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Net Neutrality, Vertical Integration, and Competition Between Content Providers

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Abstract

This paper investigates the effects of a net neutrality regulation on the competition between content providers and the investment incentives of the internet service provider. We consider a situation where the monopoly internet service provider is vertically integrated with one of the content providers, and content providers compete in prices. Without net neutrality the vertical integrated firm can prioritise the delivery of its own content. We find that, under prioritisation, the integrated internet service provider and consumers as a whole are unambiguously better off. The competing content providers might also be better off under prioritisation if the congestion intensity is high. From a social welfare perspective prioritisation is also desirable unless product differentiation and congestion intensity are low. Contrary to some claims by internet service providers, we find that investment incentives are not always higher under prioritisation.

Keywords: Vertical integration, Network neutrality, Competition, Investment

JEL Classification Numbers: L13, L41, L42, L88

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1 Introduction

Net neutrality refers to the principle that all data packets on the internet are to be treated equally, such that there is no discrimination in price and quality. It prevents last mile internet service providers from speeding up, slowing down or blocking traffic based on its source or content. While the net neutrality debate has many aspects, in this paper we focus on foreclosure by an integrated internet service provider. Ever since Madison River Communications, a small internet service provider, blocked VoIP services that competed with its own voice services, net neutrality has been subject to a fierce debate not only in the United States but also around Europe. The question arises whether and how net neutrality should be enforced. In February 2015, the Federal Communications Commission (2015) in the US adopted strong net neutrality rules. These rules prohibit internet service providers from discriminating in favour of their own services.

With the boom of broadband internet many innovative online content services have emerged. These content services stimulate the use of broadband to the benefit of internet service providers. Yet the relationship between content services and internet service providers is not unproblematic. The concern arises that vertical integrated internet service providers might have an incentive to favour their vertically affiliated content provider over unaffiliated rival services. In particular internet service providers can prioritise their own services to increase their demand and foreclose competitors.

Another key aspect of the net neutrality debate concerns the investment in the next generation network. Investments in the network are critical not only for innovation of online services but also for overall economic growth. Internet service providers, however, claim that net neutrality makes the network less profitable and therefore discourages investment in the internet infrastructure. For policy makers it is, therefore, important to assess the economic consequences of a net neutrality regulation with respect to competition and investments.

We add to this debate by developing a theoretical model that explores the competition and welfare effects of net neutrality versus prioritisation. Moreover, we analyse the incentives of the internet service provider to invest in the network capacity. Our contribution to the literature is twofold: First, our focus is on vertical integration, where the monopolistic internet service provider is integrated with one of the two competing content providers. We concentrate on potential foreclosure concerns. Without a net neutrality regulation internet service providers might use their network to discriminate unaffiliated content providers. Sec-

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1Voice over Internet Protocol
2E.g.:http://www.washingtonpost.com/wp-dyn/content/article/2006/02/06/AR2006020601624.html
ond, we introduce direct payments by consumers to content providers because, for example, online video service providers like Netflix often charge consumers a monthly subscription fee to access their content. This setup contrasts most of the existing literature, where content providers derive profits solely from advertising. Directly competing in prices allows unaffiliated content providers to better react to non-neutral behaviour by the internet service provider and adjust their prices accordingly.

Waiting time is explicitly modelled and consumers’ utility decreases in waiting time. Under net neutrality all consumers, no matter which content they consume, face the same waiting time. In the absence of net neutrality, the internet service provider prioritises the delivery of its own content. This leads to increased waiting times for consumers of the provider that is given less favourable treatment and reduced waiting times for consumers of the affiliated content. The affiliated content provider then obtains a quality of service advantage.

Comparing the two regulatory outcomes, we find that the internet service provider is always better off when it is able to prioritise its own content. In the content market profit of the integrated firm increases because it can offer a higher quality and thereby is able to increase its price. In the internet access market it has to charge lower internet subscription fees as consumers of the competing content have a lower willingness to pay due to lower quality. This loss in the internet access market is, however, more than offset by the gains in the content market.

Surprisingly, prioritisation is not always detrimental to the competing content provider although it has a quality disadvantage. Whether the competing content provider is worse off depends on the congestion intensity and the degree of content differentiation. Under low congestion intensity, the profit of the unaffiliated content provider is always lower. However, when congestion intensity is high, profits may also be higher than under net neutrality. The reason is that the integrated content provider faces a trade-off between demand and waiting time. As demand increases, waiting time for its consumers also increases. To balance this trade-off it further increase the price of its content so that demand and waiting time are reduced. Due to the strategic complementarity of the pricing strategies, prices of the unaffiliated content provider might then rise considerably compared to the drop in demand; hence, higher profits under prioritisation.

Consumers as a whole gain from a prioritisation because more consumers benefit from prioritised delivery of their content. The effect of prioritisation on social welfare compared to net neutrality depends on the congestion intensity and the degree of content differentiation. Social welfare is higher under prioritisation if the differentiation between the contents is
Finally, by analysing the effect of a net neutrality regulation on the internet service provider’s incentive to invest in its network, we find that - contrary to recent claims - internet service providers do not necessarily have higher incentives to invest under prioritisation.

Related Literature
The present paper contributes to the growing economic literature on net neutrality, which is surveyed by [Schuett (2010)](https://doi.org/10.1017/CBO9780511754798). Net neutrality is often studied in the framework of a two-sided market (see [Armstrong (2006)](https://doi.org/10.1017/CBO9780511754798)) where internet service providers are platforms that connect content providers on one side with internet consumers on the other side. Consumers and content providers generate positive network effects for each other. [Economides and Tag (2012)](https://doi.org/10.1017/CBO9780511754798) discuss net neutrality in the context of two-sided pricing. Without net neutrality regulation, internet service providers can charge the content providers additional fees for terminating their traffic to the consumers. Under net neutrality the termination fee for content providers is set to zero. They find that net neutrality can be welfare enhancing if content providers value additional consumers more than consumers value additional content providers. Consumers are clearly worse off under net neutrality because of the one sided pricing by the internet service provider.

In this paper, net neutrality is interpreted as a non-discrimination rule. Under this rule net neutrality corresponds to a situation where internet service providers cannot offer different priority lanes and, at the same time, cannot charge content providers for prioritised delivery of their traffic. Many opponents of net neutrality argue that net neutrality neglects the importance of quality of service. Some content services desire a reliable transmission of information that is time critical. For example, video on demand and email services have different quality of service requirements. From a welfare perspective, discriminatory traffic management creates the potential for allocating bandwidth in a more efficient way, thereby maximising welfare for all users of the network [Hermalin and Katz, 2007](https://doi.org/10.1017/CBO9780511754798) [Krämer and Wiewiorra, 2012](https://doi.org/10.1017/CBO9780511754798). To demonstrate this, [Krämer and Wiewiorra, 2012](https://doi.org/10.1017/CBO9780511754798) consider a continuum of non-competing congestion sensitive content providers and consumers that connect to all content providers on the network. The number of content providers that choose to join the network in equilibrium is determined endogenously. They find that a discriminatory scheme is more profitable for the internet service providers and is welfare-enhancing as long as enough content providers benefit from prioritisation. [Bourreau et al., 2014](https://doi.org/10.1017/CBO9780511754798) extend their framework by introducing competition between internet service providers as competition might mitigate the problems associated with discriminatory traffic management [Hahn and Wallsten, 2006](https://doi.org/10.1017/CBO9780511754798). They show that prioritisation leads to more investment because internet service provider
can extract more revenue from content providers and more content providers with a high congestion sensitivity enter the market. Overall, prioritisation is therefore welfare superior to net neutrality. Economides and Hermalin (2012) use a screening model to analyse the incentives of an internet service provider to provide priority lanes. Consumers adjust their behaviour based on actual transmission speeds. They conclude that net neutrality is often welfare maximising because discrimination can lead to bandwidth inefficiencies. Specifically, discrimination increases demand for content services with priority so that these services generate more traffic than under net neutrality, which in turn then re-congests the network. These papers differ from our model in that they consider a continuum of independent content providers that do not compete with each other.

