that are active in the same industry as the merging firms. Theirs is a model
of a cheap talk signaling game in which the competitor firms may try to influence the decision of the antitrust authority through strategic reports of information. Depending on circumstances, the antitrust authority may or may not be responsive to information reported to it through the competitors.

The paper is also related to Principal-Agent models of adverse selection featuring non-responsiveness. For a textbook treatment of such models see Laffont and Martimort (2002). In particular it is related to the model of a labor-managed firm in Guesnerie and Laffont (1984). For the intuition behind that paper, consider a principal wishing to procure a good from an agent who is privately informed about the cost of production. When the quality of the item is independent of the cost of production, then in the first best quantity is a decreasing function of cost. Roughly speaking, a contract in which quantity is decreasing in cost can be implemented. Hence in that instance of the problem, the first best may be implemented. However if the quality of the good depends positively on the cost of production then it may happen that in the first best quantity increases with the cost of production. The incentive constraints for the agent are however unchanged, so that in this case one obtains the result that the first best is not implementable. The solution to such a problem is then to offer a contract in which the quantity produced does not depend on the report by the agent, that is the quantity is non-responsive to the information held by the agent. Roughly speaking, the case in which quality is independent of cost corresponds to the case of weakly opposed preferences that I consider. The case in which the quality increases with the cost of production is the case of strongly opposed preferences that I consider. By assuming that the antitrust authority can investigate a merger, I can overcome the non-responsiveness that would otherwise also hold in this model.

This paper is also related to the literature on mechanism design with audits. In general, audits may be used by the mechanism designer to relax incentive constraints thereby enabling a better allocation. Baron and Besanko (1984) study a model in which a regulator may audit the cost of a firm and in case of misreports may order the firm to pay a penalty. Khalil (1997) has pointed out that the regulator will typically face a commitment problem when deciding whether to audit or not. Khalil derives the optimal incentive contract when the regulator cannot commit to an audit. The case analyzed by Khalil is close to the current paper, as here it is also assumed that the authority cannot commit to an audit strategy. Like in Khalil (1997) the authority needs to be given incentives to carry out an audit on the equilibrium path. The difference between this paper and the papers by Baron and Besanko (1984) and Khalil (1997) is that we do not consider the explicit use of transfers by the authority as part of the mechanism. In the context of auditing with limited commitment, Pollrich (2015) shows that the regulator may gain from having only an imperfect audit compared to perfect auditing. This is because the imperfect auditing technology helps in solving the regulator’s commitment problem as the audit will no longer generate positive expected revenue. This result does not hold in the current model as the antitrust authority does not face a surplus extraction problem.
2 The Model

Consider the following merger proposal game in which two firms, acting as a single unit may decide to merge or not. The benefits to the firms of merging depend on the industry structure in which the two firms operate. The (joint) gain to the merging firms from the merger is written as \( \pi(\theta) \), where \( \theta \) is a parameter related to the economic environment, which will be discussed later. It is common knowledge that \( \theta \) is distributed over the interval \([\bar{\theta}, \bar{\theta}]\) according to the continuous, strictly increasing cumulative distribution function \( F : [\bar{\theta}, \bar{\theta}] \rightarrow [0, 1] \).

Before completing the merger, the firms need to submit their merger proposal for review by the antitrust authority. Proposing a merger has a cost \( \varepsilon > 0 \). For each value of \( \theta \) let \( p(\theta) \in \{0, 1\} \) denote the choice of a firm to propose a merger, \( p(\theta) = 1 \) or not, \( p(\theta) = 0 \). The antitrust authority can block or clear a merger and the antitrust authority’s decision is final. Before doing so the antitrust authority may decide to further investigate the effects of the merger at some cost. We can think of this audit as requesting hard information from the merging firms or other firms active in the market. An investigation may involve a detailed study of consumer behavior and other evidence-gathering activities. We do not model non-merging firms or competitors as strategic agents. The cost of an investigation is denoted \( \omega \) and is distributed over the interval \([0, \bar{\omega}]\) according to the continuous and strictly increasing cumulative distribution function \( G : [0, \bar{\omega}] \rightarrow [0, 1] \). After the antitrust authority has gathered information it learns the value of \( \theta \). We denote the antitrust authority’s decision of investigating a merger by \( \alpha(\omega) \in \{0, 1\} \). Abusing notation slightly, I will also refer to \( \alpha \) as the probability of an investigation occurring while suppressing its dependence on \( \omega \).

We use \( d_0 \in \{0, 1\} \) denote the antitrust authority’s decision of blocking, \( d_0 = 1 \), or clearing a merger, \( d_0 = 0 \), when it has previously chosen not to investigate the merger. If it clears a merger then the change in welfare is \( \Delta(\theta) \). The welfare effect of the merger can be taken to be either the total welfare effect of the merger (i.e. the sum of consumer and producer surplus), the consumer surplus or some weighted sum of consumer and producer surplus. In cases in which the distinction between different welfare standards does matter, this will be pointed out. Let \( d_1(\omega, \theta) \in \{0, 1\} \) the action taken by the antitrust authority after it has learned the value of \( \theta \) from an investigation, with \( d_1(.) = 1 \) denoting that a merger has been blocked, while \( d_1(.) = 0 \) denotes that a merger has been cleared.

For the following analysis I assume that both \( \pi(\theta) \) and \( \Delta(\theta) \) are strictly monotone and continuously differentiable functions. We further assume for concreteness that \( \Delta(\theta) \) is increasing. Depending on whether \( \pi(\theta) \) is increasing or decreasing I will distinguish between two cases:

**Definition 1: Strongly Opposed Preferences** Preferences over mergers are strongly opposed

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1In practice firms could appeal the decision of the antitrust authority, but this rarely happens.
2This is a normalization. Alternatively one could have assumed \( \Delta \) to be a decreasing function.
if:

$$\text{sign}(\pi'(\theta)) \neq \text{sign}(\Delta'(\theta))$$

**Definition 2: Weakly Opposed Preferences** Preferences over mergers are weakly opposed if:

$$\text{sign}(\pi'(\theta)) = \text{sign}(\Delta'(\theta))$$

Hence preferences of the merging firms and the antitrust authority are weakly opposed when both prefer higher values of $\theta$ to lower values of $\theta$, while they are strongly opposed whenever the antitrust authority prefers higher values of $\theta$ while the merging firms prefer lower values of $\theta$. Notice that the assumption of weakly opposed preferences corresponds to assumptions made in, for example, Besanko and Spulber (1993) and Heidhues and Lagerlöf (2005). Since $\Delta' > 0$, strongly opposed preferences imply that $\pi' < 0$. Weakly opposed preferences imply that $\pi' > 0$. We let $\pi^{-1}(\cdot)$ be the inverse profit function. For $z > \max_{\theta} \pi(\theta)$ I define $\pi^{-1}(z) := \arg\max_{\theta} \pi(\theta)$. Similarly, for $z < \min_{\theta} \pi(\theta)$ I define $\pi^{-1}(z) := \arg\min_{\theta} \pi(\theta)$.

It is assumed throughout that $\Delta(\theta) < 0$ and $\Delta(\theta) - \varepsilon > 0^3$. The first assumption implies that there are some welfare-decreasing mergers which the antitrust authority would block, while the second assumption implies that there are some mergers which increase welfare even taking account of the cost of proposing the merger under a total welfare standard.

Furthermore I will assume that $\min_{\theta} \pi(\theta) > \varepsilon$ to ensure that any merger is profitable to the firms. Both the merging firms and the antitrust authority are risk-neutral.

I will make one additional assumption regarding the support of the distribution of $\omega$. Let $I(\theta) = 1$ whenever $\Delta(\theta) < 0$ and $0$ otherwise. It is then assumed that $\overline{\omega} < \int_{\theta}^{\overline{\theta}} -\Delta(\theta) I(\theta) dF(\theta)$. This assumption places an upper bound on the maximum cost of an investigation, which is assumed to be less than the expected loss from welfare-reducing mergers. This appears reasonable as an investigation should not be more costly, in terms of welfare, than the expected loss from allowing welfare-reducing mergers.

To summarize the strategic variables of both players:

- $p : \overline{\theta} \rightarrow \{0, 1\}$ is the decision of the firms whether to propose a merger or not.
- $\alpha : [0, \overline{\omega}] \rightarrow \{0, 1\}$ is the decision of the antitrust authority to investigate a merger or not.
- $d_0 : [0, \overline{\omega}] \rightarrow \{0, 1\}$ is the decision of the antitrust authority to clear or block a merger if it has not investigated it.
- $d_1 : [0, \overline{\omega}] \times \overline{\theta} \rightarrow \{0, 1\}$ is the decision of the antitrust authority to clear or block a merger if it has investigated it.

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3We do not make an assumption about the welfare effects of an average merger, that is $E[\Delta(\theta)]$ may be positive or negative.
The investigation costs \( \omega \) do not affect the effect of a merger clearance decision. So in equilibrium the \( d_0 \) and \( d_1 \) will not depend on it, so I will subsequently suppress the dependence of these two functions on the value of \( \omega \).

The timing of this game is as follows:

\[
\begin{align*}
\text{merger proposal, } p & \quad \text{merger investigation, } \alpha & \quad \text{merger clearance, } d \\
\theta \text{ realized} & \quad \omega \text{ realized} & \quad \theta \text{ learned by AA} & \quad \text{time}
\end{align*}
\]

Figure 1: Timing of the merger proposal game

The solution concept is Perfect Bayesian Nash Equilibrium\(^4\). Below I will present stylized models of competition. For each of these examples I identify a parameter which could take the role of \( \theta \) in the merger approval game presented here. We will also classify each of these parameters as either giving rise to strongly opposed preferences or to weakly opposed preferences.

