Cooperative and noncooperative R&D in an asymmetric multi-product duopoly with spillovers

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Abstract

We consider R&D investment with spillovers in a market where a multi-product firm competes with a single-product firm. We analyze whether investment incentives are higher under R&D cooperation or competition and show that this depends not only on the technology spillover but also on the degree of product differentiation. R&D investments under cooperation are lower when the products are close substitutes even if the spillover is substantial.

Keywords: R&D investments, Multi-product firms, R&D cooperation, Regulation

JEL Classification Numbers: L13, O31, O32,
1 Introduction

A firm investing in research and development (R&D) with spillovers usually imposes a positive externality on other firms which can then appropriate the results of this investment. D’Aspremont and Jacquemin (1988) show in a symmetric environment that encouraging firms to collaborate in R&D activities increases R&D investment and hence, social welfare by internalizing the externality. The European Commission has recognized these benefits of joint R&D and has thus issued revised “block exemption” regulations in 2010 that provide an automatic exemption from competition law for certain types of joint R&D agreements.

We study R&D investment in a market where a multi-product firm produces an established and an innovative product and a single-product firm only produces an innovative product. Thereby, we extend the model of D’Aspremont and Jacquemin (1988) by incorporating two additional aspects. First, we consider an asymmetric market environment where a multi-product firm competes with a single-product firm. Second, the innovative and the established goods are substitutes so that R&D investment in the innovative product might come at the expense of the sales of the established product. It is often assumed that innovative products are independent of any other products that the firms are producing. Such an assumption seems, however, rather restrictive. Hence, R&D investments have to be considered for their effects not only on the output decision of the innovative product but also on the established product.

The two extensions enable us to study asymmetric competition between multi-product and single-product firms as commonly observed in situations where “dirty” products compete with “clean”, environmentally friendly products. An example of such a market is the automobile industry. Traditional car manufactures compete with firms that specialize in the production of electric cars. For example, Tesla Motors produces exclusively electric cars and competes with more traditional firms that produce both electric and gasoline cars. The most challenging issue related to the future development of electric cars is the battery charging.
Companies invest in R&D to improve the loading time and reduce the size and cost of these batteries. In order to benefit from each other’s know-how, firms often cooperate in R&D investments. One example for such a strategic relationship is the cooperation between Daimler and Tesla Motors, which started in 2009.

Investments in R&D are strategic as they influence product market outcomes. Hence, when firms compete in R&D, in addition to the direct effect by which firms benefit from cost reductions, there are two potential strategic effects. Through a within-product competition effect a firm’s investment decision indirectly affects its own profit by its influence on its competitor’s output decision of the innovative good. Depending on the level of the spillover, this effect can be negative or positive. Particular to our asymmetric set-up is the second strategic effect, the cross-product competition effect. It states that, in addition to changes in the output of the same product, the multi-product firm also modifies its output of the established good, which in turn benefits the single-product firm as it is able to steal some business.

In contrast to R&D competition, we obtain three additional effects under cooperation. When choosing an investment level to maximize joint profit, firms internalize the effect of their R&D investment on the competitor’s profit. Because of the spillover effect, an increase in R&D investment benefits rival’s profit by decreasing also its marginal cost; hence, R&D investment is stimulated. Moreover, through the within-product cooperation effect, by investing more, a firm gains a competitive advantage over its rival in the same product, which hurts the competitor. The third effect, cross-product coordination effect, is specific to our multi-product environment. When the multi-product firm increases its R&D expenditure, it reduces its output of the substitute good to mitigate within firm cannibalization. This output reduction has a positive impact on the single-product firm’s profit and hence, increases investment incentives of the multi-product firm.

When the sum of these additional effects of cooperation is positive for a firm, its investment incentives under cooperation are higher than under competition because its investment
then benefits the other firm. We find that the additional, positive, cross-product coordination effect of the multi-product firm together with the spillover effect counteracts the negative within-product coordination effect. Hence, the profit externality conferred on the profit of the single-product firm is positive for a greater range of values of the spillover level and the degree of product differentiation in comparison to the single-product firm.

Our central result states that when the established and the innovative products are close substitutes, total R&D investment under cooperation will be lower than under competition even if the spillover is substantial. More specifically, R&D investment of the single-product firm may be higher under competition than under cooperation even if the spillover is large. Moreover, for medium spillovers and high product substitutability the multi-product firm also invests less under R&D cooperation. Thus, in contrast to standard results in D’Aspremont and Jacquemin (1988) which suggest that R&D investment under cooperation is higher than under competition when the spillover is high, we find that it not only depends on the technology spillover but also on the degree of product substitutability.

Related literature

As mentioned before, the starting point of our analysis is the study by D’Aspremont and Jacquemin (1988), which also serves as our benchmark. They analyze firms’ incentives to invest in R&D with spillovers under R&D competition and cooperation in a symmetric, homogeneous product duopoly. They show that cooperation increases R&D investment levels compared to competitive R&D only when the spillovers are sufficiently high. Kamien et al. (1992) extend their model by introducing heterogeneity among the firms. They show that the general results of D’Aspremont and Jacquemin (1988) still hold. The key intuition in that strand of the literature is that private incentives to conduct R&D are reduced when there are knowledge spillovers from one firm to another due to free-rider incentives.

Lin and Zhou (2013) analyze R&D investment incentives in a multi-product environment. They consider R&D investment in a two-product duopoly with differentiated goods, where
each firm has an initial cost advantage in one of the products. They find that when a firm invests more in one particular good, its competitor will respond by investing more in the other good. When the goods become more substitutable this effect will be stronger. Moreover, R&D coordination in R&D lower investment. In contrast to Lin and Zhou (2013), we analyze an asymmetric setting without cost advantages, but instead we allow for spillovers.

Also Kawasaki et al. (2014) consider a multi-product model, in which firms engage in R&D investment. A multi-product firm has a monopoly in one market and competes with potential entrants in a second market. Contrary to our set-up, demands for the two products are independent and R&D efforts by the multi-market firm simultaneously reduce the marginal cost of both goods. They show that entry can stimulate investment in cost-reducing R&D.

None of those studies, however, considers the interaction between an asymmetric multi-product environment and the dynamics of R&D cooperation. Bulow et al. (1985) investigate strategic interaction in an asymmetric multi-market oligopoly. They find that a shock to a firm in one market also affects its competitor’s strategy in a second market. This can be translated into our set-up as we consider R&D investment as a strategic interaction. If a firm invests in the new technology product, the competitiveness of the substitute good is reduced. Therefore, the incumbent reduces its output of the established good.

The remainder of the paper is organized as follows. Section 2 presents the theoretical model. In section 3 we analyze the retail market equilibrium. Sections 4 and 5 identify the investment incentives under competition and cooperation, respectively. In Section 6 we compare R&D investment under competition and coordination. Section 7 concludes. The proofs of all formal results are relegated to the Appendix.
2 Model

We consider a market with two firms A and B. Firm A is the single-producer of an established good (good 1), while both firms produce a substitute good (good 2), which is based on a new technology. The prices of the two products are given by the following linear inverse demand functions:

\[ p_1(q_{A1}, q_{A2}, q_{B2}) = a - q_{A1} - g(q_{A2} + q_{B2}) \]  
\[ p_2(q_{A1}, q_{A2}, q_{B2}) = a - (q_{A2} + q_{B2}) - gq_{A1} \]  

where \( a > 0 \), quantity \( q_{ji} \) is the output of good \( i \in \{1, 2\} \) produced by firm \( j \in \{A, B\} \) and \( g \in [0, 1) \) represents the degree of product substitutability between goods 1 and 2. Therefore, the two products in the market are imperfect substitutes, while good 2 is homogeneous. This asymmetric market structure exists, for example, in the automobile industry, where traditional car manufactures, producing gasoline and electric cars, compete with electric car manufactures.