Differently from the previous papers, we are interested in priority given to one content provider who is in competition with another content provider offering content with the same congestion sensitivity. Without net neutrality, the internet service provider could give priority to some content providers and thereby harm competing content providers (Economides, 2007; van Schewick, 2015). The models by Choi and Kim (2010) and Cheng et al. (2011) are conceptually the closest to ours as they also consider a monopolistic internet service provider, two content providers and consumers who only consume one content. Based on queuing theory (Kleinrock, 1975, 1976), they set up a Hotelling model of competition between content providers who, contrary to our model, derive profits solely from advertising. The main difference between the two papers is the treatment in selling priorities to the content providers. Choi and Kim (2010) only allow one content provider to acquire the priority right, while Cheng et al. (2011) allow both content providers to pay for priority.

Under prioritisation, Choi and Kim (2010) demonstrate that for the internet service provider the loss in subscription fees is offset by the gain in revenue from selling priority when advertising margins are high. While the profit of the content provider which pays for priority might increase, the profit of the non-prioritised content provider is always lower than under net neutrality. Due to the quality disadvantage, it looses market share and hence, advertising revenue. Welfare effects of imposing net neutrality largely depend on how advertising margins related to transportation costs. Regarding investment incentives, the relative value of priority becomes relatively small for higher levels of capacity. As a result, investment incentives are smaller under net neutrality in which such rent extraction effects do not exist. Cheng et al. (2011) find in their setting that the internet service provider always gains from prioritisation and content providers are left in a worse position. Consumers who consume prioritised content are better off while other consumers are worse off. Total welfare and consumer surplus increases when only one content provider pays for priority but is unchanged when both content providers pay. They further show that generally the
investment incentives are lower under prioritisation because the revenue contributions from content providers decrease in capacity expansion.

In a similar framework, Bandyopadhyay et al. (2009) incorporate vertical integration. They find that whether prioritisation is welfare enhancing depends on whether the internet service provider is integrated with the more or less effective content provider. Moreover, they argue that the integrated internet service provider may even have incentives to prioritise the competing content provider provided that it is more effective in generating advertising revenue so that the internet service provider can extract more rent.

Basically, our model builds upon Choi and Kim (2010), Cheng et al. (2011) and Bandyopadhyay et al. (2009) by introducing a different revenue model. We introduce direct payments between content providers and consumers because such a revenue model becomes increasingly popular with subscription based content. Another key difference is that we focus exclusively on vertical foreclosure related to net neutrality and do not allow for fees to be paid for prioritised delivery of data. We show that for the internet service provider the loss in subscription fees is offset by the gain in revenue in the content market due to the quality of service advantage. Even though waiting time is non-linear in our utility function, under a pure advertising based revenues model, we would obtain similar results as Choi and Kim (2010) for the non-prioritised content provider. However, with price competition between content providers, the non-prioritised content provider might even be better off under prioritisation as it can react to the non-neutral behaviour by the internet service provider and adjust its prices accordingly. Prioritisation increases the demand for the prioritised content, so that the waiting time for priority also increases. Therefore, there is a trade-off between demand and waiting time. Welfare effects depend on the trade-off between total gain in perceived quality of service and gain in total transportation costs.

Brito et al. (2013) are the first to our knowledge to introduce price competition between content providers. The key differences are that first they also consider competition between internet service providers, and second and foremost that they do not explicitly model congestion with the queuing model. They translate the quality of network service that internet service providers offer content providers into the gross utility function for consumers. Contrary to our model, quality of service does not depend on demand. The prioritised content provider, therefore, does not have to trade off quality of service with the demand. As a result, the discriminated content provider is always worse off under prioritisation. They also find that, under prioritisation, internet service providers are better off. Investment and welfare are higher under prioritisation given that internet service providers are symmetric and can allocate the level of quality of network services among the content providers freely.
Finally, by introducing vertical integration our work is also related to the large literature on vertical foreclosure. Net neutrality implies that a vertically integrated internet service provider is not allowed to artificially degrade or foreclose competing content. The Chicago School (e.g. Bowman, 1957; Posner, 1976; Bork, 1978) argues that there are no incentives to vertically foreclose a competitor when the goods are essential complements because there is only one profit to be extracted. Firms could simply use their upstream market power to extract the rent from the downstream competitors. Moreover, the monopolist even benefits from competition in the complementary market (Whinston, 1990).

Relating vertical foreclose to the net neutrality debate, Chen and Nalebuff (2006) show that a monopolistic upstream firm has no incentives to degrade the quality of service of its downstream competitors by offering its own competitive good for free and charging a higher price for the upstream product. It is further shown by Dewenter and Rösch (2014) that a vertical integrated internet service provider does not foreclose competitors in the content market if content providers are sufficiently differentiated. Economides (1998) shows that it is, however, beneficial to an upstream monopolist to raise its rivals’ cost by quality degradation. This result is also reflected in our paper. By prioritising affiliated content the monopoly input supplier artificially degrades the quality of content of its competitor in the content market. The profit of the vertically integrated firm increases.

The remainder of this paper is organised as follows. Section 2 presents the theoretical model, that is then solved in Section 3. We consider both net neutrality and prioritisation. In Section 4 we compare these two different regimes and derive welfare effects. Section 5 identifies the investment incentives of the internet service provider under the two scenarios. Section 6 concludes. The proofs of all formal results are relegated to Appendix A. In Appendix B we develop waiting times in a system based on the queueing theory.

2 The Model

We study net neutrality regulation of the internet in a market with a single internet service provider and two content providers. The internet service provider owns an internet network and provides consumers access to it. Content providers are firms that create content for the consumers on the internet. Internet access is therefore essential to the consumption of these content services. Examples of such content services are email, news, music and video services. In our scenario consider content providers as being video service providers.

We consider a situation where the monopoly internet service provider is vertically inte-
grated with one of the content providers and, therefore, effectively constitutes a single firm A. The other content provider, firm B, sells its content to consumers over the network of firm A. Consumers regard the products of the two content providers as horizontally differentiated. Adopting the Hotelling (1929) framework, consumers of mass 1 are uniformly distributed along the unit interval, while locations of the content providers are fixed. Firm A is located at point 0 and firm B is located at point 1. A consumer’s location is equivalent to his taste parameter $x \in [0,1]$. He faces total “transportation cost” of $tx$ when buying content from firm A and $t(1-x)$ when buying from firm B, where the “transportation cost” parameter $t > 0$ indicates the degree of product differentiation. To consume content on the internet, each consumer first must buy internet access from firm A at a price $p_I$ and then chooses to buy either content A at price $p_A$ or content B at price $p_B$. A consumer who purchases content from firm $i \in A,B$ thus has to pay $p_I + p_i$ in total for internet access and content. A consumer with characteristic $x$ derives utility $U_i(x,p_i,p_I,w_i)$ from consuming internet access and content $i$, where

\[
U_A(x,p_A,p_I,w_A) \equiv u + \frac{v}{w_A} - tx - p_A - p_I
\]

\[
U_B(x,p_B,p_I,w_B) \equiv u + \frac{v}{w_B} - t(1-x) - p_B - p_I
\]

The parameter $u$ is defined as $u = u_I + u_C$ such that each consumer derives a fixed utility of $u_I > 0$ from internet access and $u_C > 0$ from content consumption. The parameter $v > 0$ measures consumers’ preference for the speed of the internet connection for content consumption and $w_i$ is the waiting time until the content arrives. Therefore, $1/w_i$ represents the speed of the internet connection so that the second component, $v/w_i$, indicates the perceived quality of service. Perceived quality of service is decreasing in waiting time, which in turn depends on network capacity, demand and prioritisation.