2.1 Examples

This section presents four economic environments which could plausibly lead to the type of \( \pi(\theta) \) and \( \Delta(\theta) \) functions described above. For the examples a simple demand model with three goods will be used: A and B and an outside good. Consumers have quadratic quasilinear preferences, leading to linear demand. The detailed derivations can be found in Motta (2004). Demand for the two products is then given by:

\[
q_j(p_A, p_B) = \frac{1}{2} \left[ v - (1 + \gamma)p_j + \frac{\gamma}{2}(p_A + p_B) \right]
\]

Here \( \gamma \in (0, \infty) \) is a parameter capturing the degree of product differentiation. When \( \gamma = 0 \) then products are independent and when \( \gamma = \infty \) then products are perfect substitutes. In the following examples I will denote the merging firms as firms 1 and 2 before the merger and the merged firm as \( M \). The non-merging firms, if any, are denoted by \( O \). Firms compete in prices.

**Example 1** Prior to the merger, firm 1 produces good \( A \) while firm 2 produces good \( B \). Marginal costs for both firms before the merger are given by \( c \) such that \( v > c \). After the merger, there are some efficiency gains of \( e \) so that post-merger marginal costs are given by \( c(1 - e) \). Finding the equilibrium prices, profits and consumer surplus are standard calculations and thus omitted. The profit gain to the merging firms is:

\[
\pi(\gamma, e) = 0.25(v - (1 - e)c)^2 - \frac{1 + 0.5\gamma}{(2 + 0.5\gamma)^2}(v - c)^2
\]

\(^4\)Alternatively one could assume that the antitrust authority can commit to its decisions regarding the investigation decision before a merger is proposed. In the context of antitrust policy this assumption however seems less plausible compared to the usual context of Principal-Agent models.
The change in consumer surplus is:

$$\Delta(\gamma, e) = 0.375 (v - (1 - e)c)^2 - 0.25 \left( \frac{1 + 0.5\gamma}{(2 + 0.5\gamma)^2} \right) (v - c)^2(6 + \gamma)$$

If one takes $\theta := e$ holding $\gamma$ fixed, then differentiation of the above two equations with respect to $\theta$ shows preferences that are weakly opposed. If, holding $e$ fixed, one takes $\theta := -\gamma$ such that $\theta \in \Theta \subseteq (-\infty, 0]$ then the above two equations yield strictly opposed preferences.

The comparative statics results with respect to $\gamma$ break down when $n > 2$. The reason is that $\gamma$ in this model captures both the degree of differentiation between the merging firms’ products as well as the degree of differentiation between each of the merging firms’ products and the products of the non-merging firms. Both of these parameters have different comparative statics, so that constraining them to be equal will give non-monotone comparative statics. One way to see this is to consider the two polar cases of $\gamma = 0$ and $\gamma = \infty$ with a third, non-merging firm also present in the market. In the first case, all products are independent of each other so one would expect no effect from the merger. In the second case all goods are perfectly homogeneous so that again there would be no effect from the merger. However for intermediate values of $\gamma$ there will clearly be effects of the merger. This implies that the effect of $\gamma$ is not monotone in a model with more than two firms. To take account of this one would need to specify a model of demand in which the cross-price elasticities varied across different products. Allowing such differences in substitutability increases the computational complexity without a corresponding benefit, so that this is omitted here.

**Example 2** Suppose firms 1, 2 and O all produce good A (while I set $q_B = 0 = \gamma$) at marginal costs $c_1 = c_2 = c < c_3$. The merged firms’ cost will decrease by $e$. We further assume that $c_3 < \frac{1}{2}(v + c - e)$. This ensures that after the merger, the monopoly power of the merging firms is effectively constrained by the marginal cost of the non-merging firm. The profit gain to the merging firms is:

$$\pi(c_3, e) = (v - c_3)(c_3 - (c - e))$$

The change in total welfare is:

$$\Delta(c_3, e) = \frac{1}{2} \left( (v - c_3)^2 - (v - c_1)^2 \right) + (v - c_3)(c_3 - (c - e))$$

Setting $\theta = c_3$ preferences are strongly opposed. A higher value of $c_3$ benefits firms after the merger by allowing greater price increases. For the same reason the antitrust authority prefers lower values of $c_3$ which tend to limit price increases following the merger. By setting $\theta := e$ one obtains weakly opposed preferences.
Example 3 Suppose firms 1 and 2 produce good $A$, while firm $O$ produces good $B$. All firms have marginal cost $c$ before the merger. Before the merger Bertrand competition between firm 1 and 2 implies that $p_A = c$. Ignoring for the moment any efficiency gains from the merger, the profit gain from merger is:

$$\pi = \frac{1}{2} \frac{(v-c)^2(1+\gamma)}{(2+\gamma)^2}$$

As a proxy for the change in consumer welfare, consider the induced change in price:

$$p^M - c = \frac{2(v-c)}{4 + 2\gamma}$$

Setting $\theta = \gamma$ leads to the profit from a merger being a decreasing function in $\theta$. In turn the price increase falls with $\theta$, implying that the welfare gain of the merger is an increasing function of $\theta$. Hence preferences are strongly opposed in this case.

In Example 3 the parameter $\gamma$ quantifies the degree to which outsiders’ products are substitutes for the merging firms’ products. The higher this degree of substitution the lower is the increase in market power following the merger. While firms find mergers less profitable as the competition from outside firms increases, the higher degree of outside competition reduces the negative effect on welfare from the increase in market power following the merger.

Examples 1 to 3 highlight under what circumstances strongly opposed preferences are likely to arise. Namely this is the case when the firms have private information regarding the strength of competition between them, the degree to which other firms can produce at sufficiently close prices and the strength of competition between the merging firms’ goods and outside firms’ products. When firms compete fiercely with each other (i.e. high $\gamma$ in Example 1), then a merger allows them to overcome this competition and raise prices. Hence the fiercer the competition between firms before the merger, the more profitable the merger itself. For consumers however fierce competition is a benefit since it reduces prices. Since more competition is eliminated by a merger when there is a lot of competition before the merger, consumers a hurt more by ex ante competitive firms merging.

Similarly, when outsiders can produce at almost the same costs as the merging firms, then even if a merger eliminates all competition between the merging firms, the increase in prices is tamed after the merger by the outside producers. The more competitive these outside producers are (i.e. the lower is $c_3$ in Example 2), the lower the opportunity to increase prices after the merger and the lower the profitability of the merger. For consumers however, having outsiders restraining the merging firms price increases post-merger is a benefit.

Last when the outsiders produce a product that is not exactly the same as the merging firms’ products, but is substitutable to some extent, then the higher the degree of substitution the less able are the merging firms to increase prices after the merger. As a result, the more substitutable
are outsiders products, the less profitable a merger. For consumers however, having substitutable outside goods is a benefit, since this will restrain price increases after the merger and also means that if prices are increased, they can switch demand to the outside producers.

Note that the examples are meant to be illustrative. In reality the antitrust authority may also face some uncertainty over other model parameters, such as the marginal cost of production. What the model is meant to capture is that some factors such as $v$ in the consumers’ demand function can be seen as simple scaling factors. Similarly, when preferences are strongly opposed (so $\gamma$ from Example 1 may be private information), then this does not mean that the level of efficiency gains is perfectly known. The favored interpretation is that the efficiency gains are ex ante unknown, but that both the merging firms and the antitrust authority expect efficiency gains of around $e$.

3 Equilibrium

In what follows I will focus on equilibria of this game which involve a strictly positive probability of a merger being proposed\(^5\). For such an equilibrium there is the following preliminary result, which holds irrespective of whether preferences are strongly or weakly opposed:

**Lemma 1** In any pure strategy Bayes perfect Nash equilibrium of the merger proposal game in which there is some $\theta \in [\theta, \theta]$ such that $p(\theta) = 1$, the following must hold:

- $d_1(\theta) = 0$ if and only if $\Delta(\theta) \geq 0$
- $d_0 = 0$
- $p(\theta) = 1$ for all $\theta$ such that $\Delta(\theta) \geq 0$.

**Proof** See Appendix.

The first result in Lemma 1 simply states that after an investigation, only those mergers which raise welfare from the view of the antitrust authority will be cleared. The second result says that if the antitrust authority did not conduct an investigation, then it will clear the merger. Note that this is consistent with current practice in the sense that a merger will only be blocked after some investigation of its likely consequences, while mergers may be cleared even without an extensive investigation. The reason that mergers are cleared if there was no investigation is that if they were blocked, then welfare-decreasing mergers would never be proposed. But then it would no longer be optimal to block mergers if there was no investigation. As a consequence the expected welfare effect of any merger that is cleared is positive in equilibrium. The third result says that mergers that increase welfare will always be proposed. Hence merger policy does not deter beneficial mergers.

\(^5\)Trivially there is an equilibrium of the merger proposal game in which no mergers are proposed and in which the antitrust authority always blocks a merger that is proposed.
This result however depends on the assumption that the consequences of a merger are observed perfectly after an investigation.

Given Lemma 1 it remains to consider the behavior of firms with welfare-reducing mergers and the decision of the antitrust authority to investigate mergers or not. The decision of the antitrust authority to investigate depends on its beliefs regarding firms’ proposal behavior and its realized investigation costs. If the antitrust authority always investigated a merger, then welfare-reducing mergers would never be cleared and hence never be proposed. This would then imply that any investigation is wasteful. Hence it cannot happen that the antitrust authority investigates all proposed mergers. In equilibrium it cannot be that that the antitrust authority never investigates a merger, since then all firms would propose to merge. Under those circumstances there would be some realizations of the cost of investigation under which the antitrust authority would prefer to investigate a merger. Hence in equilibrium the antitrust authority will investigate a merger with a probability strictly between 0 and 1.