Focusing on R&D for the new technology good, we assume that the unit cost of producing the established good is fixed and equalize it to zero. Hence, only R&D investment in the new technology good is possible. The unit cost of producing the new technology good is \( c > 0 \) but each firm can invest some \( x_j > 0 \) in process R&D to reduce its unit cost:

\[ c_{j2} = c - x_j - \beta x_{-j}, \]  

where the amount \( x_j \) is the R&D investment of firm \( j \), the amount \( x_{-j} \) is the R&D investment of the rival, and \( \beta \in [0, 1] \) is the spillover of the rival’s R&D investment on firm \( j \). Hence, firms benefit from their rival’s R&D activity. We assume that the R&D cost is quadratic

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1 Derived from the utility function of a representative consumer (Dixit, 1979): \( U(q_{A1}, q_{A2}, q_{B2}) = a(q_{A1} + q_{A2} + q_{B2}) - 1/2(q_{A1}^2 + 2gq_{A1}(q_{A2} + q_{B2}) + (q_{A2} + q_{B2})^2) \)  
2 We assume that \( c \) is high enough such that the new technology is more costly than the established one.
and given by $\gamma x_j^2$, where $\gamma > 0$. Thus, the profit of the multi-product firm A is

$$\Pi_A = p_1(q_{A1}, q_{A2}, q_{B2})q_{A1} + [p_2(q_{A1}, q_{A2}, q_{B2}) - (c - x_A - \beta x_B)]q_{A2} - \gamma x_A^2$$

$$= \pi_A(q_{A1}, q_{A2}, q_{B2}, x_A, x_B) - \gamma x_A^2$$  

(4)

and the profit of the single-product firm B is

$$\Pi_B = [p_2(q_{A1}, q_{A2}, q_{B2}) - (c - x_B - \beta x_A)]q_{B2} - \gamma x_B^2$$

$$= \pi_B(q_{A1}, q_{A2}, q_{B2}, x_A, x_B) - \gamma x_B^2$$  

(5)

where $\pi_j(q_{A1}, q_{A2}, q_{B2}, x_A, x_B), j = A, B$, denotes the profit gross of R&D investment cost.

We consider the following two-stage game. In the first stage firms simultaneously choose their level of R&D investment $(x_A, x_B)$ to reduce marginal costs. We then examine R&D competition and cooperation. Based on their R&D choice, the firms compete in the second stage à la Cournot and set their production quantities simultaneously. We solve for the equilibria by backward induction.

### 3 Retail market outcomes

In the second stage, firms compete simultaneously in the product market given the R&D investment levels for the new technology good, $x_A$ and $x_B$. Firm A maximizes its profit $\pi_A$ by choosing quantities $q_{A1}$ and $q_{A2}$, while firm B maximizes its profit $\pi_B$ by only choosing quantity $q_{B2}$. From the first-order conditions $\partial \pi_A / \partial q_{A1} = \partial \pi_A / \partial q_{A2} = \partial \pi_B / \partial q_{B2} = 0$ we
obtain the equilibrium quantities as functions of the R&D investments:\(^3\):

\[
q_{A1}^*(x_A, x_B) = \frac{a(1 - g) + g(c - x_A - \beta x_B)}{2(1 - g^2)}
\]

(6)

\[
q_{A2}^*(x_A, x_B) = \frac{a(2 - 3g + g^2) - c(2 + g^2) + (4 + (2\beta - 1)g^2 - 2\beta)x_A - (2 - (2 - \beta)g^2 - 4\beta)x_B}{6(1 - g^2)}
\]

(7)

\[
q_{B2}^*(x_A, x_B) = \frac{a - c + (2\beta - 1)x_A + (2 - \beta)x_B}{3}
\]

(8)

To ensure positive output levels in the absence of R&D investments, we assume that \(a > c(2 + g^2)/(2 - g(3 - g))\). When there is no investment in R&D (\(x_A = x_B = 0\)), the multi-product firm produces more of its established good than of its new technology good because the established good has smaller marginal costs; hence, obtaining a competitive advantage. Moreover, firm B produces more of good 2 than firm A.

In the first stage, firms choose their R&D investment. We first examine how firms’ R&D investments affect the market outcomes in the second stage by differentiating expressions (6)-(8) with respect to \(x_A\) and \(x_B\). By increasing its investment in the innovative product a firm reduces its marginal cost of that product. Thereby, it always reacts with an increase in its own quantity of the innovative product,

\[
\frac{\partial q_{i2}^*}{\partial x_i} > 0.
\]

(9)

Due to the technology spillovers an increase in a firm’s R&D investment not only reduces its own marginal cost of the new technology good but also its competitor’s marginal cost of the same product. When \(\beta\) is large, the spillover effect becomes strong so that the competitor

\(^3\)The second-order conditions for a maximum are satisfied: \(D(q_{A1}, q_{A2}) = (\partial^2 \pi_A/\partial q_{A1}^2)(\partial^2 \pi_A/\partial q_{A2}^2) - (\partial^2 \pi_A/\partial q_{A2} \partial q_{A1})^2 = 4 - 4g > 0\), \(\partial^2 \pi_A/\partial q_{A1}^2 = -2 < 0\) and \(\partial^2 \pi_B/\partial q_{B2}^2 = -2 < 0\).
also reacts with an increase in its quantity of the innovative product,

\[
\frac{\partial q^*_A}{\partial x_B} = -\frac{1}{3} + \frac{\beta(4 - g^2)}{6(1 - g^2)} = \begin{cases} 
> 0, & \text{if } \beta > \hat{\beta}_B \equiv \frac{2(1-g^2)}{4-g^2} \\
< 0, & \text{if } \beta < \hat{\beta}_B \equiv \frac{2(1-g^2)}{4-g^2}
\end{cases}
\] (10)

and

\[
\frac{\partial q^*_B}{\partial x_A} = \frac{1}{3}(2\beta - 1) = \begin{cases} 
> 0, & \text{if } \beta > \hat{\beta}_A \equiv \frac{1}{2} \\
< 0, & \text{if } \beta < \hat{\beta}_A \equiv \frac{1}{2}
\end{cases}
\] (11)

As the new technology products compete directly with the established product, there are also effects of R&D investment in the output of good 1. When R&D activity in the innovative product by either firm increases, firm A responds with a reduction in its output quantity of the traditional good:

\[
\frac{\partial q^*_A}{\partial x_i} < 0, \text{ for } g > 0.
\] (12)

Moreover, by taking the derivative of (12) with respect to \( g \), it can be seen that the closer substitutes the goods are the more firm A suffers in the sales of its traditional product from investment in the new technology.