**Waiting Time and Congestion**

The internet service provider owns the network infrastructure with a capacity of $\mu > 0$. Internet capacity, also called bandwidth, is the amount of data that can be transmitted over the network from content providers to consumers in a given period of time. This capacity is shared between all subscribers of the internet connection. Due to capacity constraints, the network might suffer from congestion. If data requests are increasing, more capacity will be used up at that time, so that it will take longer until content is transmitted to the consumers. The greater the capacity, the faster data can be carried over the network.

Under net neutrality, content is transmitted on a best-effort basis over the network to the consumers. More specifically, the internet service provider treats all content the same and is
not allowed to differentiate between the transmission of its own content and the unaffiliated one. Congestion and the resulting waiting time until the content arrives is, therefore, the same for all consumers no matter which content they consume. Without a net neutrality regulation, however, the internet service provider can prioritise its own content, thereby reducing waiting time for consumers who buy the integrated content, whilst waiting time increase for consumers of content B.

As it is common in the net neutrality literature, the framework of the $M/M/1$ queuing system (e.g. Kleinrock (1975, 1976)) is adopted in order to model congestion. Congestion is measured by the waiting time $w_i$ for consumers when they request content from one of the two content providers. Waiting time depends on network capacity, total traffic in the network as well as on data prioritisation.

As Choi and Kim (2010), we assume that the content request rate of each consumer follows a Poisson process with content request rate $\lambda > 0$ corresponding to the demand intensity. Total capacity demand equals content request rate times the total number of consumers. Given full market coverage, the waiting time for a consumer to obtain the requested content under net neutrality is therefore

$$w_i^N = \frac{1}{\mu - \lambda}. \quad (2)$$

Traffic intensity $\rho$ is defined by the ratio of arrival rate to capacity and measures congestion in the system:

$$\rho = \frac{\lambda}{\mu} \quad (3)$$

As traffic intensity goes up, the amount of congestion increases and thereby consumers have longer waiting times in the system. For a stable system we need to assume that available capacity is larger than the content request rate. Otherwise, the queue will grow indefinitely long and the system will not have a stationary distribution.

**Assumption 1.** $\mu > \lambda$.

Without net neutrality, the internet service provider can offer a priority lane for consumers who buy their own integrated content and a non-priority lane for consumers who buy the competing content, thereby offering different qualities of services. Let $x_A^P$ denote the market share of content provider A under priority. Then, given full market coverage, the waiting time for consumers of content A, who are in the priority lane, is given by

$$w_A^p = \frac{1}{\mu - \lambda x_A^p}. \quad (4)$$

\(^4\)see appendix B for details
whereas the waiting time for consumers of content $B$, who are in the non-priority lane, is given by

$$w_B^P = \frac{\mu}{(\mu - \lambda)(\mu - \lambda x_A^P)}$$ (5)

Consumers buying non-prioritised content face higher waiting times since the relative ratio of $w_B^P$ to $w_A^P$ is greater than one, i.e. $w_B^P/w_A^P = \mu/(\mu - \lambda) > 1$. As a consequence, $w_B^P > w_i^N > w_A^P$ for $\mu > \lambda$.

The perceived quality of service is decreasing in waiting time indicating that consumers suffer from congestion. More specifically, waiting time is decreasing in capacity $\mu$ and increasing in content request rate $\lambda$. Under prioritisation, waiting times are defined by equations (4) and (5). The number of priority consumers exhibits negative externalities on consumers of both priority lanes. The larger the market share of content provider $A$, the higher the waiting time for everyone. Moreover, this functional form exhibits a diminishing marginal disutility of waiting. The marginal negative impact of waiting on consumers’ utility decreases as the waiting time increases so that there is a much greater loss in marginal utility with short waiting times.

Demand for internet access equals 1 since full market coverage is assumed for tractability. The market shares of content providers $A$ and $B$ are given by $x_A(p_A, p_B)$ and $x_B(p_A, p_B)$, respectively. Both firms have identical marginal costs equal to zero. Hence, firm $A$’s profit is the sum of the profits from internet access, $\Pi_{AI}$, and content services, $\Pi_{AC}$, that is

$$\Pi_A = \Pi_{AI} + \Pi_{AC} = p_I + p_A x_A(p_A, p_B).$$

Content provider $B$ makes profit only from content services, that is

$$\Pi_B = p_B x_B(p_A, p_B).$$

Competition then proceeds as follows:

Stage 1: In the absence of net neutrality regulation, the internet service provider chooses whether to prioritise its own content. Under net neutrality, there is no stage 1.

Stage 2: The internet service provider sets the subscription fee $p_I$ for internet access.

Stage 3: Content providers compete simultaneously in the content market by setting prices $p_A$ and $p_B$ and consumers decide whether to subscribe to the internet and choose which content to buy.
3 Equilibrium Outcomes

We next solve for the equilibria under net neutrality and under prioritisation by backward induction.

3.1 Network Neutrality

Under net neutrality, all content has to be treated equally. The internet service provider, therefore, cannot prioritise its own content; hence, waiting times are the same for all consumers no matter whose content is consumed. Provided that all consumers buy internet access, waiting times are given by (2) and hence,

$$w^N_A = w^N_B = \frac{1}{\mu - \lambda}.$$

Each consumer chooses whether to buy content from firm $A$ or firm $B$. The consumer indifferent between the two firms, denoted by $\hat{x}$, is defined by $U_A(\hat{x}, p_A, p_I, w^N) = U_B(\hat{x}, p_B, p_I, w^N)$. This yields

$$\hat{x}(p_A, p_B) = \frac{1}{2} + \frac{p_B - p_A}{2t} \quad (6)$$

Since we assume that the market for content is covered, the market shares for content provider $A$ and content provider $B$ are $x_A^N(p_A, p_B) = \hat{x}(p_A, p_B)$ and $x_B^N(p_A, p_B) = 1 - \hat{x}(p_A, p_B)$, respectively. The content providers compete by setting prices to the consumers. Firm $A$ maximises its profit from content services, $\Pi_{AC}^N(p_A, p_B)$, and content provider $B$ maximises its profit, $\Pi_{AC}^N(p_A, p_B)$. From the first-order conditions $\partial \Pi_{AC}^N(p_A, p_B)/\partial p_A = 0$ and $\partial \Pi_{AC}^N(p_A, p_B)/\partial p_B = 0$ we obtain the equilibrium prices

$$p_A^N = p_B^N = t. \quad (7)$$

By substituting (7) into (6), we obtain the equilibrium market shares of the content providers

$$x^N_A = x^N_B = \frac{1}{2}. \quad (8)$$

Due to the fact that waiting times under net neutrality are the same for all consumers independent of the content they consume, content providers are symmetric and therefore share the market equally.