While welfare-increasing mergers will always be proposed, firms with a value of $\theta$ such that $\Delta(\theta) < 0$ consider their expected gain from a merger given the probability that the merger is investigated and subsequently blocked. In equilibrium the investigation probability will deter some welfare-reducing mergers from being proposed. The deterred mergers are those which are relatively less profitable to the firms.

Since, by our normalization $\Delta(\theta)$ is an increasing function, there exists some $\theta_0$ such that $\Delta(\theta_0) = 0$ and for all $\theta \geq \theta_0$ it holds that $\Delta(\theta) > 0$. Hence $\theta_0$ is a cut-off determining which firms are the welfare-increasing ones. Behavior for firms with $\theta \geq \theta_0$ does not depend on the investigation decision taken by the antitrust authority. For firms with $\theta \leq \theta_0$ the decision of whether to propose or not does depend on the investigation decision taken by the authority. Behavior differs according to whether preferences are strongly or weakly opposed. There will be a second cut-off $\theta^*$ denoting the firm which is just indifferent between proposing to merge and not proposing to merge. Clearly $\theta^* < \theta_0$, since only mergers with $\theta \geq \theta_0$ will always be (weakly) profitable as these are always cleared with probability one by the antitrust authority.

Depending on whether preferences are either strongly or weakly opposed, firms to the left or to the right of $\theta^*$ will propose to merge in equilibrium. The reason that under strongly opposed preferences, firms to the left of $\theta^*$ propose is that these mergers are the most profitable ones to the firms. This implies however that for $\theta \in (\theta^*, \theta_0)$ one obtains $p(\theta) = 0$. This is why the set of merging firms under strongly opposed preferences is disjoint. Under weakly opposed preferences firms to the right of $\theta^*$ will propose to merge and hence the set of merging firms is then convex.

Given the antitrust authority’s belief about which firms will merge, the expected gain from an investigation will be fixed. The antitrust authority will choose to investigate if and only if the cost of doing so, which is given by $\omega$ lies below the expected gain of an investigation, which given the firms’ behavior is constant. We denote this cut-off by $\omega^*$. Hence the antitrust authority will use a cut-off strategy for its investigation decision. The gain from an investigation depends on
the behavior of firms and on whether preferences are strongly opposed or weakly opposed. The equilibrium cut-offs \((\theta^*, \omega^*)\), solve the following equations:

\[
\begin{align*}
\omega & \leq \omega^* = E[G(\theta)|p(\theta) = 1; \theta^*] \\
\pi(\theta^*)(1 - G(\omega^*)) - \varepsilon &= 0
\end{align*}
\]  

(1)  
(2)

In the next two subsections I find the equilibrium under strongly and weakly opposed preferences.

### 3.1 Equilibrium under Strongly Opposed Preferences

Under strongly opposed preferences all firms to the right of \(\theta_0\) and all firms to the left of \(\theta^*\) will propose to merge. Let \(\theta_e\) denote the antitrust authority’s belief of the cut-off below which firms will propose to merge. Then the level of the investigation cost below which the authority will investigate given its beliefs over the set of merging firms is given by:

\[
\omega^*(\theta_e) = \int_{\theta_0}^{\theta_e} -\Delta(\theta) \frac{f(\theta)}{1 - F(\theta_0) + F(\theta_e)} d\theta
\]  

(AA-SOP)

The gain from an investigation consists of preventing those welfare-reducing mergers which are being proposed, i.e. those with \(\theta \in [\theta, \theta_e]\). The gain needs to be evaluated conditional on a merger having been proposed. The probability of a merger being proposed is \(1 - F(\theta_0) + F(\theta_e)\). Given \(\omega^*\) the probability of an investigation is then given by \(G(\omega^*(\theta_e))\) which, given our assumptions, can be shown to be weakly increasing in \(\theta_e\) (See the Appendix).

Let \(\omega_e\) be the belief of the merging firms regarding the threshold used by the antitrust authority and let \(\alpha_e = G(\omega_e)\) be the associated belief of the firms regarding the probability of an investigation. The cut-off \(\theta^*(\omega_e)\) below which welfare-reducing mergers will be proposed is implicitly defined by the value \(\theta^*\) that solves:

\[
\pi(\theta^*)(1 - G(\omega_e)) - \varepsilon = 0
\]  

(3)

Rearranging to write the expected probability of an investigation such that type \(\theta\) is indifferent between proposing and not proposing a merger yields:

\[
\alpha_e(\theta) = \frac{\pi(\theta) - \varepsilon}{\pi(\theta)}
\]  

(F-SOP)

From our assumptions it follows that \(\alpha^*(\theta)\) is a decreasing function.

Equilibrium is then given by \((\alpha^*, \theta^*)\) such that \(\theta_e = \theta^*\) and \(\alpha_e = \alpha^*\) that solve equations AA-SOP and F-SOP. For most of the discussion I will discuss equilibrium in terms of the associated probability \(\alpha^* = G(\omega^*)\) with which an investigation will occur. Figure 2 plots the equilibrium when
preferences over mergers are strongly opposed. The blue curve represents the antitrust authority’s decision to merge, while the red curve represents the firm’s decision whether to propose a merger or not. The dashed green lines indicate the types of mergers that will be proposed, while the dashed orange line is the set of mergers that will not be proposed. The equilibrium is given by \((\theta^*, \omega^*)\), where the AA-SOP and F-SOP curves cross. Notice that in equilibrium the welfare increasing mergers are proposed as well as those welfare reducing mergers which reduce social welfare the most.

![Figure 2: Equilibrium under Strongly Opposed Preferences](image)

We have the following result:

**Lemma 2** Under strongly opposed, there is a Bayesian Perfect Equilibrium of the merger proposal game, given by the solution to AA-SOP and F-SOP. Within the set of pure strategy equilibria in which mergers are proposed with strictly positive probability, this is the only equilibrium.

**Proof** See Appendix.

Knowing what the equilibrium will be, it is possible to derive some comparative statics results with regards to some of the model’s primitives. To do so, I write the profit from the merger as \(\pi(\theta; e)\) and the welfare effect of the merger as \(\Delta(\theta; e)\) where \(e\) is a parameter known to the firms and the antitrust authority. This parameter can be interpreted as measuring the strength of efficiency gains from the merger. Therefore I suppose that \(\pi_e > 0\) and \(\Delta_e > 0\). Note that this case arises naturally under the set-up of Example 1, but is not restricted to it.
Proposition 1  Consider the equilibrium \((\theta^*, \alpha^*)\) of the merger proposal game with \(\Delta(\theta; e)\) and \(\pi(\theta; e)\) as above. Further suppose that \(G\) is the uniform distribution over the interval \([0, \bar{\omega}]\), then

- an increase in \(\varepsilon\) leads to a decrease in both \(\theta^*\) and \(\alpha^*\)
- an increase in \(\bar{\omega}\) (weakly) leads to a decrease in \(\alpha^*\) and an increase in \(\theta^*\)
- an increase in \(e\) leads to an increase in \(\theta^*\) and has an ambiguous effect on \(\alpha^*\)

Proof  See Appendix.

The result follows by totally differentiating the equilibrium conditions. The intuition for the result is as follows. Suppose the value of \(\varepsilon\), the cost of proposing a merger, increases. Then a firm which was previously indifferent between proposing a merger or not, at the equilibrium value of \(\alpha^*\) will find a merger to be unprofitable. Hence fewer firms will decide to merge and this will lead to a reduction in \(\theta^*\). The antitrust authority’s decision does not depend on \(\varepsilon\), but as fewer mergers are proposed it will find an investigation to be less profitable than previously and consequently investigate less frequently. Hence the value of \(\alpha\) falls. So as mergers are more costly to propose fewer welfare-reducing mergers will be proposed.

An increase in the value of \(\bar{\omega}\) means that investigations are more costly for the antitrust authority. As a result, holding the firms’ strategies constant, the authority will investigate less frequently. Hence the value of \(\alpha^*\) falls. Firms respond to this by proposing a merger more frequently, so the value of \(\theta^*\) increases. As a result, as investigations are more costly, more welfare-reducing mergers are proposed and fewer investigations are carried out.

An increase in the level of efficiencies \(e\) affects both the pay-off of the antitrust authority and the profit of the firms directly. As the antitrust authority expects mergers to lead to greater efficiencies, the gain from an investigation falls holding firms’ strategies fixed. A greater level of efficiencies means that firms find mergers to be more profitable holding the probability of an investigation fixed. Both of these effects imply that \(\theta^*\) will increase. The effect on the probability of an investigation is ambiguous because while the antitrust authority expects mergers with a given \(\theta\) to be less harmful to welfare, which tends to reduce the gain from an investigation, firms are now more likely to propose to merge even when those mergers are harmful, which tends to increase the gain from an investigation. In equilibrium it is not clear which of these effects is greater, so that the effect on \(\alpha^*\) is ambiguous.

3.2 Equilibrium under Weakly Opposed Preferences

Under weakly opposed preferences all firms to the right of \(\theta^*\) will propose to merge. Again, let \(\theta_e\) be the antitrust authority’s belief regarding the indifferent firm type. Then the threshold level
of investigation cost $\omega^*$ below which there will be an investigation is given by:

$$\omega^*(\theta_e) = \int_{\theta_e}^{\theta_0} -\Delta(\theta) \frac{f(\theta)}{1 - F(\theta_e)} d\theta$$  \hspace{1cm} \text{(AA-WOP)}$$

The gain from an investigation on the right hand side consists of preventing those mergers which would reduce welfare if they were cleared, i.e. those with $\theta \in [\theta^*, \theta_0)$. Again this needs to be evaluated conditional on a merger having been proposed, which happens with probability $1 - F(\theta^*)$. Given $\omega^*$ the probability of an investigation is then given by $G(\omega^*(\theta_e))$ which, given our assumptions, can be shown to be weakly decreasing in $\theta_e$ (See the appendix).