4 R&D competition

In the first stage, firms invest in R&D taking into account the optimal output strategy in stage two. Both firms decide on their R&D investments to maximize their respective profit. Firm \( i \) chooses \( x_i \) to maximize its total profit

\[
\Pi_i = \pi_i(x_i, x_j, q_{i1}^*(x_i, x_j), q_{i2}^*(x_i, x_j), q_{j2}^*(x_i, x_j)) - \gamma x_i^2
\] (13)
where \( i \in \{A, B\} \) and \( j \neq i \). The first-order condition for maximizing firm \( i \)'s profit in expression (13) is

\[
\frac{d\Pi_i}{dx_i} = \frac{d\pi_i}{dx_i} - 2\gamma x_i = 0.
\] (14)

In what follows we assume that \( \gamma \) is large enough so that all second-order conditions are satisfied in order to have interior solutions.

**Assumption 1.**

\[
\gamma > \gamma_{\text{min}} \equiv \frac{11g^2 - 20 + \beta(1 - g^2)(32 - 20\beta)(g^2 - 1)}{36(1 - g^2)}
\]

The relevant first-order conditions then lead to the R&D best-response functions \( R^N_A(x_B) \) and \( R^N_B(x_A) \) for firm A and B, respectively\(^4\). The reaction functions are downward sloping for low values of \( \beta \) and upward sloping for higher values of \( \beta \), as illustrated in Figure 1.

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\(^4\)The closed forms are provided in the appendix by formulas (26) and (27).
Lemma 1. The slope of

(i) \( R_N^A(x_B) \) is negative if \( 0 < \beta < 1/2 \) and \( 0 < g < \bar{g}_N(\beta) \) and positive elsewhere, where 
\[
\bar{g}_N(\beta) \equiv \sqrt{4(\beta - 2)(2\beta - 1)/(8 + \beta(8\beta - 11))}
\]

(ii) \( R_N^B(x_A) \) is negative if \( 0 < \beta < 1/2 \) and \( 0 < g < 1 \) and positive elsewhere.

Proof. See Appendix \( \square \)

Lemma 1 shows that R&D investments are strategic substitutes when spillovers are low. Intuitively, an increase in R&D investment by one firm leads to a decrease in the output of the competitor. As spillovers intensify and products are more substitutable, R&D investments are strategic complements. An increase in R&D by one firm leads to a decrease in the competitor’s marginal cost due to the technology spillover. This reduction in marginal cost has a positive impact on competitor’s output decision and thereby increases its incentives to invest in R&D. In order to stay competitive with firm B, firm A may also increase its R&D investment when spillovers are low. This is the case when the products are closer substitutes.

We next derive the marginal benefit of investment for each firm and identify different strategic effects that arise under R&D competition.

By applying the Envelope Theorem to \( d\pi_A/dx_A \) in (14) we obtain

\[
\frac{d\pi_A}{dx_A} = \frac{\partial \pi_A}{\partial x_A} + \underbrace{\frac{\partial \pi_A}{\partial q_{A1}^*} \frac{\partial q_{A1}^*}{\partial x_A}}_{\text{direct effect}} + \underbrace{\frac{\partial \pi_A}{\partial q_{A2}^*} \frac{\partial q_{A2}^*}{\partial x_A}}_{\text{(envelope theorem) \ (=0)}} + \underbrace{\frac{\partial \pi_A}{\partial q_{B2}^*} \frac{\partial q_{B2}^*}{\partial x_A}}_{\text{(envelope theorem) \ (=0)}} - \frac{\partial q_{A1}^*}{\partial x_A}
\]

Recall that \( \partial \pi_A/\partial q_{Ai}^* = 0 \) when evaluated in the optimum \( q_{Ai}^* \), \( i = 1, 2 \), as a result of profit maximization in the second stage. By applying the Envelope Theorem to \( d\pi_B/dx_B \) in (14)
we obtain
\[
\frac{d\pi_B}{dx_B} = \frac{\partial \pi_B}{\partial x_B} + \left(\frac{\partial \pi_B}{\partial q^*_B} \frac{\partial q^*_B}{\partial x_B}\right)_{=0} + \frac{\partial \pi_B}{\partial q^*_A} \frac{\partial q^*_A}{\partial x_B} + \frac{\partial \pi_B}{\partial q^*_A} \frac{\partial q^*_A}{\partial x_B}
\]
(16)

Recall that \(\partial \pi_B/\partial q^*_B = 0\) when evaluated in the optimum \(q^*_B\) as a result of profit maximization in the second stage.

Each firm invests in R&D to reduce the costs of its innovative product. Those investments affect firms’ profits in different ways. First of all, there is a direct effect of R&D investment, but additionally we identify two types of strategic effects:

(i) within-product competition effect. - A firm’s investment decision indirectly affects its own profit through its influence on its competitor’s output decision of the same product.

(ii) cross-product competition effect. - A firm’s investment decision indirectly affects its own profit through its influence on its competitor’s output decision of the substitute product.

We summarize the effects of R&D investments under R&D competition on the firms’ gross profits (i.e. excluding R&D costs) in the following proposition.

**Proposition 1.** An increase in firm j’s R&D investment \(x_j\) affects its profits

(i) positively through the direct effect

(ii) positively for \(0 \leq \beta < \hat{\beta}_j\) and negatively otherwise through the within-product competition effect. ⁵

Additionally,

(iii) an increase in firm B’s R&D investment also influences its own profit positively through the cross-product competition effect.

⁵The critical spillover \(\hat{\beta}_j\), \(j \in \{A, B\}\) is defined by equations (10) and (11)
Proof. See Appendix

Intuitively, the direct effect is always positive. This results from the fact that an increase in the level of R&D investment leads to a reduction in firm’s marginal cost of the new technology good, which in turn leads to an increase in its profit.

Due to knowledge spillovers an increase in a firm’s R&D investment also reduces its rival’s marginal cost. However, only when spillovers are large, the competitor’s marginal cost is reduced substantially so that it also reacts more aggressively and increases its output level. Then profit of the investing firm is reduced. Hence, the within-product competition effect increases R&D incentives of a firm when spillovers are low and decreases them for high spillovers. Only small spillovers create a real competitive advantage for an investing firm because high spillovers create greater potential for free-riding.

The cross-product competition effect is specific to the single-product firm. It only prevails in our asymmetric environment. As the multi-product firm produces two substitute goods, it influences the single-product firm’s optimal R&D decision not only through its response regarding its output decision of the new technology product but also regarding its output decision of the traditional product. Clearly, the multi-product firm lowers its output of the traditional product because that good loses its competitive advantage. The single-product firm then benefits from that output reduction as its competitiveness towards the established good increases. The cross-product competition effect, therefore, always raises investment incentives for firm B.

The marginal benefit of firm A’s cost-reducing investment depends only on the relative magnitudes of the direct effect and the within-product competition effect. Formally,

\[
\frac{d\pi_A}{dx_A} = \frac{1}{3}g(1 - 2\beta)q_{A1} + \frac{2}{3}(2 - \beta)q_{A2}
\]

The overall marginal benefit of firm B’s cost-reducing investment under R&D competition
additionally depends on the cross-product competition effect. Hence, we obtain

\[ \frac{d\pi_B}{dx_B} = \frac{2}{3}(2 - \beta)q_{B2}. \]  

(18)

As the latter is positive, firm B always has an incentive to invest in R&D. On the contrary, firm A’s investment incentives can be negative if \( \beta > 1/2 \) and \( g \) is very large. Under such a parameter constellation, firm A would not invest at all so that then \( x_A = 0 \).