For the rest of the paper we assume for simplicity that (i) fixed utility from content
consumption, \( u_C \), is high enough so that all consumers obtain a positive net utility from content consumption and (ii) the utility from internet access, \( u_I \), is sufficiently high compared to the consumer surplus in the content market so that it is not profitable for the internet service provider to exclude some consumers from the market. Then, as internet access is essential to the consumption of content services, the internet service provider can exploit its market power in the access market and extract some of the surplus consumers gain in the content market. The subscription fee \( p_I \) is set so that all consumers connect to the internet and the indifferent consumer receives zero overall utility:

\[
\max_{p_I} \Pi_{AI} = p_I \text{ s.t. } U_A(\hat{x}(p_A^N, p_B^N), p_A^N, p_I, w) \geq 0
\]

which leads to equilibrium access fee

\[
p_I^N = u + v(\mu - \lambda) - \frac{3t}{2}.
\]

The above analysis implies that the integrated firm A’s overall profit, \( \Pi_A = \Pi_{AC} + \Pi_{AI} \), in the content and internet access market is given by

\[
\Pi_A^N = u + v(\mu - \lambda) - t
\]

and firm B’s profit by

\[
\Pi_B^N = \frac{t}{2}.
\]

Under net neutrality, profits in the content market are independent of capacity and content request rate because of the equal treatment of consumers of both content provider. However, an increase in network capacity increases firm A’s profit in the internet access market. This is due to the fact that it can charge a higher subscription fee to consumer because congestion is reduced thereby reducing consumer’s waiting time and increasing their willingness to pay. Because internet is essential to the consumption of content, firm A can extract all this surplus. On the other hand, when there is an increase in the content request rate, congestion is increased so that the willingness to pay decreases and firm A’s profit will be smaller.

### 3.2 Prioritisation

Without net neutrality regulation the integrated internet service provider is able to offer priority and non-priority lines for content transmission. The internet service provider there-
fore prioritises transmission for consumers who have bought its own integrated content $A$. Consequently, consumers face a different waiting time depending on the choice of content service. By (4) and (5), waiting time in the system for consumers buying content $A$ is

$$w_A^P = \frac{1}{\mu - \lambda x_A}$$

and for consumers of content $B$

$$w_B^P = \frac{\mu}{\mu - \lambda} \frac{1}{\mu - \lambda x_A}.$$

The consumer $\tilde{x}$, who is indifferent between the two content providers, is defined by $U_A(\tilde{x}, p_A, p_I, w^P) = U_B(\tilde{x}, p_B, p_I, w^{NP})$. We assume consumers’ expectations regarding the demand for content $A$ is fulfilled, so that $\tilde{x} = x_A(p_A, p_B)$. From the indifference condition we then obtain

$$\tilde{x}(p_A, p_B) = \frac{\mu(t + v\lambda + p_B - p_A)}{2\mu + v\lambda^2}.$$  \hspace{1cm} (12)

As before, the market for content is covered so that the market shares of content provider $A$ and content provider $B$ are given by $x_A^P(p_A, p_B) = \tilde{x}(p_A, p_B)$ and $x_B^P(p_A, p_B) = 1 - \tilde{x}(p_A, p_B)$, respectively. The content providers compete in prices by maximising their profits. Profit from content services for firm $A$ is $\Pi^P_{AC}(p_A, p_B)$ and profit for content provider $B$ is $\Pi^P_B(p_A, p_B)$.

Solving the first-order conditions, $\partial\Pi^P_{AC}(p_A, p_B)/\partial p_A = 0$ and $\partial\Pi^P_B(p_A, p_B)/\partial p_B = 0$, yields the equilibrium prices

$$p_A^P = t + \frac{v\lambda}{3\mu} (\mu + \lambda)$$  \hspace{1cm} (13)

$$p_B^P = t - \frac{v\lambda}{3\mu} (\mu - 2\lambda).$$  \hspace{1cm} (14)

Substituting these equilibrium prices into the demand functions, we obtain the market shares for the content providers

$$x_A^P = \frac{3\mu t + v\lambda(\mu + \lambda)}{6\mu t + 3v\lambda^2}$$

$$x_B^P = 1 - x_A^P.$$  \hspace{1cm} (15)

When the internet service provider prioritises its own content, perceived quality for consumers of content $A$ is higher than perceived quality of content $B$. As a result firm $A$ always obtains a larger market share than firm $B$.

**Proposition 1.** Under prioritisation content provider $A$ always covers more than half of the
market, that is \( x_A^P > 1/2 \) and therefore also has a larger market share than content provider \( B \), that is \( x_A^P > x_B^P \).

In order to ensure an interior solution where both firm sell strictly positive quantities of their services, one needs \( x_i^P \in (0,1) \). Therefore, we assume in what follows that the differentiation parameter \( t \) is high enough.

**Assumption 2.** \( t > \frac{1}{3} \equiv \frac{|v\lambda(\mu - 2\lambda)|}{(3\mu)} \)

Differentiation is high enough so that even with prioritisation of its own content, firm \( A \) cannot attract the entire market for content.

Anticipating the prices that will be set in the content market, the internet service provider sets the access fee \( p_I \) such that all consumers subscribe to the internet. It extracts all utility from the indifferent consumer:

\[
\max_{p_I} \Pi_{AI} = p_I \text{ s.t. } U_A(\tilde{x}(p_A^P, p_B^P), p_A^P, p_I, w^P) \geq 0
\]

This yields the equilibrium access fee

\[
p_I^P = u + \frac{6vt\mu(\mu^2 - 2\lambda - \lambda^2) + v^2\lambda^2(2\mu^2 - 2\lambda\mu - \lambda^2) - 9t^2\mu^2}{3\mu(2t\mu + v\lambda^2)}.
\]

Using the results (13) – (16) we can calculate the profits of the firms. The profit of the integrated firm \( A \) from sales of internet access as well as content services, \( \Pi_A = \Pi_{AC} + \Pi_{AI} \), is

\[
\Pi_A^P = u + \frac{6vt\mu(3\mu^2 - 2\lambda\mu - 2\lambda^2) + v^2\lambda^2(7\mu^2 - 4\lambda\mu - 2\lambda^2) - 18t^2\mu^2}{9\mu(2t\mu + v\lambda^2)}
\]

and the profit of firm \( B \) is

\[
\Pi_B^P = \frac{(3t\mu + 2v\lambda^2 - v\lambda\mu)^2}{9\mu(2t\mu + v\lambda^2)}.
\]

### 4 Network Neutrality vs Prioritisation

To study the impact of a net neutrality regulation, we now compare the two regulatory alternatives, net neutrality versus prioritisation, with respect to profits of the two firms, consumer surplus and social welfare. We take the capacity level \( \mu \) as constant.

#### 4.1 Firms’ Prices and Profits

We first analyse the effects of prioritisation on the price-setting behaviour of the firms.
Proposition 2. (i) Compared to net neutrality, under prioritisation the integrated firm A always sets higher prices in the content market, that is $p_A^P > p_A^N$, and lower subscription fees in the internet service market, that is $p_I^P < p_I^N$.

(ii) The effect on firm B’s prices depends on traffic intensity:

(a) If $0 < \lambda < \mu / 2$, then $p_B^P < p_B^N$.

(b) If $\mu / 2 < \lambda < \mu$, then $p_B^P > p_B^N$.

(iii) Under prioritisation, content prices of firm A are always higher than prices of firm B, that is $p_A^P > p_B^P$.