Let $\omega^*_e$ be the belief of the merging firms regarding the threshold used by the antitrust authority. The cut-off $\theta^*(\omega_e)$ above which welfare-reducing mergers will be proposed is implicitly defined by:

$$\pi(\theta^*)(1 - G(\omega_e)) - \varepsilon = 0$$  \hspace{1cm} \text{(F-WOP)}$$

From our assumptions it follows that $\theta^*(\omega_e)$ is an increasing function.

Equilibrium is then given by $(\omega^*, \theta^*)$ that simultaneously solve equations AA-WOP and F-WOP. We will reformulate the equilibrium strategies in terms of the probability of an investigation, $\alpha^*$ rather than the highest level of the investigation cost for which the antitrust authority investigates a merger. Figure 3 graphically shows the equilibrium. In equilibrium some welfare reducing mergers are proposed, but unlike in the case of strongly opposed preferences, these are not the mergers which reduce welfare the most.

Figure 3: Equilibrium under Weakly Opposed Preferences

We have the following result:
Lemma 3 \textit{Under weakly opposed preferences, there is a Bayesian Perfect Equilibrium of the merger proposal game, characterized by the solutions to equations AA-WOP and F-WOP. Within the set of pure strategy equilibria in which mergers are proposed with strictly positive probability, this is the only equilibrium.}

\textbf{Proof} See Appendix.

As for the case of strongly opposed preferences one can derive some comparative statics result regarding changes in parameter values and their effect on equilibrium values. Again I modify the profit and welfare functions slightly: \( \pi(\theta; \gamma) \) and \( \Delta(\theta; \gamma) \) where both functions are such that \( \pi_\gamma > 0 \) and \( \Delta_\gamma < 0 \). Note that these assumptions are satisfied by the functions in Example 1, with \( \theta \) being the level of efficiency gains from the merger. Within the context of Example 1, \( \gamma \) represents the degree of product differentiation between the merging firms’ products. The result however is applicable for any such functions.

\textbf{Proposition 2} \textit{Consider the equilibrium \((\theta^*, \alpha^*)\) of the merger proposal game with \( \Delta(\theta; \gamma) \) and \( \pi(\theta; \gamma) \) as above. Further suppose that \( G \) is the uniform distribution over the interval \([0, \omega]\), then}

\begin{itemize}
  \item an increase in \( \varepsilon \) leads to a an increase in the value of \( \theta^* \) and a decrease in the value of \( \alpha^* \)
  \item an increase in \( \omega \) (weakly) leads to a decrease in both \( \theta^* \) and \( \alpha^* \)
  \item an increase in \( \gamma \) leads to an increase in \( \alpha^* \) and has an ambiguous effect on \( \theta^* \)
\end{itemize}

\textbf{Proof} See Appendix.

The intuition of the result can be explained as follows. Consider an increase in the value of \( \varepsilon \). This increases the firms’ cost of proposing to merge so that some firms which before the increase proposed to merge, for a given value of \( \alpha_e \) no longer find it optimal to do so. As a result, the value of \( \theta^* \) will increase (remember that firms with a value of \( \theta \) greater than \( \theta^* \) will merge in equilibrium under weakly opposed preferences). Since fewer welfare-reducing mergers are proposed, the antitrust authority’s gains from investigations fall and hence \( \alpha^* \) will fall.

Next suppose that \( \omega \) increases. Holding firms’ strategies fixed, this implies that the antitrust authority will find it optimal to investigate a merger less frequently than before, as investigations are now more expensive. As a result, more firms will propose to merge as the risk of a merger being blocked is reduced. Overall, the increase in the costs of investigations leads to fewer investigations and more welfare-reducing mergers being proposed. So there is both a deterrence effect and a direct effect of this increase in \( \omega \) working to make antitrust policy less beneficial.

Last consider the increase in the value of \( \gamma \), which means that products are less closely related. The increase in \( \gamma \) means that the value of \( \Delta(\theta) \) now increases for each value of \( \theta \). However, the profit to be gained from a merger is now lower since there is less competition between the merging
firms for the merger to eliminate. This affects the antitrust authority’s and the firms’ decisions in the following way. The gains from an investigation, holding firms’ strategies fixed, is now uniformly lower. Hence fewer investigations will occur. Holding the antitrust authority’s strategy fixed, firms now find mergers less profitable so to keep the set of merging firms the same, fewer investigations will need to occur. Overall the effect of the increase in $\gamma$ is then to reduce the likelihood of an investigation. The effect on the set of firms which will propose to merge is however ambiguous. While the effect of a greater $\gamma$ would lead the antitrust authority to be more lenient and let more mergers pass, at the same time fewer firms will find it profitable to propose to merge, which works in the opposite direction. Note further that the set of welfare-increasing mergers changes after the increase in $\gamma$, so that now more mergers are welfare-improving.

4 Applications of the Model

4.1 Imperfect Investigation Technology

So far it was assumed that the antitrust’s investigation power was perfect. Suppose now that after an investigation the antitrust authority only learns the value of $\theta$ with probability $\rho \in [0, 1]$. When $\rho = 1$, one obtains the original model. When $\rho = 0$ then we are in the environment of Besanko and Spulber (1993). In this subsection I will assume that $E[\Delta(\theta)] < 0$ as in Besanko and Spulber (1993).

We have the following result:

**Proposition 3** If the probability of a investigation success falls from $\rho_0 = 1$ to $\rho_1 < 1$ such that under $\rho_1$ it still holds that

$$\rho_1 \int_{\theta_0}^{\theta} -\Delta(\theta)f(\theta)d\theta < \omega$$

then the equilibrium probability of an investigation falls and the measure of welfare-reducing mergers that are proposed increases.

**Proof** See the Appendix.

The effect of a greater probability of investigation failure is two-fold. The direct effect of the greater likelihood of an unsuccessful investigation is to reduce the probability, from the point of view of the firms, of a merger being blocked holding constant the gain the antitrust authority expects from an investigation. This change increases the set of welfare-reducing firms that will propose to merge. The second, indirect effect is that investigations are less beneficial to the antitrust authority. Hence it will no longer investigate for relatively high realizations of the cost of investigation, $\omega$ since it expects to learn the true effect of the merger only with probability $\rho_1$. Both of these effects operate independently of whether preferences are strongly or weakly opposed.
In the extreme case, when $\rho = 0$ we have the following results.

**Proposition 4 (Besanko and Spulber, 1993)** When $\rho = 0$ and preferences are weakly opposed, then the antitrust authority will randomize its decision of whether to clear or block a merger. The cut-off level of $\theta$ above which firms choose to merge is such that the antitrust authority is indifferent between clearing and blocking a merger.

While the above result comes directly from Besanko and Spulber (1993), that paper did not consider the possibility of strongly opposed preferences. In that case we have the following result:

**Proposition 5** When $\rho = 0$ and preferences are strongly opposed, then the antitrust authority blocks all mergers.

Suppose to the contrary that the antitrust authority randomized the decision of whether to clear or block a merger. Then only the relatively profitable mergers will be proposed. But the expected welfare change from this restricted set of mergers is lower than that of all mergers, which is by assumption negative. Therefore the antitrust authority must have strictly preferred blocking the proposed merger. Allowing all mergers can also not be optimal, since the unconditional expectation of the welfare change from mergers is assumed to be negative. Hence the only remaining possibility is that the antitrust authority blocks all mergers. The equilibrium in the original Besanko and Spulber model thus significantly depends on whether preferences over mergers are strongly or weakly opposed. As Rey (2002) suggested, in the case of strongly opposed preferences the antitrust authority essentially does not use the information held by the firms but bases its decision entirely on its prior beliefs.

### 4.2 Filing fees

It was assumed so far that the firms bear a sunk cost of $\varepsilon$ of proposing a merger. Suppose that in addition, the antitrust authority can choose a fee, $t \geq 0$ to be paid by the firms when proposing a merger. We interpret the fee as a transfer between firms and the authority, so there is no welfare loss associated with it.

**Proposition 6** Suppose that preferences are weakly opposed. Then by choosing the value of $t$ appropriately, the antitrust authority can implement the first best. Moreover when doing so, the antitrust authority will never use an investigation.

Before constructing the optimal filing fee, I clarify what is the first best. A merger brings a social benefit of $\Delta(\theta)$ but assuming temporarily a total welfare standard at the social cost of $\varepsilon$. Hence only mergers with $\Delta(\theta) - \varepsilon \geq 0$ should be implemented\(^6\). Let $\theta^{fb}$ be defined by

\[^6\text{In case of a pure consumer standard, the condition would be } \Delta(\theta) \geq 0\]
\[ \Delta(\theta^{fb}) - \varepsilon = 0^7. \] Then under weakly opposed preferences all firms with \( \theta \geq \theta^{fb} \) should merge. Note also that in the first best, the authority never investigates a merger since investigations are costly. Now the filing fee that implements the first best is constructed. The optimal filing fee is given by \( t = t^{fb} = \pi(\theta^{fb}) - \varepsilon. \) Then because \( \pi(\theta) \) is an increasing function, all firms with \( \theta \geq \theta^{fb} \) will propose to merge, which is exactly the first best. Furthermore, since conditional on a merger being proposed the welfare change expected from a merger is positive, there is no gain to the antitrust authority of investigating a merger.