5 R&D cooperation

We next consider cooperation in R&D investment while the second stage remains competitive. In the first stage firms choose investment levels \( x_A \) and \( x_B \) by maximizing their joint profits given by (13):

\[
\max_{x_A, x_B} \Pi_A + \Pi_B = \pi_A(x_A, x_B, q_{1A}^*(x_A, x_B), q_{2A}^*(x_A, x_B), q_{1B}^*(x_A, x_B), q_{2B}^*(x_A, x_B)) \\
+ \pi_B(x_A, x_B, q_{1A}^*(x_A, x_B), q_{2A}^*(x_A, x_B), q_{1B}^*(x_A, x_B), q_{2B}^*(x_A, x_B)) \\
- \gamma x_A^2 - \gamma x_B^2 
\]  

(19)

The first-order condition for investment under joint profit maximization for firm \( i \) in expression (19) is

\[ \frac{d(\Pi_A + \Pi_B)}{dx_i} = \frac{d(\pi_A + \pi_B)}{dx_i} - 2\gamma x_i = 0 \]  

(20)

where \( d(\pi_A + \pi_B)/dx_A \) and \( d(\pi_A + \pi_B)/dx_B \) are net marginal increases in the firms’ joint
profit. This then leads to

\[ R_i^C(x_j) = \arg\max_{x_i} [\Pi_A + \Pi_B]. \] (21)

For convenience and abusing somewhat usual conventions we call \( R_i^C(x_j) \), in what follows, reaction functions under cooperation\(^6\).

**Lemma 2.** The slope of \( R_i^C(x_j) \) under R&D cooperation is negative if \( 0 \leq \beta < 1/2 \) and \( 0 \leq g \leq \bar{g}_C(\beta) \) and positive otherwise, where \( \bar{g}_C(\beta) \equiv \sqrt{8(\beta - 2)(2\beta - 1)/(16 + \beta(16\beta - 31))} \).

**Proof.** See Appendix \( \Box \)

Specifically, for the R&D investment level of firm A, by applying the Envelope Theorem, we obtain the following marginal benefit of joint profit maximization:

\[
\frac{d(\pi_A + \pi_B)}{d x_A} = \frac{d\pi_A}{d x_A} + \frac{d\pi_B}{d x_A} = \frac{d\pi_A}{d x_A} + \frac{\partial\pi_B}{\partial x_A} + \frac{\partial q^*_A}{\partial x_A} \left( \frac{\partial q^*_A}{\partial x_A} - \frac{\partial q^*_A}{\partial q^*_A} \right) + \frac{\partial\pi_B}{\partial q^*_A} \left( \frac{\partial q^*_A}{\partial x_A} - \frac{\partial q^*_A}{\partial q^*_A} \right) \] (22)

Similarly, we obtain the following result for firm B’s investment:

\[
\frac{d(\pi_A + \pi_B)}{d x_B} = \frac{d\pi_B}{d x_B} + \frac{d\pi_A}{d x_B} = \frac{d\pi_B}{d x_B} + \frac{\partial\pi_A}{\partial x_B} + \frac{\partial q^*_B}{\partial x_B} \left( \frac{\partial q^*_B}{\partial x_B} - \frac{\partial q^*_B}{\partial q^*_B} \right) \] (23)

Cooperation may increase the incentive to conduct R&D by internalizing spillovers across the firms. However, R&D investment makes firms tougher competitors, hence, the effect of

\(^6\)The closed forms are given in the Appendix by formulas (43) and (44).
cooperation may be to reduce the incentive to conduct R&D.

The (strategic) effects of the first term, \( d\pi_i/dx_i, i = A, B \), are derived and analyzed under R&D competition in Section 4 above. Under cooperation each firm, in addition, cares about how its choice of R&D investment affects the profit of its competitor. Hence, we identify a \textit{spillover effect} and two further strategic effects under R&D cooperation:

(i) \textit{within-product coordination effect}. - A firm’s investment decision is influenced by the effect on its competitor’s profit through changes in its own output decision of the same product.

(ii) \textit{cross-product coordination effect}. - A firm’s investment decision is influenced by the effect on its competitor’s profit through its own output decision of the substitute product.

We summarize the additional effects of R&D investment due to cooperation on the firms’ joint gross profit in the following proposition.

\textbf{Proposition 2.} \textit{When} \( x_i \text{ increases, firm } j \text{’s profit is influenced}

(i) \textit{positively through the spillover effect} \\

(ii) \textit{negatively through the within-product coordination effect}. \\

Additionally,

(iii) \textit{when firm A increases its R&D investment, it also influences firm B’s profit positively through the cross-product cooperation effect}.

\textit{Proof.} See Appendix

When R&D by one firm spills over to the other firm, private incentives to conduct R&D are reduced due to potential free-riding. Yet, if firms choose R&D investment levels cooperatively, then these spillover externalities are internalized and R&D investment is stimulated.
An increase in one firm’s R&D investment reduces the other firm’s cost due to the technology spillover; hence, lower costs increase rivals profit.

The *within-product coordination effect* decreases a firm’s investment incentives as it also cares about its rival’s profit under cooperation. When a firm invests more in the new technology, it increases its output of the innovative good, as seen in expression (9). As a result the competitor faces a decline in its market share and suffers from a loss in its profit.

The *cross-product coordination effect* only exists for the multi-product firm in this asymmetric set-up as it internalizes its positive impact on the single-product firm’s profit when it reduces its output of the traditional good to mitigate within-firm cannibalization. The *cross-product coordination effect* is always positive because the new technology good becomes more competitive towards the established good by approaching the cost level of the established good. This effect is strengthened if the products are closer substitutes because then the multi-product firm will lose significant competitive advantage in the established good.

Within-product competition and cooperation effects indicate how R&D investment influence profits through the new technology good, whereas the cross-product competition and coordination effects indicated how R&D investment influence profits through the traditional good.

The overall marginal benefit of a firm’s cost-reducing investment under R&D cooperation depends on the relative magnitudes of the direct and spillover effects and three strategic effects, where some of the strategic effects differ for the multi-product and the single-product firm.

6 R&D competition vs. cooperation

We next analyze the equilibrium R&D investment levels of each firm under competition and cooperation. In order to do so, we compare the reaction functions $R_i^C(x_j)$ and $R_i^N(x_j)$ in
the $x_A - x_B$-diagram. Whether $R_i^C(x_j)$ under cooperation lies above or below $R_i^N(x_j)$ under competition depends only on the sign of the profit externality, $d\pi_j/dx_i$, in equations (22) and (23). If $R_i^C(x_j)$ is above (below) $R_i^N(x_j)$, a firm will respond with a higher (lower) investment level under cooperation than under competition.

By adding the spillover-, within-product coordination- and cross-product coordination-effects of expression (22), we obtain the profit externality conferred by A’s R&D investment on the profit of firm B:

$$
\frac{d\pi_B}{dx_A} = \frac{2}{3}(2\beta - 1)q_{B2}
$$

(24)

Since $q_{B2}$ is always positive, the position of $R_A^C(x_B)$ depends only on the level of the spillover.

**Lemma 3.** For all $0 \leq g < 1$, $R_A^C(x_B)$ lies below $R_A^N(x_B)$ if $0 \leq \beta < 1/2$ and above $R_A^N(x_B)$ if $1/2 < \beta \leq 1$.