Proof. See Appendix A \(\square\)

Intuitively, this result can be explained as follows. First, the subscription fee is always lower under prioritisation than under net neutrality. By implementing priority, waiting time for consumers of content B increases, thereby reducing their willingness to pay for internet access. Thus to ensure full market coverage, the internet provider needs to reduce the price it charges to consumers for access.

Next, under prioritisation content provider A has a perceived quality of service advantage relative to content provider B due to lower waiting times. This allows content provider A to charge a higher price for its content than content provider B. Despite of charging a higher price than its competitor, it also has a larger market share (see Proposition [1]). Moreover, for the same reason its price and also its market share is higher than under net neutrality. Note that consumers of content A exhibit a negative network effect not only on consumers of content A but also on consumers of content B. Thus by maintaining a high price, firm A takes into account that a lower market share increases the utility of its consumers.

As mentioned before, content provider B charges a lower price compared to A to compensate its consumers for the lower quality of service. Depending on traffic intensity, however, the price charged by content provider B is higher or lower than its price under net neutrality.

Firm A has a quality of service advantage over firm B and therefore obtains a greater demand. This high demand increases, however, the waiting time for its consumers. Therefore, it needs to counterbalance the increases in waiting time for its consumers by increasing its price. As content providers’ pricing strategies are strategic complements, a price increase of one firm leads to a price increase of the competitor. Moreover, content prices under prioritisation are increasing in the content request rate $\lambda$. Thus, the higher the congestion intensity the higher both content prices. It is therefore optimal for content provider B to charge a higher price, the higher the price of the competitor even though its consumers have a lower
perceived quality of service. As a result, for $\mu/2 < \lambda < \mu$ its price is higher compared to net neutrality.

We next compare firm’s profits between prioritisation and net neutrality.

**Proposition 3.** (i) The integrated firm $A$ always has higher profit under prioritisation compared to net neutrality, that is $\Pi_A^P > \Pi_A^N$.

(ii) The effect on firm $B$’s profit depends on traffic intensity and product differentiation:

(a) If $0 < \lambda < \mu/2$, then $\Pi_B^P < \Pi_B^N$ for all $t > \frac{t}{2}$.

(b) If $\mu/2 < \lambda < 4\mu/5$, then there exists a critical value $t_1^*$, for which $t_1^* > \frac{t}{2}$ such that $\Pi_B^P > \Pi_B^N$ only if $t < t_1^*$.

(c) If $4\mu/5 < \lambda < \mu$, then $\Pi_B^P > \Pi_B^N$ for all $t > \frac{t}{2}$.

**Proof.** See Appendix A □

Prioritisation of traffic involves a trade-off for the vertical integrated firm $A$: By Propositions 1 and 2 firm $A$ clearly has smaller subscription fees from internet access but higher prices and more sales from content services. The analysis shows that firm $A$ can always compensate for losses in the access market by gains in the content market. If firm $A$ has the choice, it will always choose to prioritise its own content over the rival content.

The effect of prioritisation on firm $B$’s profit is not clear-cut and depends on the traffic intensity and the differentiation parameter. When congestion intensity is low, that is $\lambda < \mu/2$, it follows immediately from Propositions 1 and 2 that profit of firm $B$ is lower under prioritisation. When traffic intensity is high, such that $\mu/2 < \lambda < \mu$, there are two opposing effects on content provider $B$’s profit. On one hand the price it charges is higher but on the other hand its market share is lower than under net neutrality. The overall effect on profit therefore depends on the relative magnitude of these opposing effects. We distinguish two cases. First, when $\mu/2 < \lambda < 4\mu/5$, traffic intensity is moderately high. The higher $t$, the more differentiation there is between the content providers. Additionally, the difference in perceived quality between contents is larger the higher $t$ is. When product differentiation is sufficiently large, the perceived quality of service advantage of $A$ becomes large, so that for content provider $B$ even the increase in its price $p_B^P$ cannot offset the loss in demand so that overall profit of firm $B$ is lower under prioritisation. Secondly, when $4\mu/5 < \lambda < \mu$, traffic intensity is extremely high, so that congestion matters a lot and all consumers have long waiting times. Nonetheless, the increase in the price $p_B^P$ offsets the loss in market share; hence higher profit of $B$. 

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4.2 Consumer Surplus and Total Welfare

For a policy maker who has to decide whether to allow the integrated internet service provider to prioritise its own content or to implement a net neutrality rule, welfare effects are an important consideration. We now compare consumer surplus and social welfare under priority and under net neutrality.

First, we determine consumer surplus. Adding up the net utilities of all consumers buying content $A$ and content $B$, we obtain the consumer surplus of the regulatory regime $j \in \{N, P\}$:

$$CS^j = \int_0^{x_A} U_A(x, p_A^j, p_I^j, w_A^j)dx + \int_{x_A}^1 U_B(x, p_B^j, p_I^j, w_B^j)dx$$

$$= \frac{t}{2} - tx_A^j + t(x_A^j)^2$$

(19)

Under both regimes, consumer surplus only depends on the differentiation parameter and the location of the consumer indifferent between the two content providers. This is due to the fact that the internet access price $p_I^j$ captures all the surplus from the indifferent consumer. All other consumers get excess surplus that depends on their location. From (8) consumer surplus under net neutrality is

$$CS^N = \frac{t}{4}$$

and from (15) consumer surplus under prioritisation is

$$CS^P = \frac{t(18\mu(t\mu + v\lambda^2) + v^2\lambda^2(5\lambda^2 - 2\lambda\mu + 2\mu^2))}{18(2\mu t + v\lambda^2)^2}.$$ (21)

Under net neutrality content providers are symmetric, so that the location of the indifferent consumer is fixed in the middle of the Hotelling line. Under prioritisation, however, content providers are asymmetric. The strength of the asymmetry and also the location of the indifferent consumer depend on the congestion intensity.

**Proposition 4.** Consumers as a whole benefit from prioritisation, i.e. $CS^P > CS^N$.

**Proof.** See Appendix A

For those consumers opting for prioritised content, on one hand prioritisation positively affects consumers’ utility since it reduces their waiting time and thus increases their utility. On the other hand, prioritisation increases the price they have to pay for content. In the
Hotelling framework, individual consumer surplus increases linearly with the distance of his location from the marginal consumer who is indifferent between the two content providers. Under prioritisation, the marginal consumer is located to the right of the middle. Consumers are, therefore, redistributed towards content provider $A$ with the perceived quality advantage. As a result, the average perceived quality of service increases. This increase in average perceived quality of service as well as the reduction in the subscription fee more than compensates the increase in transportation costs and the increases in the price of content service.

To determine the overall effect of prioritisation by the integrated firm, we now look at total welfare of the regulatory regime $j \in \{N,P\}$, which is defined as the sum of profits of both firms and consumer surplus.

$$TS^j = \Pi_A^j + \Pi_B^j + CS^j$$

$$= \int_0^{x_A^j} (u + \frac{v}{w_A^j} - tx)dx + \int_{x_A^j}^1 (u + \frac{v}{w_B^j} - t(1-x))dx$$

From (10), (11) and (20) we obtain total welfare under net neutrality

$$TS^N = u + v(\mu - \lambda) - \frac{t}{4}$$

and from (17), (18) and (21), we derive total welfare under prioritisation

$$TS^P = u + \frac{1}{18\mu(2\mu t + v\lambda^2)^2}(-18t^3 \mu^3 + 72vt^2 \mu^3(\mu - \lambda)$$

$$+ v^2 \lambda^2 t \mu(70\mu^2 - 70\lambda \mu + 13\lambda^2) + 4v^3 \lambda^4(2\mu - \lambda)^2)$$

Proposition 5. The impact of prioritisation on total welfare depends on traffic intensity and product differentiation:

(a) If $0 < \lambda < \mu/2$, then there exists a critical value $t^*_2$, for which $t^*_2 > \underline{t}$ such that $TS^P > TS^N$ only if $t > t^*_2$

(b) If $\mu/2 < \lambda < \mu$, then $TS^P > TS^N$ for all $t > \underline{t}$.