This proposition derived here differs from a similar proposition in Besanko and Spulber (1993) which only considered a small change in the filing fee paid by firms such that the initial mixed strategy equilibrium would persist with the increased fee. The result here is also applicable in Besanko and Spulber (1993).

While filing fees are highly effective when preferences are weakly opposed, it remains to consider the case of strongly opposed preferences. In that case however the following negative result holds.

**Proposition 7** Suppose preferences are strongly opposed. Then an optimally chosen filing fee cannot implement the first best. Moreover, investigations will still be used even when filing fees are available to the antitrust authority. The optimal filing fee \( t^* \) lies in the interval \([\pi(\theta^{fb}) - \varepsilon, \pi(\theta_0) - \varepsilon]\).

The intuition behind this result is as follows. An increase in the filing fee from 0 reduces the profit that firms can make from a merger. In particular an increase in \( t \) starting from 0 is akin to an increase in \( \varepsilon. \) This has the effect of reducing \( \theta^* \) and \( \alpha^* \), so fewer bad mergers are proposed and fewer investigations are conducted. Both of these effects raise the pay-off to the antitrust authority. However an increase in \( t \) also affects the relatively unprofitable mergers, which are socially desirable. This is an example of the adverse deterrence effect. Raising the filing fee above the profit of the least profitable firm induces some socially desirable mergers to not be proposed. At such a level of the filing fee, the welfare-reducing mergers will still be profitable and continue to be proposed in equilibrium. The optimal filing fee is bounded from below by \( \pi(\theta) \), the profit of the least profitable merger, since below that level increases in the filing fee \( t \) only have a beneficial deterrence effect. To see that it is bounded from above by \( \pi(\theta_0) \), consider setting \( t = \pi(\theta_0) \). Then no socially desirable merger will be proposed in equilibrium. But then, the antitrust authority will clear no mergers, implying a pay-off of 0, which is less than the pay-off when there is no filing fee (since conditional on a merger being proposed the authority expects it to have a positive effect on welfare).

Consider now the ex-ante pay-off of the antitrust authority when choosing the level of the filing fee. We will denote by \( \theta^+(t) : [\pi(\theta) - \varepsilon, \pi(\theta_0) - \varepsilon] \to [\theta_0, \overline{\theta}] \) the function associating for each level of the filing fee, the welfare-increasing merger type such that the firms make zero profit with this merger after paying the filing fee. Note that \( \theta^+(t) \) is a strictly decreasing function and given

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7In case of a pure consumer standard one would have \( \theta^{fb} = \theta_0 \)
by: \( \theta^+(t) = \pi^{-1}(\varepsilon + t) \). Note that I will make use of the equilibrium cut-off investigation cost, \( \omega^* \) in what follows, rather than the equilibrium investigation probability \( \alpha^* = G(\omega^*) \) for ease of exposition. We also suppress the dependence of \( \theta^* \) and \( \omega^* \) on the filing fee.

\[
E[W] = (1 - G(\omega^*)) \int_{\theta^*}^{\theta^+} \Delta(\theta) dF(\theta) + \int_{\theta^*}^{\theta^+} \Delta(\theta) dF(\theta) \\
- (F(\theta^*) + F(\theta^+) - F(\theta_0)) \left( \varepsilon + \int_{0}^{\omega^*} \omega dG(\omega) \right)
\]  

(4)

The first term is the expected loss from welfare decreasing mergers being allowed. The second term is the expected gain from allowing welfare-increasing mergers. The third term is the expected cost of merger proposals and investigating mergers. Note that by assumption the filing fee does not enter directly into the antitrust authority’s expected welfare. Differentiating with respect to the filing fee and simplifying using the fact that \( \omega^* \) is chosen optimally by the antitrust authority then yields:

\[
\frac{dE[W]}{dt} = (1 - G(\omega^*)) \Delta(\theta^*) f(\theta^*) \frac{d\theta^*}{dt} + \Delta(\theta^+) f(\theta^+) \frac{d\theta^+}{dt} \\
- \left( \varepsilon + \int_{0}^{\omega^*} \omega dG(\omega) \right) \left( f(\theta^*) \frac{d\theta^*}{dt} + f(\theta^+) \frac{d\theta^+}{dt} \right)
\]  

(5)

From Lemma 7 in the Appendix it is known that \( \theta^* \) and \( \alpha^* \) are decreasing in the level of the filing fee \( t \). It is thus possible to put signs on each of the effects an increase in the filing fee in the relevant range has. The first term reflects the fact that a greater filing fee will deter some welfare-reducing mergers from being proposed, which is the beneficial deterrence effect. The second term reflects the fact that a high filing fee will deter some welfare-increasing mergers, which is the adverse deterrence effect. The third term captures the fact that a greater filing fee will reduce the expected cost of investigations and the cost of merger proposals, since fewer mergers will be proposed. At the optimal level of the filing fee, \( \frac{dE[W]}{dt} \leq 0 \) will hold.

Under strongly opposed preferences I thus conclude that filing fees are a useful policy instrument, which is however limited in its effectiveness by the informational asymmetries. The optimal filing fee trades off the gain from having fewer bad mergers proposed and a reduced frequency of investigations with the loss imposed from making some welfare-enhancing mergers unprofitable to the firms.

4.3 Consumer v Welfare Standard

The original paper by Besanko and Spulber (1993) showed that an antitrust lacking the power to commit to a probability of blocking mergers, could implement the full commitment solution by
having the government choose the welfare standard. When the true welfare standard is a total welfare standard, then the implication of their analysis is that the government should give the antitrust authority a welfare standard putting less weight on producer surplus and move closer to a consumer welfare standard. This issue can also be analyzed using the model developed in this paper. For briefness we will only consider the case of strongly opposed preferences in this subsection.

We change notation slightly to write the welfare-impact of a merger as \( \Delta(\theta, \beta) = CS(\theta) + \beta \pi(\theta) \). We assume that for all \( \beta \in [0, 1] \), \( \Delta(\theta, \beta) \) is an increasing function of \( \theta \) and that all the assumptions used previously for the analysis of the equilibrium of our model continue to hold for all values of \( \beta \).

The parameter \( \beta \) captures the weight that the social welfare function puts on the profits generated by the firms. Again, the dependence of \( \theta^* \), \( \theta_0 \) and \( \omega^* \) on \( \beta \) is suppressed in the notation for simplicity. Mechanically, an increase in the value of \( \beta \) reduces the value of \( \theta \) for which mergers raise welfare. This means that \( d\theta_0/d\beta < 0 \). The signs of \( d\theta^*/d\beta \) and \( d\omega^*/d\beta \) are given in the following lemma.

**Lemma 4** Given that the government has set the weight of producer surplus in the antitrust authority’s welfare function to \( \beta \), it holds that \( d\theta^*/d\beta > 0 \) and \( d\omega^*/d\beta < 0 \).

The intuition behind the result is as follows. An increase in \( \beta \) reduces the gain of an investigation to the antitrust authority, *ceteris paribus*. This in turn reduces the probability of an investigation, leading to more welfare-reducing mergers being proposed which offsets the reduction in the gain of an investigation to the authority, but not completely. Overall, more bad mergers are proposed and the probability of an investigation is reduced.

The expected welfare to the government of the antitrust authority using its true welfare standard of \( \Delta(\theta, 1) \) can be written as follows:

\[
E[W] = (1 - G(\omega^*)) \int_{\theta_0}^{\theta^*} \Delta(\theta, 1) dF(\theta) + \int_{\theta_0}^{\theta^*} \Delta(\theta, 1) dF(\theta) \\
- \left( F(\theta^*) + 1 - F(\theta_0) \right) \left( \varepsilon + \int_{0}^{\omega^*} \omega dG(\omega) \right)
\]

The derivative with respect to \( \beta \) is given by:

\[\text{This follows from totally differentiating } \Delta(\theta, \beta) = 0, \text{ noting that both first-order derivatives are positive.}\]
\[
\frac{dE[W]}{d\beta} = \Delta(\theta^*, 1)(1 - G(\omega^*)) f(\theta^*) \frac{d\theta^*}{d\beta} - \Delta(\theta_0, 1) f(\theta_0) \frac{d\theta_0}{d\beta} \\
- \left( \varepsilon + \int_0^{\omega^*} \omega dG(\omega) \right) \left( f(\theta^*) \frac{d\theta^*}{d\beta} - f(\theta_0) \frac{d\theta_0}{d\beta} \right) \\
- g(\omega^*) \frac{d\omega^*}{d\beta} \left( \int_0^{\theta^*} \Delta(\theta, 1) dF(\theta) + \omega^*(F(\theta^*) + 1 - F(\theta_0)) \right)
\]

Evaluating this expression at \( \beta = 1 \), which implies that \( \Delta(\theta_0, 1) = 0 \) and that \( \int_0^{\theta^*} \Delta(\theta, 1) dF(\theta) + \omega^*(F(\theta^*) + 1 - F(\theta_0)) = 0 \), then yields:

\[
\frac{dE[W]}{d\beta} = \Delta(\theta^*, 1)(1 - G(\omega^*)) f(\omega^*) \frac{d\theta^*}{d\beta} - \left( \varepsilon + \int_0^{\omega^*} \omega dG(\omega) \right) \left( f(\theta^*) \frac{d\theta^*}{d\beta} - f(\theta_0) \frac{d\theta_0}{d\beta} \right) < 0
\]

We thus have the following proposition:

**Proposition 8** The government optimally endows the antitrust authority with a welfare standard putting less weight on producer surplus relative to a total welfare standard.