**Proof.** See Appendix

![Figure 2: Positions of reaction functions](image)

The result follows from the fact that the negative within-product coordination effect outweighs the sum of the positive spillover- and cross-product cooperation effects when the
spillover is small, as illustrated in Figure 2. However, when the spillover is large the opposite is true.

From expression (23) we derive the profit externality of firm B’s R&D investment on the profit of firm A:

$$
\frac{d\pi_A}{dx_B} = \frac{2}{3}(2\beta - 1)q_{A2} - \frac{1}{3}g(2 - \beta)q_{A1}
$$

(25)

The first term, \(2(2\beta - 1)q_{A2}/3\), is positive if and only if \(\beta > 1/2\), whereas the second term, \(-g(2 - \beta)q_{A1}/3\), is always negative. Hence, the position of \(R^C_B(x_A)\) of firm B under cooperation depends not only on the knowledge spillover but also on the degree of product differentiation between goods 1 and 2. The reaction function \(R^C_B(x_A)\) lies above \(R^N_B(x_A)\) whenever the second term is negligible; that is if \(g\) approaches zero.

**Lemma 4.** There is an upward sloping function \(g_C^B(\beta) : [1/2, 1] \rightarrow (0, 1)\) such that \(R^C_B(x_A)\) lies above \(R^N_B(x_A)\) if \(1/2 < \beta \leq 1\) and \(0 \leq g \leq g_C^B(\beta)\), and below otherwise.

**Proof.** See Appendix

Firm B also internalizes the impact of its R&D on the other firm through cooperation. However, as firm B only produces the innovative good, the positive cross-product coordination effect on firm A’s established product does not exist. Hence, there are only two opposing effects of R&D cooperation on the investment incentives of firm B. On one hand, if firms choose R&D investment levels cooperatively, then the spillover externalities are internalized and R&D investment is stimulated. On the other hand, the within-product coordination effect counteracts this positive effect on R&D investment incentives and may even dominate it when spillovers are high and product substitutability is high. This happens when firm A’s loss in profit due to a decline in its market share in both products will not be offset by an increase in its profit due to reduced marginal cost. If the products are close substitutes firm A will loose significant market power in product 1 following R&D investment by firm
B which cannot be offset by the spillover effect.

Having analyzed the effect of cooperation on the incentives to invest in R&D we are now able to determine the equilibrium R&D investment levels of both firms. The directions of the slopes of the reaction functions under R&D competition and under R&D cooperation are known from Lemmas 1 and 2, respectively. In addition, Lemmas 3 and 4 describe the positions of the respective reaction functions under R&D cooperation compared to R&D competition. We note that these functions are linear\(^7\).

If spillovers are sufficiently low (i.e. \(0 \leq \beta < 1/2\)), we know from Lemmas 3 and 4 that both \(R^C_A(x_B)\) and \(R^C_B(x_A)\) are below the reaction functions under competition for all degrees of product substitutability. Hence, it follows directly that, in this case, R&D investment levels under cooperation are lower than under R&D competition. Figure 3 illustrates this in the \(x_A - x_B\)-diagram.

\[
0 < \beta < 1/2, \ 0 < g < \bar{g}_N(\beta)
\]

Figure 3: Optimal R&D investment levels under competition and cooperation

Similarly, if spillovers are sufficiently high (i.e. \(1/2 \leq \beta \leq 1\)) and at the same time, the degree of product substitutability is low (i.e. \(0 \leq g < g^C_B(\beta)\)), then both \(R^C_A(x_B)\) and \(R^C_B(x_A)\) lie above the reaction functions under competition. It is thus straightforward to see

\(^7\)This is easily seen from equations (26) - (27) and (43) - (44).
that then R&D investment levels under cooperation are higher for both firms.

When the positions are in the same direction, it is straightforward to determine the effect of cooperation on R&D levels. However, when this is not the case, it becomes more cumbersome. We illustrate all different cases subsequently.

Let us consider the case when $1/2 \leq \beta \leq 1$ and $g_B^C(\beta) < g < 1$, which is illustrated in Figure 4. By Lemmas 1 and 2, slopes of all reaction functions are upward sloping. By Lemma 3 $R_A^C(x_B)$ lies above $R_A^N(x_B)$, while by Lemma 4 $R_B^C(x_A)$ lies below $R_B^C(x_A)$.

We observe two possible scenarios. First, the difference between $R_i^C(x_j)$ and $R_i^N(x_j)$ is of similar size and second, the difference between $R_B^C(x_A)$ and $R_B^N(x_A)$ is significantly larger than the difference between $R_A^C(x_B)$ and $R_A^N(x_B)$. The latter one occurs when the spillover approaches $1/2$ and the degree of substitutability is positive. Then, we observe from equation (24) that the difference $(R_A^C(x_B) - R_A^N(x_B))$ is approaching zero. Additionally, the difference $(R_B^C(x_A) - R_B^N(x_A))$ is significant as the first term in equation (25) is negligible, while the second term increases as $g$ increases.

![Figure 4: Optimal R&D investment levels under competition and cooperation](image)

From the two diagrams in Figure 4 it can be seen that in the observed interval, the R&D investment level of firm B is lower under cooperation than under competition and the R&D
investment level of firm A could also be lower under cooperation than under competition when $\beta \to 1/2$ and $g \neq 0$.

In the following Proposition, we summarize the effects of R&D cooperation on the investment levels of both firms when all parameter constellations are taken in account. Figure 5 illustrates these results graphically.

**Proposition 3.** There is an upward sloping function $g^C_A(\beta) : [0, 1] \to (0, 1)$ such that:

(i) $x^C_A < x^N_A$ if $0 < \beta \leq 1/2$ and $0 < g \leq 1$ and $1/2 < \beta < 1$ and $g^C_A(\beta) < g < 1$; otherwise $x^C_A > x^N_A$.

(ii) $x^C_B < x^N_B$ if $0 < \beta \leq 1/2$ and $0 < g \leq 1$ and $1/2 < \beta < 1$ and $g^C_B(\beta) < g < 1$; otherwise $x^C_B > x^N_B$.

**Proof.** See Appendix

![Figure 5: Comparison of R&D investments under competition and cooperation](image-url)

We find that whether the level of R&D investment, $x_i$, increases or decreases following cooperation depends not only on the technological spillover but also on the degree of substitution between the two products. If spillovers are sufficiently high and the degree of
substitution is relatively low, R&D investment levels under cooperation exceed those of competition. The internalization leads to an increase in R&D because the positive effect of the spillover on firm $j$’s profit is higher than the negative effect of the reduction in the marginal cost on firm $j$’s profit.

If the products are perfect substitutes ($g = 0$), our result replicates the standard result by D’Aspremont and Jacquemin (1988), as seen in Figure 5. Due to the positive product differentiation in our analysis, we find that the R&D investment of firm B is higher under competition than under cooperation even when the spillover is high.

From a policy perspective it is important to determine the overall effect of cooperation on total R&D due to the marginal cost reduction. Total R&D depends on the sum of the changes in $x_A$ and $x_B$. Let total competitive R&D investment be $x^N \equiv x^N_A + x^N_B$ and total cooperative R&D investment be $x^C \equiv x^C_A + x^C_B$. The next proposition directly follows from Proposition 3. Figure 6 illustrates the result.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure6.png}
\caption{Comparison of total R&D investment}
\end{figure}

**Proposition 4.** Total investment under cooperation is lower than under competition if $0 < \beta \leq 1/2$ and $g > 0$ and if $1/2 < \beta < 1$ and $g$ high enough.