Proof. See Appendix A

Since prices are simple transfers from consumers to the firms, total welfare only depends on the utility and perceived quality of service of the content as well as transportation costs. Since the fixed utility of content $u$ is the same under both regimes, the difference in total welfare only depends on the perceived quality of service.
welfare is determined by the total perceived quality of service and the total transportation cost.

First, more than one half of the consumers obtain a higher perceived quality of service and less than one half obtain lower perceived quality of service under prioritisation of content A than under net neutrality. As more consumers benefit from higher quality of service than consumers lose from lower quality of service, average and total perceived quality of service increases compared to net neutrality. Second, total transportation costs are minimised when the marginal consumers is located at the midpoint so that prioritisation with $\tilde{x} > 1/2$ is inefficient in terms of transportation cost minimisation. Hence, total transportation costs increase under prioritisation. In terms of total welfare, there is a trade-off between gains from quality of services and losses from transportation cost. If the gain in quality of service is large relative to the degree of product differentiation, prioritisation is preferred.

If $\mu/2 < \lambda < \mu$ congestion intensity is high, consumers of content A gain relatively more than consumers of content B loose in terms of quality of service. Hence, the average gain in quality of service is higher and always more than offsets the increase in transportation costs. If $\lambda < \mu/2$, total welfare is higher only for higher degrees of product differentiation. As the product differentiation parameter $t$ increases, the difference in quality of service increases more than the additional costs.

5 Investment Incentives

In a dynamic setting, the internet service provider can invest into the network capacity. Investment in capacity decreases congestion and increases the relative perceived quality differential of the content services. We denote by $C(\mu)$ the fixed cost of investing into a network with capacity level $\mu$ with $C'(\mu) \geq 0$ and $C''(\mu) \geq 0$. The optimal investment level is determined at the point where the marginal profit (or benefit), $d\Pi_A(\mu)/d\mu$, equals the marginal cost of investment, $dC(\mu)/d\mu$. The higher the marginal profit the larger is the incentive to invest for the internet service provider. Profit under net neutrality is

$$\Pi_A^N = p_I^N(\mu) + p_A^N x_A^N$$

(26)

Using (10), firm A’s marginal profit of capacity investment under net neutrality is given by

$$\frac{d\Pi_A^N}{d\mu} = \frac{dp_I^N}{d\mu} = v$$

(27)
Under net neutrality perceived quality for all consumers is the same for all levels of capacity so that consumers’ demand decisions in the content market and hence, equilibrium content prices and demand are independent of changes in capacity. Profits in the content market are, therefore, unaffected by investment. Yet investment in capacity speeds up the delivery of content for all consumers leading to an increase in their willingness to pay for internet access; hence the internet service provider can increase the subscription fee by \( v \) per unit of additional capacity.

Profit under prioritisation is

\[
\Pi_A^P = p_I^P(\mu) + p_A^P(\mu)x_A^P(\mu)
\]  

(28)

Using (17), the marginal profit of capacity investment under prioritisation is given by

\[
\frac{d\Pi_A^P}{d\mu} = \frac{dp_I^P}{d\mu} + \frac{d\Pi_A^P}{d\mu} = [v(1 - \lambda\frac{dx_A^P}{d\mu}) - t\frac{dx_A^P}{d\mu} - \frac{dp_A^P}{d\mu}] + \left[ \frac{dp_A^P}{d\mu}x_A^P(\mu) + p_A^P(\mu)\frac{dx_A^P}{d\mu} \right]
\]  

(29)

We note that \( \frac{dp_A^P}{d\mu} < 0 \) and \( \frac{dx_A^P}{d\mu} > 0 \) due to the trade-off between demand and waiting time under prioritisation. As capacity increases congestion becomes less important, therefore, the negative externality of more consumers of content \( A \) on waiting time is reduced.

Under prioritisation perceived quality differs across consumers, so that an investment in capacity has not only an effect on the subscription fee but also on competition in the content market. The parameter \( v \) measures the increase in the willingness to pay for access of a consumer buying prioritised content through the faster delivery of content, under the condition that demand is fixed. We next note that investment in capacity reduces congestion on the network. Less congestion increases the quality advantage of the content provider with priority, hence its demand increases which in turn increases congestion. Therefore, \( -v\lambda(dx_A^P/d\mu) \) indicates the decrease in the willingness to pay of a consumer in the priority class due to increased congestion induced by additional demand. As investment changes the location of the marginal consumer, this investment in capacity increases the transportation cost of the marginal consumer who consumes prioritised content. This decreases the marginal consumer’s willingness to pay for internet subscription, captured by \( -t(dx_A^P/d\mu) \). Moreover, an increase in capacity leads to a decrease in the price of \( A \) by the law of demand. This effect, denoted by \( -(dp_A^P/d\mu) \), increases consumers’ willingness to pay for access. The relative effect on investment in capacity on consumers’ subscription fee depends on parameter values.

The second square bracket represents the effect of capacity investment on firm \( A \)’s profit in
the content market. There are two opposing effects. Capacity investment increases demand for content $A$ but at the same time decreases the price of content $A$. Therefore, the effect on profit depends on parameter values.

When evaluating whether incentives to invest in capacity are higher under prioritisation or under net neutrality, we consider the difference between them:

$$\Delta = \frac{d\Pi_{PA}}{d\mu} - \frac{d\Pi_{NA}}{d\mu} = (p_{PA}^* - v\lambda - t)\frac{dx_{PA}^*}{d\mu} - (1 - x_{PA}^* \frac{dp_{PA}^*}{d\mu}).$$

(30)

**Proposition 6.** The impact of prioritisation on the investment incentives of the internet service provider depends on product differentiation: There exists a critical value $t_{3}^*$, with $t_{3}^* < t$ such that $d\Pi_{PA}/d\mu > d\Pi_{NA}/d\mu$ only if $t > t_{3}^*$.

**Proof.** See Appendix A.

Whether the internet service provider has higher incentives to invest in capacity under prioritisation or net neutrality depends on the relative magnitudes of the indirect effect of investment through changes in market shares and the indirect effect of investment through changes in the price of content. When product differentiation $t$ is sufficiently high, the indirect effects through changes in market shares become negligible as $dx_{PA}^*/d\mu$ is decreasing in $t$ and approaching zero. The contents become differentiated enough so that demand is not affected by an increase in capacity. Then investment incentives under prioritisation are higher than under net neutrality as the total indirect effect through changes in the content price is positive and large for high $t$.

6 Conclusion

This paper provides an economic analysis of a net neutrality regulation when the internet service provider is vertically integrated into content provision. We have investigated the effect of such a regulation on the price competition of content providers, on social welfare and on the internet service provider’s incentive to invest in its network. We have considered a situation in which consumers pay directly to the content providers for content.