When \( \beta \) is close to unity, a further increase has the effect of increasing the loss from welfare-decreasing mergers, as more such mergers will be proposed. This effect is captured in the first term. Furthermore, a rise in \( \beta \) will increase the expected cost of merger proposals and investigating mergers, as more mergers are proposed in total (both welfare-decreasing and welfare-increasing mergers, as seen from a total welfare standard). This effect is captured in the second term. Hence, also in this model the antitrust authority may choose to adopt for itself a welfare standard that does not put full weight on firms’ profits. Note that when \( \beta \) is not close to one, there may be countervailing effects so that a consumer-surplus standard is not necessarily optimal. For \( \beta \) away from unity, the antitrust authority will investigate more often than the government would want. Since the antitrust authority, with a low \( \beta \) sees mergers as less favorable than the government, it will have a greater benefit to investigating mergers. Another negative effect from the view of the government is that with a low \( \beta \), the antitrust authority’s tougher stance on mergers will prevent some mergers that increase social welfare even after accounting for the merger proposal cost. Such mergers would be allowed by an antitrust authority with a total welfare standard, but when profits only have a weight of \( \beta \), then these mergers are seen as reducing social welfare. Hence such mergers are blocked by the antitrust authority following an investigation, which potentially is the adverse screening effect. However since investigations perfectly reveal the value of \( \theta \) those mergers which would be subject to the adverse screening effect are not profitable to propose by
the firms. Hence only the adverse deterrence effect and not the adverse screening effect will be present in equilibrium when $\beta$ is strictly less than unity.

In any case, it can be seen that giving the antitrust authority a tougher welfare standard is beneficial to the government’s objective.

5 Policy Relevance

The original analysis Besanko and Spulber (1993) assumed that firms were privately informed about the efficiency gains resulting from a merger. As discussed above, most of the literature has followed this assumption. One purpose of this paper is to point out that one can also consider a different case, namely that of strongly opposed preferences, with markedly different results in some areas. The case of strongly opposed preferences was shown to arise in a number of stylized models of competition, when the parameter that the firms had private information about, was chosen appropriately. So far I have investigated the theoretical relevance and plausibility of the assumption of strongly opposed preferences. The question then is whether strongly and weakly opposed preferences are of empirical relevance. A priori it appears prudent to believe that either of the cases may be relevant in practice. This leaves open the question of which of these cases is more relevant, which of course should in general depend on the particulars of a case and an industry. Since so far the literature has implicitly focused on the case of weakly opposed preferences, I shall for argument’s sake attempt to make the case that strongly opposed preferences should be expected to be present in most cases. Notice that this does not imply that efficiency gains from a merger are not important. Rather I would argue that information concerning efficiency gains is less asymmetrically present. One reason is that efficiency gains, by their very nature, appear only in the future. Any information concerning efficiency gains is thus necessarily subject to forecasting errors and not verifiable, except by implementing the merger. Both the firm and the antitrust authority must face the inherent uncertainty of forecasting the level of efficiency gains.

On the other hand information concerning say the substitutability of products is something that can in principle be observed at the time of the merger proposal. In fact, companies should have an interest in knowing the degree of substitutability of their products even without reference to mergers, since this information is needed to optimally set prices. Hence one should expect firms to be relatively well informed in those cases. On the other hand, the antitrust authority at the time it receives a merger proposal did not previously have the need to know such information. Such information has to be carefully collected and evaluated by the antitrust authority, sometimes making use of sophisticated econometric techniques to estimate demand functions of the products concerned.

One indication that antitrust authorities do not only consider efficiencies can be obtained from a reading of the guidelines issued by leading antitrust authorities. Considering the US Guidelines, [12], it can be seen that the chapter discussing efficiencies has 3 pages, while other
topics, for example the definition of the relevant market (7 pages), take up the bulk of the remainder of the guidelines. Similarly, the European Commission’s [17] guidelines discuss the evaluation of efficiencies in 12 paragraphs, while spending 42 paragraphs outlining the potential anti-competitive effects of mergers. Rather than seeing these documents as representing an unfortunate focus on negative aspects of mergers at the expense of their efficiency gains, the perspective put forth here is that this simply is reflective of the focus of the investigations conducted by the authorities up to this point. The reason then that more space is devoted to issues pertaining to market definition and similar issues would simply be the fact that in these areas, asymmetries of information are relatively more severe.

Before discussing one particular case of interest one should also note the difference between efficiency gains and, for example, product substitutability. The realized level of efficiency gains can only be known for sure in the future. Substitutability of products affects firms competing in the market today. For example, firms in the benchmark, linear, differentiated goods industry of Example 1 had to know the value of \( \gamma \) to set their prices in equilibrium. If they did not know this, then their profits could not be set optimally. The same does not apply to the value of efficiency gains. These do not need to be known by the firms’ decision makers in their current operations, but are only estimated prior to a merger. Similarly, firms that are active in a market have an interest in knowing which other companies constrain their exercise of market power, be it because they are able to produce the same product at almost the same level of marginal cost (as in Example 2) or be it because their products are closely substitutable to the merging firms’ products (as in Example 3). The antitrust authority on the other hand typically will not possess this information immediately, but will need to spend some resources on learning which are the relevant competitors.

While the above arguments suggest for why one should at least take seriously the notion that strongly opposed preferences are practically relevant, let us consider a specific case. The difficulty in discussing what type of information firms have before proposing a merger is precisely that the firms know this privately and often do not have incentives to reveal this information. However in one notable case there is some evidence to suggest that the firm had some prior knowledge of the likely consequences of a merger in terms of the strength of competition between the merging parties and outside producers, corresponding roughly to a situation as in Example 3. The discussion will be focused on a case study of the proposed Staples-Office Depot Merger by Dalkir and Warren-Boulton appearing in Kwoka and White (2008), which the reader may consult for more details.

**The Staples - Home Depot Merger** This case involved the proposed merger of two Office Super Stores (OSS), selling office supplies to small businesses and consumers at discounts otherwise only available to larger business. The case revolved around the question of whether supermarkets, such as Walmart, which had a limited assortment of office supplies, were in the same relevant market as staples and Home Depot. If the market were confined only to OSS, then the merger of Staples and Home Depot would be a merger from 3 firms to 2 firms in the market, with potentially
adverse economic consequences. If the market were wider however, then the competition from the supermarkets would constrain prices of the merging firms, so that consumers would not be harmed. Note here, the two stylized potential cases that could occur. Either, the market does include supermarkets. Then a merger would not allow firms to raise prices substantially, implying a low, if any, harm to consumers and low gains from the merger to the merging firms. On the other hand, if the market were defined narrowly to include only OSS, then a merger would allow firms to increase prices by large amounts, implying a substantial harm to consumers and large profit gains for the merging firms.

During the litigation the FTC used econometric evidence to establish that the two merging firms’ prices differed between geographic markets depending on whether a third OSS (MaxOffice) was also present in that area, with higher prices when MaxOffice was absent. This clearly pointed towards the narrow market definition. Furthermore, it was also discovered that the merging firms were in fact aware of the possibility of raising prices after the merger in cities in which they were the only competitors, suggesting that firms, when proposing their merger, were relatively better informed than the antitrust authority prior to its investigation.

One criticism of the model of merger policy that is put forward in this paper is that it is unrealistic to assume that the antitrust authority can perfectly predict the consequences of a merger. In fact, a number of studies focus precisely on that question. This assumption is however made mainly for convenience. One could alternatively interpret the model as allowing for uncertainty and errors with regard to the assessment of the consequences of a merger. In that case one would view \( \theta \) as capturing all information that can be had at the time of the merger regarding its likely consequences. The assumption that \( \theta \) may be perfectly learned by the authority would then imply that the asymmetry of information between the merging firms and the antitrust authority can be eliminated by an investigation, which is a more reasonable hypothesis.

5.1 Policy Implications

If as argued above, the case of strongly opposed preferences is practically relevant, then this provides one explanation for the fact that antitrust authorities do not actively use filing fees as an instrument in merger control, since the gains of doing so are limited. Such behavior would be highly inefficient in a context of weakly opposed preferences. So if one maintains that weakly opposed preferences are indeed the empirically more relevant case then transfers should be used strategically by antitrust authorities. The model in that case would also imply that no investigations need to take place concerning possible efficiencies. If efficiencies are strong enough, then firms would be willing to pay a high price for going ahead with the merger. If the efficiencies were too small, then the firms would not. In practice there is not going to be a filing fee that is optimal for all cases.

\(^9\text{See Section 1.1.}\)
Indeed, the optimal level of the filing fee would need to depend on other observable characteristics of a case, so that investigations would still need to be carried out.

The view taken here is that the case of strongly opposed preferences is empirically relevant at least in some cases, implying that the first best cannot be obtained from an introduction of filing fees in current antitrust practice. The fact that the gains from an active use of filing fees are limited under strongly opposed preferences notwithstanding, this paper has shown that some welfare gains may be achieved by setting filing fees appropriately. In the model of Besanko and Spulber (1993) a marginal increase in the filing fee did not affect the type of firms proposing to merge. This paper shows however that the use of filing fees will have an effect on which mergers are proposed in equilibrium, implying that the way this instrument is used in practice should be carefully considered.

Another application of the model is that the effectiveness of the antitrust authority’s screening tools matters for the type of merger that are proposed in equilibrium and those which are finally allowed to pass. In the extreme case when investigations never yield information concerning the effects of a merger, then the optimal antitrust policy under strongly opposed preferences is to ban all mergers or to allow all mergers, depending on prior beliefs. In the case of weakly opposed preferences, when investigations never yield information, merger policy will affect which mergers are proposed. One could thus expect that investigations are relatively more valuable in the case of strongly opposed preferences, as compared to the case of weakly opposed preferences.