*Proof.* See Appendix
When $\beta > 1/2$ and $g_B^C(\beta) < g < g_A^C(\beta)$, cooperation will reduce $x_B$ but increase $x_A$. It is therefore unclear whether R&D competition or R&D cooperation lead to a higher overall investment level. The net effect on total R&D will depend on the magnitudes of these changes.

We simulate the overall effect of cooperation on total R&D. We use the following specification: $a = 1000$, $c = 50$ and $\gamma = 60$. The resulting Figure 6 shows that, indeed, there exists a function $g^C(\beta)$, such that for every $g > g^C(\beta)$ and $\beta > 1/2$ the overall investment in R&D under competition is greater than under R&D cooperation.

### 7 Conclusion

In this paper we study strategic R&D investment between a multi-product firm and a single-product firm. Investigating whether such asymmetric firms should be allowed to coordinate their decisions at the R&D stage, as in D’Aspremont and Jacquemin (1988), we find that R&D investment levels under cooperation are lower when the established and the innovative product are close substitutes even if the spillover is substantial. Hence, the asymmetry between the firms leads to higher R&D investment levels under competition than under cooperation for many values of the technology spillovers and degrees of product substitution. Our results, therefore, indicate that regulators need to be more cautious about allowing R&D joint ventures in an asymmetric context.

In addition, we also identify several strategic effects that are incorporated under R&D cooperation. For the multi-product firm, investment incentives are lower under cooperation when spillovers are low because the negative *within-product coordination effect* then dominates the positive spillover and *cross-product coordination* effects. For the single-product firm, if product substitutability is high, investment incentives are also lower under cooperation even when spillovers are high. Following R&D investment by the single-product firm, the multi-product firm would lose significant market share in the established good if the
products are close substitutes. Then this loss cannot be offset by the spillover effect.

It would seem natural to assume that the spillovers of the two firms are not identical, given that large multi-product firms are able to protect their patents better than smaller firms. Hence, we can extend our analysis for asymmetric spillovers, namely $\beta_A < \beta_B$. However, our main result (Proposition 4) still holds.

8 Appendix

Proof of Lemma 1:

From the optimal quantities (6)-(8) in the second stage and profit maximization of (13) with respect to $x_i$ the relevant first-order conditions lead to the following R&D best-response functions

$$R^N_A(x_B) = \frac{a(g-1)(g-8+4\beta(1+g))-c(8+g^2+4\beta(g^2-1))+(4(2-\beta)(2\beta-1)+(8+\beta(8\beta-11))\gamma^2\gamma x_B}{36\gamma(1-g^2)-16+7g^2+4\beta(\beta-4)(g^2-1)} \quad (26)$$

$$R^N_B(x_A) = \frac{(2-\beta)(a-c-(1-2\beta)x_A)}{9\gamma-(2-\beta)^2} \quad (27)$$

The derivative with respect to the strategic variable of the competitor, $x_j$, yields the slope of the reaction function. For firm A, we obtain from (26)

$$\frac{dR^N_A(x_B)}{dx_B} = \frac{(8 + \beta(8\beta - 11))g^2 - 4(\beta - 2)(2\beta - 1)}{36\gamma(1-g^2)-16+7g^2+4\beta(\beta-4)(g^2-1)} \quad (28)$$

To determine the slope of $R^N_A(x_B)$ we need the sign of (28). By assumption 1 the denominator is always positive. Hence, it remains to show the sign of the numerator of (28), which has two components. The first component, $(8 + \beta(8\beta - 11))g^2$ is positive for all $0 < \beta < 1$ and $0 < g < 1$. The second component, $-4(\beta - 2)(2\beta - 1)$, is, for all $g \geq 0$, positive if $1/2 < \beta < 1$ and negative if $0 < \beta < 1/2$.

Therefore, the derivative in expression (28) is always positive if $1/2 \leq \beta < 1$. Moreover,
if \( 0 < \beta < 1/2 \), it is also positive if substitutability is high:

\[
g > \bar{g}_N(\beta) \equiv \sqrt{\frac{4(\beta - 2)(2\beta - 1)}{8 + \beta(8\beta - 11)}}
\]  

(29)

For \( 0 < \beta < 1/2 \) and \( 0 < g < \bar{g}_N(\beta) \), the derivative is negative.

For firm B, we obtain from (27)

\[
\frac{dR_N^B(x_A)}{dx_A} = \frac{(2 - \beta)(2\beta - 1)}{9\gamma - (2 - \beta)^2}
\]  

(30)

The sign of sign of (30) determines the slope of \( R_N^B(x_A) \). Substituting \( \gamma > \gamma_{\text{min}} \) (assumption 1) into the denominator we find that the denominator is always positive. Then it is easy to see that for all \( g > 0 \) the derivative in (30) is positive if \( 1/2 < \beta < 1 \) and negative if \( 0 < \beta < 1/2 \).

Hence, this concludes the proof of Lemma 1. \( \square \)

**Proof of Proposition 1:**

First, we derive each effect in (15) for firm A separately:

(i) From (1), (2) and (4), the direct effect

\[
\frac{\partial \pi_A}{\partial x_A} = q_{A2} > 0
\]  

(31)

is always positive.

(ii) The within-product competition effect consists of two components. From (1), (2) and (4), the first component

\[
\frac{\partial \pi_A}{\partial q_{B2}} = -(gq_{A1} + q_{A2}) < 0
\]  

(32)

is always negative. The second component derived from (8)

\[
\frac{\partial q_{B2}}{\partial x_A} = \frac{2\beta - 1}{3}
\]  

(33)
is positive if $1/2 < \beta \leq 1$, zero if $\beta = 1/2$ and negative otherwise. Hence, the within-product competition effect

$$\frac{\partial \pi_A}{\partial q_{\text{A}2}} \frac{\partial q_{\text{B}2}}{\partial q_B} \frac{\partial q_{\text{B}2}}{\partial x_A} = -\frac{2\beta - 1}{3}(g q_{\text{A}1} + q_{\text{A}2})$$

(34)

is negative if $1/2 < \beta \leq 1$, zero if $\beta = 1/2$ and positive otherwise.

Second, we derive each effect in (16) for firm B:

(i) From (1), (2) and (5), the direct effect

$$\frac{\partial \pi_B}{\partial x_B} = q_{\text{B}2} > 0$$

(35)

is always positive.

(ii) The within-product competition effect consists of two components. Also from (1), (2) and (5), the first component

$$\frac{\partial \pi_B}{\partial q_{\text{A}2}} = -q_{\text{B}2} < 0$$

(36)

is always negative. The second component derived from (7) is given by

$$\frac{\partial q_{\text{A}2}^*}{\partial x_B} = \frac{4\beta + (2 - \beta)g^2 - 2}{6(1 - g^2)}$$

(37)

The denominator is always positive for $0 < g < 1$. Hence, the whole term is positive if

$$\beta > \frac{2g^2 - 2}{g^2 - 4} \equiv \beta$$

(38)

Hence, the within-product competition effect

$$\frac{\partial \pi_B}{\partial q_{\text{A}2}} \frac{\partial q_{\text{A}2}^*}{\partial x_B} = -\frac{4\beta + (2 - \beta)g^2 - 2}{6(1 - g^2)} q_{\text{B}2}$$

(39)

is negative if $\beta < \beta < 1$ and $0 \leq g < 1$ and positive otherwise.