Compared to net neutrality, we find that prioritisation generally has positive short-run efficiency effects. The integrated internet service provider has an incentive to favour vertically affiliated content over unaffiliated rival services. This is, however, not always detrimental.
to the rival content provider. Consumer surplus is higher, while the effect on social welfare depends on congestion intensity and the degree of product differentiation. In the long run, prioritisation does not always guarantee dynamic efficiency, as investment incentives might be lower when the degree of product differentiation is small. Consequently, enforcing net neutrality by law seems to be unnecessary.

Future research can relax the assumption of full market coverage. The internet service provider might find it profitable to increase the subscription fee such that some consumers are excluded from the market. Further, one can look at asymmetries in the content market. It might be interesting to see how the results change depending on whether the integrated firm is more or less efficient. Another important extension is to explore the effects of net neutrality with competition in the internet service market. Competition is said to mitigate the problems associated with a violation of net neutrality.

A Proofs of Propositions

Proof of Proposition 1 By (8) and (15), the difference in equilibrium demands for content A is given by

$$\Delta x_A = x_A^P - x_A^N = \frac{v\lambda(2\mu - \lambda)}{6(2\mu t + v\lambda^2)} > 0$$

(31)

which always holds under Assumption 1. This implies that $x_A^P > 1/2$.

Since we assume that the content market is covered, by (31) it must also hold that

$$\Delta x_B = x_B^P - x_B^N < 0.$$ (32)

This proves that $x_B^P < x_B^N = 1/2 = x_A^N < x_A^P$.

Proof of Proposition 2 By (9) and (16), the difference in internet subscription fees is given by

$$\Delta p_I = p_I^P - p_I^N = -\frac{v\lambda^2(3\mu t + 2v\mu^2 + 2v\lambda^2 - 2v\lambda\mu)}{6\mu(2\mu t + v\lambda^2)}$$

(33)

We now have to prove that $\Delta p_I < 0$. This is equivalent to

$$3\mu t + 2v\mu^2 + 2v\lambda^2 - 2v\lambda\mu = 3\mu t + 2v(\mu^2 + \lambda^2 - \lambda\mu) > 0.$$ (34)

Since $\mu^2 + \lambda^2 > \lambda\mu$, the inequality in (34) must hold, and therefore $\Delta p_I$ is always negative.
This proves that \( p_I^P < p_I^N \).

Next we consider the difference in equilibrium content prices to consumers of content A and content B as we move away from net neutrality to a prioritisation scheme. That is

\[
\Delta p_A = p_A^P - p_A^N = \frac{v\lambda(\mu + \lambda)}{3\mu}
\]

(35)

\[
\Delta p_B = p_B^P - p_B^N = -\frac{v\lambda(\mu - 2\lambda)}{3\mu}
\]

(36)

Equation (35) is always positive while equation (36) is negative for \( \lambda < \mu/2 \) and positive for \( \mu/2 < \lambda < \mu \). This proves that \( p_A^P > p_A^N \). Moreover it shows that \( p_B^P < p_B^N \) if \( \lambda < \mu/2 \) and \( p_B^P > p_B^N \) if \( \mu/2 < \lambda < \mu \). Finally, we show that (iii) holds. By (13) and (14), the difference in content prices under prioritisation is

\[
\Delta p_C = p_A^P - p_B^P = \frac{v\lambda(2\mu - \lambda)}{3\mu}
\]

(37)

By Assumption 1, (37) is strictly positive. This concludes the proof of Proposition 2.

**Proof of Proposition 3** We first prove statement (i). From Propositions 1 and 2, it follows directly that firm A’s profit in the content market is greater under prioritisation, that is \( \Pi_A^P > \Pi_A^N \) and firm A’s profit from internet access is lower under prioritisation, hence \( \Pi_AI^P < \Pi_AI^N \). The total impact on A’s profit, therefore, depends on the magnitude of these opposing effects.

By (10) and (17), the difference in total profits of firm A is

\[
\Delta \Pi_A = \Pi_A^P - \Pi_A^N = \frac{v\lambda(2\mu - \lambda)(3\mu t + 2v\lambda^2 - v\lambda\mu)}{9\mu(2\mu t + v\lambda^2)} > 0
\]

(38)

Indeed this inequality must hold because \( 2\mu - \lambda > 0 \) by Assumption 1 and \( 3\mu t + 2v\lambda^2 - v\lambda\mu > 0 \) by Assumption 2. This proves that the profit for the integrated firm A is higher under prioritisation; hence, \( \Pi_A^P > \Pi_A^N \).

We now prove statement (ii). First, consider statement (a) if \( \lambda \leq \mu/2 \). It follows directly from Propositions 1 and 2 that the difference in profits for firm B is negative because \( p_B^P < p_B^N \) and \( x_B^P < x_B^N \).

Considering next \( \mu/2 < \lambda < \mu \), it depends on the relative magnitudes of \( p_B^P \) and \( x_B^P \) whether profit is higher or lower under prioritisation. From (11) and (18), the difference in
equilibrium profits of firm $B$ is
\[
\Delta \Pi_B = \Pi_B^P - \Pi_B^N = \frac{v \lambda (-12 \mu^2 t + 8 v \lambda^3 - 8 v \lambda^2 \mu + \lambda \mu (15 t + 2 v \mu))}{18 \mu (2 \mu t + v \lambda^2)} \quad (39)
\]

For profit to be higher under prioritisation, the numerator in the RHS of equation (39) has to be greater than zero. This is the case if
\[
-12 \mu^2 t + 8 v \lambda^3 - 8 v \lambda^2 \mu + \lambda \mu (15 t + 2 v \mu) = 2 v \lambda (4 \lambda^2 - 4 \lambda \mu + \mu^2) + 3 \mu (5 \lambda - 4 \mu) > 0 \quad (40)
\]

The first term of the LHS of (40) is always positive. The second term of the LHS of (40) is positive if $(5 \lambda - 4 \mu) > 0$. This is true for $4 \mu / 5 < \lambda < \mu$. Under this condition, the profit of firm $B$ is higher under prioritisation. This proves statement (b).

It remains to consider statement (c) if $\mu / 2 < \lambda < 4 \mu / 5$. Thus, $5 \lambda - 4 \mu$ is now negative. Rearranging the LHS of (40), we obtain the critical value $t_1^*$ for which the LHS of (40) is negative:
\[
t < t_1^* \equiv -\frac{2 v \lambda (\mu - 2 \lambda)^2}{3 \mu (5 \lambda - 4 \mu)}. \quad (41)
\]

To be a feasible solution $t_1^*$ has to be greater than $t$; otherwise, profit under prioritisation is always smaller if $\mu / 2 < \lambda < 4 \mu / 5$. Therefore, we have to check that $t_1^* > t$, that is
\[
t_1^* = -\frac{2 v \lambda (\mu - 2 \lambda)^2}{3 \mu (5 \lambda - 4 \mu)} > t \quad (42)
\]

which is true under Assumption 1. Hence, there are some values of $t$ such that $t_1^* > t$ and profit is higher under prioritisation in the given parameter range. This then concludes the proof of Proposition 3.

**Proof of Proposition 4** From (20) and (21), we derive the difference in consumer surplus under the two alternatives.
\[
\Delta CS = CS^P - CS^N = \frac{v^2 \lambda^2 t (\lambda - 2 \mu)^2}{36 (2 \mu t + v \lambda^2)^2} > 0 \quad (43)
\]

The LHS of (43) is always positive. Therefore, consumer surplus is higher under prioritisation, that is $CS^P > CS^N$.