In light of the recent ”More Economic approach” to competition policy enforcement in the EU, the analysis in this paper suggests two types of welfare gains from the increasing use of economics in merger policy. First, conditional on a merger being investigated, it is more likely that the welfare-decreasing mergers are identified and subsequently blocked. Second, some welfare-decreasing mergers which otherwise would have been proposed given small chances of being blocked are no longer being proposed, since their chances of clearance will have dropped. A simple across-the-board reduction in the likelihood of merger approval on the other hand would not have a beneficial selective effect.

The model also clarifies that the relationship between the toughness of antitrust policy and the observable decisions is not straightforward. For example, Seldeslachts, Clougherty and Barros (2009) analyze the effect of mergers being blocked on subsequent merger proposal frequencies. The model introduced here however makes clear that the likelihood of a prohibition depends on what share of proposed mergers consists of welfare-decreasing merger. This probability increases when fewer welfare-decreasing mergers are deterred. A high probability of a prohibition, rather than suggesting to firms that future policy may be more stringent would then simply reflect that policy in the past was less stringent as more welfare-decreasing mergers were proposed by the firms. Incorporating the insights of this model may thus be of value in the assessment of antitrust policy measures.
6 Conclusion

This paper has proposed and solved a simple model of merger policy, in which firms can be privately informed about the consequences of their merger. The behavior of the antitrust authority in the model is roughly comparable with that of actual antitrust authorities. When receiving a merger proposal the antitrust authority first needs to decide whether to consider the merger further or clear it straight away. After an investigation the antitrust authority then needs to decide whether to clear or block the merger. In practice antitrust authorities usually clear mergers in Phase I investigations unless they refer them to more in-depth Phase II investigations during which more extensive evidence is collected.

The paper distinguishes two cases: strongly and weakly opposed preferences. Strongly opposed preferences were seen to arise when the firms are privately informed about non-merging firms’ costs, substitutability of products or of the merging firms’ products’ substitutability. Weakly opposed preferences were shown to arise when merging firms are privately informed about efficiency gains from a merger.

The use of filing fees as an instrument of merger policy was shown to be capable of implementing the first best under weakly opposed preferences, but not under strongly opposed preferences. Reductions in the effectiveness of the investigations by the antitrust authority were shown to allow more welfare-decreasing mergers to be proposed and cleared. This implies that there are welfare gains to be had from better analysis of the welfare effects of mergers.

The antitrust authority in this paper is a benevolent social planner, who in particular, could perfectly assess the consequences of a merger, once all information was known. However in practice antitrust authorities might incorrectly assess the consequences of a merger. When an antitrust authority makes mistakes in its assessments of mergers then it may happen that antitrust authority affects the decision of firms who would propose a welfare-increasing merger. Some of these mergers may be deterred if the firms correctly predict that the antitrust authority will wrongly believe the merger to reduce welfare. Such effects may be of practical relevance.

While the examples in this paper considered mainly the merger of two firms competing in the same industry, one can also apply it to the analysis of vertical mergers.
References


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A Omitted Proofs

Proof (Lemma 1) After an investigation, the authority knows the value of $\theta$. If it allows the merger the authority obtains a pay-off of $\Delta(\theta) - \omega$. If it blocks the merger it obtains a pay-off of $-\omega$. Hence allowing the merger is optimal whenever $\Delta(\theta) \geq 0$. For the second statement, we show that when there is a gain from a merger exceeds be that $g$. Note however that this is not sufficient, since even if the gain from an investigation decreases, it may be that $\Delta(\theta) < 0$. Then we have that $d_1(\hat{\theta}) = 1$. The pay-off of proposing such a merger is then $-\epsilon$ to the firms, as this merger will never be cleared. So $p(\hat{\theta}) = 0$ must hold. But then the antitrust authority believes that no welfare-decreasing mergers are proposed, implying that $d_0 = 1$ cannot be optimal, which yields the desired contradiction. Since $d_0 = 0$ and $d_1(\theta) = 0$ for $\theta$ such that $\Delta(\theta) \geq 0$, a merger with $\Delta(\theta) \geq 0$ will always be cleared. Hence the pay-off from proposing such a merger is $\pi(\theta) - \epsilon > 0$, from the assumption that all mergers are inherently profitable to the firms. It follows that $p(\theta) = 1$ if $\Delta(\theta) \geq 0$.

Lemma 5 Under strongly opposed preferences the function $G'(\omega^*(\theta))$ is weakly increasing in $\theta_e$ on the interval $[\tilde{\theta}, \theta_0]$.

Proof The function $G'(\omega^*(\theta))$ is given by:

$$G'(\omega^*(\theta)) = G \left( \int_\theta^{\theta_e} -\Delta(\theta) \frac{f(\theta)}{1 - F(\theta_0) + F(\theta_e)} d\theta \right)$$

Using Leibnitz’ rule of differentiation then gives:

$$\frac{dG(\omega^*(\theta))}{d\theta_e} = g(.) \left( -\Delta(\theta_e) \frac{f(\theta_e)}{1 - F(\theta_0) + F(\theta_e)} + \int_{\theta}^{\theta_e} \Delta(\theta) \frac{f(\theta) f(\theta_e)}{(1 - F(\theta_0) + F(\theta_e))^2} d\theta \right)$$

Re-arranging we get:

$$\frac{dG(\omega^*(\theta))}{d\theta_e} = g(.) \frac{f(\theta_e)}{(1 - F(\theta_0) + F(\theta_e))^2} \left( -\Delta(\theta_e) (1 - F(\theta_0) + F(\theta_e)) + \int_{\theta}^{\theta_e} \Delta(\theta) f(\theta) d\theta \right)$$

First, note that $\frac{dG(\omega^*(\theta))}{d\theta_e}|_{\theta} = g(.) \left( -\Delta(\theta) \frac{f(\theta)}{1 - F(\theta_0)} \right) > 0$ since $g(.)$ is assumed to be strictly positive and since $\Delta(\theta) < 0$. Since $g(.)$ is assumed to be continuously differentiable, so is this derivative. For $G'(\omega^*(\theta))$ to be decreasing over the range $[\tilde{\theta}, \theta_0]$ it is needed that $\frac{dG(\omega^*(\theta))}{d\theta_e}$ takes a negative value. Note however that this is not sufficient, since even if the gain from an investigation decreases, it may be that $g(.) = 0$ so that the probability of an investigation remains constant. This occurs when the gain from a merger exceeds $\omega$. Differentiating the term $-\Delta(\theta_e) (1 - F(\theta_0) + F(\theta_e)) + \int_{\theta}^{\theta_e} \Delta(\theta) f(\theta) d\theta$ using Leibnitz’ rule yields:

$$-\Delta'(\theta_e) (1 - F(\theta_0) + F(\theta_e)) + \Delta(\theta_e) f(\theta_e) < 0$$
since $\Delta(\theta)$ is an increasing function. Hence whenever the marginal gain from an investigation is negative, it will remain negative for higher values of $\theta_e$. But by our assumption that when all welfare-reducing mergers are proposed, the antitrust authority will always find it optimal to merge, it follows that the marginal gain from an investigation can only be negative whenever the gain of an investigation exceeds $\omega$.

**Proof (Lemma 2)** Setting $\theta^* = \theta_e$ and $\alpha^* = \alpha_e$, rearrange equations AA-SOP and F-SOP to yield, respectively:

$$
\alpha^* = x(\theta^*) = G \left( \int_{\theta_e}^{\theta^*} -\Delta(\theta) \frac{f(\theta)}{1 - F(\theta_0) + F(\theta^*)} d\theta \right) 
$$

$$
\alpha^* = y(\theta^*) = \frac{\pi(\theta^*) - \varepsilon}{\pi(\theta^*)} 
$$

Note that $x(\theta) = 0$ and $x(\theta_0) = 1$ from the assumption that the maximum cost of an investigation is lower than the expected loss from welfare-reducing mergers, while $x' \geq 0$ by Lemma 5. In addition, $y(\theta) > 0$ due to the assumption that $\pi(\theta) > \varepsilon$ and $y(\theta_0) < 1$, while $y' < 0$ which can be obtained by differentiation and $d\pi(\theta)/d\theta < 0$. Note furthermore that both $x$ and $y$ are continuous functions. Therefore the above equations will cross once, but no more than that. Hence an equilibrium exists. This is also the only equilibrium in which mergers are proposed with positive probability.