(iii) The cross-product competition effect consists of two components. Also derived from
(1), (2) and (5), the first component
\[
\frac{\partial \pi_B}{\partial q_{A1}} = -gq_{B2} < 0 \quad (40)
\]
is negative. The second component derived from (6)
\[
\frac{\partial q^*_{A1}}{\partial x_B} = -\frac{\beta g}{2(1 - g^2)} < 0 \quad (41)
\]
is also negative. Hence, the cross-product competition effect
\[
\frac{\partial \pi_B}{\partial q_{A1}} \frac{\partial q^*_{A1}}{\partial x_B} = \frac{\beta g^2}{2(1 - g^2)} q_{B2} > 0 \quad (42)
\]
is positive ∀β, g.

**Proof of Lemma 2:**

From optimal second stage quantities (6)-(8) and joint profit maximization of (19) with respect to \(x_i\) the relevant first-order conditions yield \(x_i\) as a function of \(x_j\):

\[
R^C_i(x_j) = \frac{-4(1+\beta)c+(4\beta - 5)c g^2 - a(g-1)(4-5g+4\beta(1+g))+(16+\beta(16\beta - 31))g^2 x_B}{36\gamma(1-g^2)-20+11g^2+4\beta(5\beta - 8)(g^2-1)} \quad (43)
\]

\[
R^C_i(x_j) = \frac{-c(4(1+\beta)+(5\beta - 4)g^2)+a(g-1)(-4(1+g)+\beta(5g-4))+(16+\beta(16\beta - 31))g^2 x_A}{-32\beta(g^2-1)+\beta^2(11g^2-20)-4(\beta - 1)(9\gamma - 5)} \quad (44)
\]

The derivative of \(R^C_i(x_j)\) with respect to the strategic variable of the competitor, \(x_j\), yields the slope of \(R^C_i(x_j)\). For firm A, we obtain from (43)
\[
\frac{dR^C_i(x_B)}{dx_B} = \frac{(16 + \beta(16\beta - 31))g^2 - 8(\beta - 2)(2\beta - 1)}{36\gamma(1-g^2) - 20 + 11g^2 + 4\beta(5\beta - 8)(g^2-1)} \quad (45)
\]

In order to determine the slope, we need the sign of (45). By assumption 1 the denominator is always positive. It remains to show the sign of the numerator of (45), which has two components. The first component, \((16 + \beta(16\beta - 31))g^2\) is positive for all \(0 \leq \beta \leq 1\) and \(0 \leq g < 1\). The second component, \(-8(\beta - 2)(2\beta - 1)\), is positive if \(1/2 < \beta \leq 1\) and
negative if \(0 \leq \beta < 1/2\). Therefore, the derivative in expression (45) is always positive if \(1/2 \leq \beta \leq 1\). Moreover, if \(0 \leq \beta < 1/2\), it is also positive if substitutability is high:

\[
g > \bar{g}_C(\beta) \equiv \sqrt{\frac{8(\beta - 2)(2\beta - 1)}{16 + \beta(16\beta - 31)}}
\]  

(46)

For \(0 \leq \beta \leq 1/2\) and \(0 \leq g \leq \bar{g}_C(\beta)\), the derivative is negative.

For firm B, we obtain from (44)

\[
\frac{dR_B(x_A)}{dx_A} = \frac{(16 + \beta(16\beta - 31))g^2 - 8(\beta - 2)(2\beta - 1)}{-32\beta(g^2 - 1) + \beta^2(11g^2 - 20) - 4(g^2 - 1)(9\gamma - 5)}
\]  

(47)

In order to determine the slope of \(R^C_B(x_A)\), we need to determine the sign of (47). By assumption 1 we find that the denominator is always positive. It remains to show the sign of the numerator of (47), which is equivalent to the numerator of (45). Hence, for \(0 \leq \beta < 1/2\) and \(0 \leq g \leq \bar{g}_C(\beta)\), the derivative is negative. This concludes the proof of Lemma 2. □

**Proof of Proposition 2:**

First, we derive each part in (22) for firm A. The first term, \(d\pi_A/dx_A\) is derived in proposition 1. We use (1), (2) and (5) to determine some of the following components.

(i) The spillover effect given by

\[
\frac{\partial \pi_B}{\partial x_A} = \beta q_{B2} > 0
\]  

(48)

is positive.

(ii) The within-product coordination effect consists of two components. The first component

\[
\frac{\partial \pi_B}{\partial q_{A2}} = -q_{B2} < 0
\]  

(49)
is always negative. The second component derived from (7) is given by

\[ \frac{\partial q^*_A}{\partial x_A} = \frac{4 + (2\beta - 1)g^2 - 2\beta}{6(1 - g^2)} \]  

(50)

The term is always positive for all \( \beta, g \in [0, 1] \). Hence, the within-product coordination effect

\[ \frac{\partial \pi_B}{\partial q^*_A} \frac{\partial q^*_A}{\partial x_A} = \frac{-4 + (2\beta - 1)g^2 - 2\beta}{6(1 - g^2)} q_{B2} \]  

is always negative.

(iii) The cross-product coordination effect consists of two components. The first component

\[ \frac{\partial \pi_B}{\partial q_{A1}} = -g q_{B2} < 0 \]  

(52)

is negative. The second component derived from (6)

\[ \frac{\partial q^*_{A1}}{\partial x_A} = -\frac{g}{2(1 - g^2)} < 0 \]  

(53)

is also negative. Hence, the cross-product coordination effect

\[ \frac{\partial \pi_B}{\partial q_{A1}} \frac{\partial q^*_{A1}}{\partial x_A} = \frac{g^2}{2(1 - g^2)} q_{B2} > 0 \]  

(54)

is positive \( \forall \beta, g \).

Second, we derive each effect of firm B in (23). The first term, \( d\pi_B/dx_B \) is derived in proposition 1. For the following components, we use (1), (2) and (4).

(i) The spillover effect given by

\[ \frac{\partial \pi_A}{\partial x_B} = \beta q_{A2} > 0 \]  

(55)

is positive.
(ii) The within-product coordination effect consists of two components. The first component
\[
\frac{\partial \pi_A}{\partial q_{B2}} = -(gq_{A1} + q_{A2}) < 0 \quad (56)
\]
is always negative. The second component derived from (8) is given by
\[
\frac{\partial q^*_B}{\partial x_B} = \frac{2 - \beta}{3} > 0 \quad (57)
\]
is always positive. Hence, the within-product coordination effect
\[
\frac{\partial \pi_A}{\partial q_{B2}} \frac{\partial q^*_B}{\partial x_B} = -\frac{2 - \beta}{3} (gq_{A1} + q_{A2}) \quad (58)
\]
is always negative.

**Proof of Lemma 3:**
As seen in (22), the marginal benefit for firm A under cooperation can be decomposed into two components, where \(d\pi_B/dx_A\) is the additional component under cooperation. Whether \(R^C_A(x_B)\) lies above or below \(R^N_A(x_B)\) under competition depends only on the sign of this additional component. If the additional component is positive, then \(R^C_A(x_B) > R^N_A(x_B)\). When substituting (48), (51) and (54) into the additional component of (22) we obtain (24).