**Proof of Proposition 5** Given (24) and (25), we derive the difference in total welfare under
the two alternatives:

$$\Delta TS = TS^P - TS^N = \frac{v^2 \lambda^2(-4v\mu^3t + 8v^2\lambda^4 + v\lambda^2\mu(35t + 4v\lambda) + 4\mu^2(9t^2 + v\lambda t - v^2\lambda^2))}{36\mu(2\mu + v\lambda^2)^2}$$  \hfill (44)

For total welfare to be higher under prioritisation we look at the sign of the large bracket in numerator of the RHS of equation (44). Rearranging this term yields

$$\phi_1(t) \equiv 36\mu^2t^2 + v\mu t(-2\mu + 7\lambda)(2\mu + 5\lambda) + 4v^2\lambda^2(2\lambda - \mu)(\lambda + \mu)$$  \hfill (45)

For this to be positive for sure, both \((-2\mu + 7\lambda)\) and \((2\lambda - \mu)\) have to be positive. This is satisfied if \(\mu/2 \leq \lambda < \mu\). This proves statement (b).

Next we prove statement (a) if \(\lambda < \mu/2\). By collecting terms in \(t\), \(\phi_1(t)\) is a quadratic function of \(t\). First, we determine the sign of \(\phi_1(t)\), which is

$$\phi_1(t) = \frac{v^2\lambda(\lambda - 2\mu)^2(2\lambda - \mu)}{3} < 0$$  \hfill (46)

for \(\lambda < \mu/2\). Total welfare is lower under prioritisation if \(t = t^*\). Therefore, we now look at the derivatives \(\phi'_1(t)\) and \(\phi''_1(t)\) to determine the shape of the function \(\phi_1(t)\).

$$\phi'_1(t) = 72\mu^2t + v\lambda(7\lambda - 2\mu)(5\lambda + 2\mu)$$  \hfill (47)

$$\phi''_1(t) = 72\mu^2 > 0$$  \hfill (48)

Since \(\phi_1(t)\) is a quadratic and convex function and \(\phi_1(t) < 0\), there exists a unique \(t^*_2\) such that \(\phi_1(t^*_2) = 0\). If \(t < t^*_2\), then \(\phi_1(t) < 0\) and if \(t > t^*_2\), then \(\phi_1(t) > 0\). The sign of expression (45) and hence the effect of prioritisation on total welfare is positive under the condition \(\lambda < \mu/2\) provided that \(t\) is large enough, that is \(t > t^*_2\). This then concludes the proof of Proposition 5. 

Proof of Proposition 6 From (24) and (25) we obtain the difference in the marginal benefits

$$\Delta = \frac{d\Pi^P_A}{d\mu} - \frac{d\Pi^N_A}{d\mu} = (p^P_A - v\lambda - t) \frac{dx^P_A}{d\mu} - (1 - x^P_A) \frac{dp^P_A}{d\mu}.$$

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Next we derive the signs of the derivatives \( \frac{dx_P}{d\mu} \) and \( \frac{dp_P}{d\mu} \):

\[
\frac{dx_P}{d\mu} = \frac{v\lambda^2(t + v\lambda)}{3(2\mu + v\lambda^2)^2} > 0
\]

\[
\frac{dp_P}{d\mu} = -\frac{v\lambda^2}{3\mu^2} < 0
\]

It is easy to see that \( \frac{dx_P}{d\mu} > 0 \) and \( \frac{dp_P}{d\mu} < 0 \). Using (13) as well as the LHS of (50) and (51) we can write the difference in (49) as

\[
\Delta = \frac{2v\lambda^2(3\mu^2t^2 + 2v\lambda\mu(2\lambda - \mu) + v^2\lambda^2(\lambda^2 - \mu^2))}{9\mu^2(2\mu + v\lambda^2)^2}
\]

For investment incentives to be higher under prioritisation we look at the sign of the large bracket in numerator of the RHS of (44). Let us define

\[
\phi_2(t) = 3\mu^2t^2 + 2v\lambda\mu(2\lambda - \mu) + v^2\lambda^2(\lambda^2 - \mu^2)
\]

which is a quadratic function in \( t \). We first determine the sign of \( \phi_2(t) \), which is

\[
\phi_2(t) = -\frac{v^2\lambda^2(\lambda - 2\mu)^2}{3} < 0
\]

by Assumption 1. Investment incentives are lower under prioritisation if \( t = t_3 \). We now look at the derivatives \( \phi_2'(t) \) and \( \phi_2''(t) \) to determine the shape of the function \( \phi_2(t) \).

\[
\phi_2'(t) = 6\mu^2t + 2v\lambda\mu(2\lambda - \mu)
\]

\[
\phi_2''(t) = 6\mu^2 > 0
\]

Since \( \phi_1(t) \) is a quadratic and convex function and \( \phi_2(t) < 0 \), there exists a unique \( t_3^* > t \) such that \( \phi_2(t_3^*) = 0 \). If \( t < t_3^* \), then \( \phi_2(t) < 0 \) and if \( t > t_3^* \), then \( \phi_2(t) > 0 \). The sign of expression (53) and hence the effect of prioritisation on the investment incentives is positive provided that \( t \) is large enough, that is \( t > t_3^* \). This then concludes the proof of Proposition 6.
B Queuing Theory: Waiting Time in a System

Queueing theory is the mathematical way of studying waiting times in a system. The M/M/1 queuing model has a single server and both the arrival rate ($\lambda$) and the service rate ($\mu$) are exponentially distributed. Arrival and service rates are independent and identically distributed. More specifically, consumers arrive according to a Poisson process at an average rate of $\lambda$ per time period. On average one consumer appears every $1/\lambda$ time periods. Moreover, there is a single server with an exponential service rate of $\mu$ consumers per time period. To ensure that the queue will not grow infinitely, it must be $\lambda < \mu$.

Related to the transmission of data packets in the internet, $\lambda$ refers to the rate of packets that arrive per time period and measures the expected capacity demand. $\mu$ packets is the available service capacity, i.e. bandwidth, that the server can serve per time period.

Traffic intensity $\rho$ is defined by the ratio of arrival rate to service rate, hence

$$\rho = \frac{\text{capacity demand}}{\text{available capacity}} = \frac{\lambda}{\mu}.$$  

It measures congestion in the system. As the traffic intensity increases the amount of congestion increases and thereby consumers have longer waiting times in the system.

Consumers move from the queue into service on a first-come-first-served principle. The consumer that has been waiting the longest is served first. The expected number of consumers in the entire system is

$$L = \frac{\rho}{1 - \rho} = \frac{\lambda}{\mu - \lambda}.$$  

By Little’s law [1961] the average number of consumers in the system $L$ is the effective arrival rate $\lambda$ times the average time that a consumer spends in the system $W$; put simply $L = \lambda W$. As a result, waiting time spent in the entire system is

$$W = \frac{1}{\mu - \lambda}.$$  

Under a priority scheme, consumers with priority are served first. We consider preemptive queues where a job in service without priority can be interrupted by one with priority. Hence, the priority class has absolute priority over the non-priority class. Therefore, for consumers with priority the consumers without priority do not exist. Hence we immediately have the
expected waiting time of the priority class

\[ W_P = \frac{1}{\mu - \lambda_P} \]

given an arrival rate of \( \lambda_P \) for the priority consumers. Once there are no more consumers with priority in the system, the server proceeds with serving the non-priority consumers. Expected waiting time without priority is given by

\[ W_{NP} = \frac{\mu}{\mu - \lambda} W_P = \frac{\mu}{(\mu - \lambda)(\mu - \lambda_P)} \]

where \( \lambda \) is the sum of the arrival rates from priority and non-priority consumers.
References