**Lemma 6** Under weakly opposed preferences the function $G(\omega^*(\theta_e))$ is weakly decreasing in $\theta_e$.

**Proof** The function $G(\omega^*(\theta_e))$ is given by:

$$
G(\omega^*(\theta_e)) = G \left( \int_{\theta_e}^{\theta_0} -\Delta(\theta) \frac{f(\theta)}{1 - F(\theta_0) + F(\theta^*)} d\theta \right)
$$

Using Leibnitz’ rule of differentiation then gives:

$$
\frac{dG(\omega^*(\theta_e))}{d\theta_e} = g(.) \left( \Delta(\theta_e) \frac{f(\theta_e)}{1 - F(\theta_e)} + \int_{\theta_e}^{\theta_0} -\Delta(\theta) \frac{f(\theta)f(\theta_e)}{(1 - F(\theta_e))^2} d\theta \right)
$$

Re-arranging yields:

$$
\frac{dG(\omega^*(\theta_e))}{d\theta_e} = g(.) \frac{f(\theta_e)}{(1 - F(\theta_e))^2} \left( \Delta(\theta_e)(1 - F(\theta_0)) + \int_{\theta_e}^{\theta_0} \left( \Delta(\theta) - \Delta(\theta_e) \right) f(\theta) d\theta \right)
$$

The third factor on the right-hand side of the above equation always takes a negative sign. For $\theta_e$ close to $\theta$ the value taken by $g(.)$ will be equal to zero, by our assumption that an investigation will always occur when all firms propose to merge. Hence $G(\omega^*(\theta_e))$ is a weakly decreasing function when preferences are weakly opposed.
Proof (Lemma 3) Setting $\alpha_e = \alpha^*$ and $\theta_e = \theta^*$ and rearranging the equilibrium conditions AA-WOP and F-WOP become:

$$
\alpha^* = x(\theta^*) = G \left( \int_{\theta^*}^{\theta_0} -\Delta(\theta) \frac{f(\theta)}{1 - F(\theta^*)} d\theta \right) 
$$

(12)

$$
\alpha^* = y(\theta^*) = \frac{\pi(\theta^*) - \varepsilon}{\pi(\theta^*)}
$$

(13)

By Lemma 6 we have that $x' < 0$. Furthermore we have that $x(\theta) = 1$ from the assumption that the maximal investigation cost is lower than the expected loss from welfare-reducing mergers and $x(\theta_0) = 0$. Since $\pi$ is increasing in $\theta$ under weakly opposed preferences, we have that $y' > 0$. Furthermore, $y(\theta) < 1$ and $y(\theta_0) > 0$. Again both $x$ and $y$ are continuous functions. Therefore the system of equations has a unique solution.

Proof (Proposition 1) Consider the re-written equilibrium condition:

$$
\Psi(\theta^*, e, \varepsilon, \bar{w}) = 1 - \frac{\varepsilon}{\pi(\theta^*, e)} - G \left( \int_{\theta^*}^{\theta_0} -\Delta(\theta, e) \frac{f(\theta)}{1 - F(\theta_0) + F(\theta^*)} d\theta; \bar{w} \right) = 0
$$

(14)

Using Lemma 5 above and the fact that, $\pi_\theta < 0$, we get $\Psi_{\theta^*} < 0$. Furthermore we obtain $\Psi_\varepsilon < 0$. Using $\pi_e > 0$, $\Delta_e > 0$ as well as $d\theta_0/de < 0$ we get $\Psi_e > 0$. Last, using $G$ being uniform over $[0, \bar{w}]$ we get $\Psi_{\bar{w}} \geq 0$. Therefore we obtain: $\frac{\partial \Psi}{\partial e} = \frac{-\Psi_\varepsilon}{\Psi_{\theta^*}} > 0$, $\frac{\partial \Psi}{\partial \theta^*} = \frac{-\Psi_e}{\Psi_{\theta^*}} < 0$ and $\frac{\partial \Psi}{\partial \bar{w}} = \frac{-\Psi_{\bar{w}}}{\Psi_{\theta^*}} \geq 0$. For the effects on $\alpha$ note that in equilibrium, $\alpha^* = \frac{\pi(\theta^*, e) - \varepsilon}{\pi(\theta^*, e)}$, which is decreasing in $\theta^*$ and not directly affected by $\bar{w}$. Hence $\frac{d\alpha^*}{d\bar{w}} \leq 0$. Note that the the left hand term $1 - \frac{\varepsilon}{\pi(\theta^*, e)}$ is the equilibrium probability of an investigation. The right hand term is a negative function of $\theta^*$ but is independent of $\varepsilon$. This implies that since $d\theta^*/d\varepsilon < 0$, $d\alpha^*/d\varepsilon < 0$. The effect of $e$ on $\alpha^*$ is however ambiguous, since $e$ affects both $\pi$ and $\Delta$.

Proof (Proposition 2) Consider the re-written equilibrium condition:

$$
\Psi(\theta^*, \gamma, \varepsilon, \bar{w}) = 1 - \frac{\varepsilon}{\pi(\theta^*, \gamma)} - G \left( \int_{\theta^*}^{\theta_0} -\Delta(\theta, \gamma) \frac{f(\theta)}{1 - F(\theta_0) + F(\theta^*)} d\theta; \bar{w} \right) = 0
$$

(15)

Using Lemma 6 above and the fact that $\pi_\theta > 0$, we get $\Psi_{\theta^*} > 0$. Furthermore we obtain $\Psi_\varepsilon < 0$. Using $\pi_e > 0$, $\Delta_e < 0$ the effect of $\gamma$ on $\Psi$ cannot be signed. Hence $\gamma$ will have an ambiguous effect on $\theta^*$. Last, using $G$ being uniform over $[0, \bar{w}]$ we get $\Psi_{\bar{w}} \geq 0$. Therefore we obtain $\frac{\partial \Psi}{\partial e} > 0$ and $\frac{\partial \Psi}{\partial \theta^*} \leq 0$. From that it also follows that $\frac{d\alpha^*}{d\theta^*} < 0$ and $\frac{d\alpha^*}{d\gamma} \leq 0$. To obtain $d\alpha^*/d\gamma > 0$ we re-write equations AA-WOP and F-WOP such that:

$$
\tilde{\Psi}(\alpha^*, \gamma) = \tilde{x}(\alpha^*, \gamma) - \tilde{y}(\alpha^*, \gamma) = 0
$$

(16)

where $\tilde{x}$ and $\tilde{y}$ are the inverses of the functions defined in the proof of Lemma 3. Then we have that $\tilde{\Psi}_{\alpha^*} > 0$ and $\tilde{\Psi}_\gamma < 0$ so that $\frac{d\alpha^*}{d\gamma} > 0$. 

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Proof (Proposition 3) For the case of strongly opposed preferences consider the equilibrium condition given by:

$$
\Psi(\theta^*, \rho) \equiv 1 - \frac{\varepsilon}{\pi(\theta^*)} - \rho G \left( \rho \int_{\theta}^{\theta^*} -\Delta(\theta) \frac{f(\theta)}{1 - F(\theta_0) + F(\theta^*)} d\theta \right) = 0
$$

(17)

As before, we have $\Psi_{\theta^*} < 0$. Furthermore we have $\Psi_\rho < 0$. Therefore we obtain $\frac{d\theta^*}{d\rho} < 0$. As the threshold for proposing a merger falls, fewer welfare-decreasing mergers will be proposed when $\rho$ increases. We have that $1 - \frac{\varepsilon}{\pi(\theta^*)}$ again represents the probability of an investigation, which does not depend directly on $\rho$. Hence we have that $\frac{d\alpha^*}{d\rho} > 0$.

For the case of weakly opposed preferences consider the equilibrium condition given by:

$$
\Psi(\theta^*, \rho) \equiv 1 - \frac{\varepsilon}{\pi(\theta^*)} - \rho G \left( \rho \int_{\theta}^{\theta^*} -\Delta(\theta) \frac{f(\theta)}{1 - F(\theta_0) + F(\theta^*)} d\theta \right) = 0
$$

(18)

As before we have that $\Psi_{\theta^*} > 0$. Furthermore we have $\Psi_\rho < 0$. Therefore we obtain $\frac{d\theta^*}{d\rho} > 0$. As the threshold for proposing a merger rises, fewer welfare-decreasing mergers will be proposed when $\rho$ increases. We have that $1 - \frac{\varepsilon}{\pi(\theta^*)}$ again represents the probability of an investigation, which does not depend directly on $\rho$. Hence we have that $\frac{d\alpha^*}{d\rho} > 0$.

**Lemma 7** In the equilibrium of the merger proposal game with a filing fee we have that: $\frac{d\theta^*}{dt} < 0$ and $\frac{d\alpha^*}{dt} < 0$.

**Proof (Lemma 7)** Consider the equilibrium condition of the merger proposal game in which the filing fee is set at $t$:

$$
\Psi(t, \theta^*) \equiv 1 - \frac{\varepsilon + t}{\pi(\theta^*)} - G \left( \int_{\theta}^{\theta^*} -\Delta(\theta) \frac{f(\theta)}{F(\theta^*) - F(\theta_0) + F(\theta^*)} d\theta \right)
$$

(19)

Noting that $\frac{d\theta^*}{dt} < 0$ we can see that the partial derivative with respect to $t$ is:

$$
\frac{\partial \Psi}{\partial t} = \frac{-1}{\pi(\theta^*)} \frac{g(.)}{\Delta(\theta)} \frac{f(\theta)}{F(\theta^*) - F(\theta_0) + F(\theta^*)} d\theta \left( -f(\theta^*) \right) \frac{d\theta^*}{dt} < 0
$$

(20)

Using $\frac{\partial \Psi}{\partial \theta^*} < 0$ it follows that $\frac{d\theta^*}{dt} < 0$ and also $\frac{d\alpha^*}{dt} < 0$.

**Proof (Lemma 4)** Consider the equilibrium condition given by:

$$
\Psi(\theta^*, \beta) \equiv 1 - \frac{\varepsilon}{\pi(\theta^*)} - G \left( \int_{\theta}^{\theta^*} -\Delta(\theta, \beta) \frac{f(\theta)}{1 - F(\theta_0) + F(\theta^*)} d\theta \right) = 0
$$

(21)

As usual we have that $\Psi_{\theta^*} < 0$. Since $\Delta(\theta, \beta)$ is increasing in $\beta$ for all values of $\theta$, we also have that $\Psi_\beta > 0$, implying that $\frac{d\theta^*}{d\beta} > 0$. A change in $\beta$ does not affect the firm’s decision directly, hence we have that $\frac{d\alpha^*}{d\beta} < 0$ and thus also $\frac{d\omega^*}{d\beta} < 0$. 37