It is easy to see that whenever \(\beta > 1/2\), then \(d\pi_B/dx_A > 0\). Moreover, if \(\beta < 1/2\), then \(d\pi_B/dx_A < 0\) and if \(\beta = 1/2\), then \(d\pi_B/dx_A = 0\). This concludes the proof.

**Proof of Lemma 4:** In (23) the marginal benefit under cooperation for firm B can be decomposed into two components, where \(d\pi_A/dx_B\) is an additional component under cooperation. Whether \(R^C_B(x_A)\) lies above or below \(R^N_B(x_A)\) under competition depends only on the sign of the additional component. If the additional component is positive, then \(R^C_B(x_A) > R^N_B(x_A)\). When substituting (55) and (58) into the additional component of (23) we obtain (25). It is easy to show that if \(0 \leq \beta < 1/2\) and any \(g \geq 0\), then equation (25)
will be negative.

Next, if \( g = 0 \) (as in D’Aspremont and Jacquemin (1988)) and \( \beta = 1/2 \), we have

\[
\frac{d\pi_A}{dx_B} = 0. \tag{59}
\]

This implies that then \( R^C_A(x_B) = R^N_A(x_B) \).

If we now keep \( \beta = 1/2 \) and increase \( g \), we have

\[
\frac{d\pi_A}{dx_B} = -\frac{g q_{A1}}{2} < 0. \tag{60}
\]

This means that for \( \beta = 1/2 \) and every \( g > 0 \) the reaction function under coordination is below the one under competition.

It remains to show what happens if \( \beta > 1/2 \) and \( g > 0 \). Expression (25) is positive if

\[
g < g^C_B(\beta) \equiv \frac{2(2\beta - 1)q_{A2}}{(2 - \beta)q_{A1}} \tag{61}
\]

and negative otherwise.

Given that the quantities \( q_{A1} \) and \( q_{A2} \) depend on the parameter \( g \) themselves, we cannot obtain the closed form for \( g^C_B(\beta) \). However, when \( 1/2 < \beta \leq 1 \), it is \( 2\beta - 1 > 0 \) and also \( q_{A1} > 0 \) and \( q_{A2} > 0 \), \( g^C_B(\beta) \in (0, 1) \)

**Proof of Proposition 3:** In order to compare R&D investment levels under competition and cooperation, we need to analyze the positions of \( R^C_A(x_B) \) and \( R^C_B(x_A) \) under R&D cooperation compared to \( R^N_A(x_B) \) and \( R^N_B(x_A) \) under R&D competition.

We observe that all relevant reaction functions, (26), (27), (43) and (44), are linear. Hence, it is convenient to compare R&D investment levels under cooperation and cooperation graphically in a \( x_A - x_B \)-diagram.
**Case 1:** $0 \leq \beta < 1/2$. According to Lemmas 3 and 4, both $R_i^C(x_j)$ under R&D cooperation lie below $R_i^N(x_j)$, hence R&D investment levels under cooperation are lower than under competition. Figure 3 illustrates the case when $0 < g < \bar{g}_N(\beta)$. For this parameter range, from Lemmas 1 and 2, we know that the slopes of $R_i^k(x_j)$ for $k \in C, N$ are negative. The cases $\bar{g}_N(\beta) < g < \bar{g}_C(\beta)$ and $\bar{g}_C(\beta) < g < 1$ are depicted in Figure 7.

![Figure 3: Optimal R&D investment levels under competition and cooperation](image)

$0 < \beta < 1/2, \bar{g}_N(\beta) < g < \bar{g}_C(\beta)$

**Figure 7:** Optimal R&D investment levels under competition and cooperation

We conclude that when $0 \leq \beta < 1/2$ and $0 \leq g < 1$, both firms invest more under R&D competition than under R&D cooperation, hence $x_A^N > x_A^C$ and $x_B^N > x_B^C$.

**Case 2:** $1/2 \leq \beta < 1$. When $g_C^C(\beta) < g < 1$, by Lemma 3, $R_A^C(x_B) > R_A^N(x_B)$, while by Lemma 4, $R_B^C(x_A) < R_B^N(x_A)$. According to Lemmas 1 and 2 the slopes of the reaction functions under both regimes are positive. This is illustrated in Figure 4. The R&D investment level of firm B is lower under cooperation than under competition and the R&D investment level of firm A could also be lower under cooperation than under competition when the difference $|R_B^C(x_A) - R_B^N(x_A)|$ is significantly greater than the difference $|R_A^C(x_B) - R_A^N(x_B)|$. This scenario happens when $\beta \to 1/2$ and $g \neq 0$. There exists a function $g^A(\beta) : [1/2, 1] \to (0, 1)$ with $g_B^C(\beta) \leq g^C_A(\beta) \leq 1$ such that $x_A^N > x_A^C$, when $g > g^C_A(\beta)$ and $x_A^N < x_A^C$, when $g < g^C_A(\beta)$. 

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Moreover, when $\beta = 1/2$ and $g \in (0, 1)$ by Lemma 3, $R_C^A(x_B) = R_N^A(x_B)$, while, by Lemma 4, $R_C^B(x_A) < R_N^B(x_A)$. This case is depicted in the left diagram of Figure 8. It is easy to see that $x_N^A > x_C^A$ and $x_N^B > x_C^B$. Only if $g = 0$, both (24) and (25) are the same as under competition such that $x_N^A = x_C^A$ and $x_N^B = x_C^B$.

It remains to analyze the case when $0 < g < g_C^B(\beta)$. According to Lemmas 3 and 4, $R_i(x_j)$ of both firms under cooperation lie above those under competition. According to Lemmas 1 and 2, the slopes are all positive. This is illustrated in the right diagram of Figure 8. It is easy to see that $x_N^A < x_C^A$ and $x_N^B < x_C^B$. $\square$

**Proof of Proposition 4:** From proposition 3, it follows directly that when $0 < \beta \leq 1/2$ and $0 < g < 1$ and when $\beta > 1/2$ and $g_C^A(\beta) < g < 1$:

$$x_C^A < x_N^A \quad (62)$$

$$x_C^B < x_N^B \quad (63)$$

Hence, for these parameter values, $x_C = x_C^A + x_C^B < x_N^A + x_N^B = x_N$. 

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Further on, in the area $1/2 < \beta < 1$ and $0 < g < g_B^C(\beta)$ the following holds:

\[
x^C_A > x^N_A \tag{64}
\]
\[
x^C_B > x^N_B \tag{65}
\]

Thus, for these parameter values, $x^C = x^C_A + x^C_B > x^N_A + x^N_B = x^N$.

In the remaining area, i.e. $1/2 < \beta < 1$ and $g_B^C(\beta) < g < g_A^C(\beta)$, we have

\[
x^C_A > x^N_A \tag{66}
\]
\[
x^C_B < x^N_B \tag{67}
\]

hence, there exists a function $g^C(\beta) : [0, 1] \to (0, 1)$ with $g_B^C(\beta) \leq g^C(\beta) \leq g_A^C(\beta)$, such that $x^C > x^N$, when $g > g^C(\beta)$ and $x^C < x^N$, when $g < g^C(\beta)$. We, however, do a numerical analysis for this special case. See Figures 6 and ?? for clarification. \(\square\)

References


