Working Paper Series
2015-19

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First Version: January, 2016
Revisited: July, 2018

Abstract

This paper examines under which conditions the puzzling observation of capital flows from poor to rich countries and accompanying changes in domestic economic development can be explained by the presence of financial market imperfections. Motivated by the mixed results from the literature, we employ an incomplete-markets model in which entrepreneurs face capital risk, earn risky profits and receive riskless wage income. Moreover, borrowing constraints simultaneously impede consumption smoothing and limit the access to external funds for scaling up production. We find that in the presence of uninsurable risk only and for plausible parameter values, capital does flow from poor to rich countries and that an increase in the interest rate leads to higher levels of capital and output in the steady state under financial integration. However, we also find that tight borrowing constraints and high persistence of shocks strongly affect the model predictions and lead to significantly tighter parameter restrictions. With these findings we contribute to the ongoing debate on the consequences of financial integration.

JEL classification: D52; E22; F41; G11

Keywords: Incomplete markets; Borrowing constraints; Financial integration

*Financial support from the German Research Foundation (Deutsche Forschungsgemeinschaft) within the priority program 1578 "Financial Market Imperfections and Macroeconomic Performance" is gratefully acknowledged. An earlier version of this paper was named: Idiosyncratic Risk, Borrowing Constraints and Financial Integration - A Discussion of Ambiguous Results.

The authors would like to especially thank Christiane Clemens, Marius Clemens, Ulrich Eydam, Frank Heinemann, Jean Imbs, Vahagn Jerbashian, Tom Krebs, Philipp Pfeiffer, Wolfgang Strehl and Lutz Weinke for valuable comments and suggestions. We also thank various participants at the Royal Economic Society conference in Manchester, the Verein für Socialpolitik conference in Münster, the ZEW conference in Mannheim, the ISNE conference in Galway, the DFG workshop in Konstanz, the Brown Bag Seminar at HU Berlin and the Potsdam Research Seminar. Any remaining errors are ours.

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1 Introduction

The topic of international integration has clearly gained in importance over the last decades. Among others, Prasad et al. (2006, 2007) and Mendoza et al. (2009a) document a persistent increase in the volume of cross-border capital flows, which reflects the profound developments in the process of financial integration. However, existing discrepancies between the observed pattern on the one hand and the predictions from standard theory on the other hand have not been wiped away by the larger amount of capital flows and further, striking observations have emerged over time. In this respect, the so-called 'Lucas paradox' is one of the most prominent examples. In his influential paper, Lucas (1990) points to the fact that the share of capital flowing from rich to poor countries is significantly smaller compared to the predictions of the frictionless, neoclassical model. In recent times, this paradox has even intensified. Prasad et al. (2006, 2007) show that since the end of the 20th century, the average income of countries running current account surpluses has fallen below the average income of countries running current account deficits, implying that capital nowadays even flows from poor to rich countries.

In addition to the Lucas paradox, domestic economic development in the light of capital outflows challenges conventional wisdom. While the neoclassical model predicts a decrease of the domestic capital stock and output if the interest rate increases under financial integration, triggering capital outflows, Prasad et al. (2006, 2007) find periods of capital outflows from high-growth nonindustrial countries. Similarly, Sandri (2010) notes improvements in the current accounts of developing countries during periods of high per capita income growth. Finally, Gourinchas and Jeanne (2013) find a negative correlation between total factor productivity growth and net capital inflows for developing countries, which they call the 'allocation puzzle'.

The obvious failure of standard theory to explain the empirical pattern has given rise to a voluminous literature, seeking for explanations beyond the common assumptions of the frictionless, neoclassical model. Recently, special attention has been paid to the influence of financial market imperfections in the form of borrowing constraints or the presence of uninsurable idiosyncratic risks following from incomplete insurance markets. As pointed out by Mendoza et al. (2009a), countries still differ significantly in the level of financial development despite the persistent increase in the volume of cross-border capital flows. However, the questions whether and under what conditions the presence of uninsurable risks and borrowing constraints contributes to explain the empirical findings have not been completely resolved yet. In fact, the literature shows that the results are extremely diverse and strongly vary with the underlying source of risk (capital vs. income risk), the exact specification of borrowing constraints (household vs. production side), and further model assumptions. Hence, while progress has been made to understand some of the individual effects, we are still lacking a clear understanding of the rich interactional effects of the different types of financing constraints and risky income components and therefore, of the joint overall effect of financial market imperfections on the process of financial integration. This, however, is important as the simultaneous presence of risks and different types
of financing constraints can hardly be rejected based on available measures of financial market development. In this paper, we aim at filling this remaining gap in the literature. Our work relates to the line of research including Gertler and Rogoff (1990), Matsuyama (2005), Aoki et al. (2009), Buera and Shin (2009), Mendoza et al. (2009a,b), Sandri (2010), Angeletos and Panousi (2011), Song et al. (2011), Clemens and Heinemann (2013) and von Hagen and Zhang (2014).

The model we consider is an incomplete-markets model with two sectors of production and heterogeneous entrepreneurs. The model captures essential features from the related literature while providing a richer representation of the different effects of uninsurable risk and financing constraints. The model structure can be outlined as follows. In the final good sector, a homogenous good is produced under perfect competition with intermediate goods and labor as input factors. In the intermediate goods sector, firms operate under monopolistic competition and each firm, producing a single intermediate good, is owned and managed by one entrepreneur. The economy is populated by a continuum $[0,1]$ of infinitely-lived households. Each household consists of one entrepreneur and is endowed with one unit of labor that is supplied inelastically to the perfectly competitive labor market. Entrepreneurs can invest in the own firm and can trade a riskless bond subject to a borrowing constraint. Idiosyncratic risk is introduced by stochastic fluctuations in the entrepreneur’s productivity, capturing different kinds of business risk. The chosen model structure guarantees the existence of capital risk since investment has to be chosen before the idiosyncratic shock is realized, of a risky income component since profits fluctuate as well, and of a riskless income component given by the riskless wage income. Moreover, the borrowing constraints entrepreneurs face simultaneously impede consumption smoothing and restrict the access to external funds for scaling up individual production.

Contrary to main parts of the literature, we do not aim for a specific calibration strategy, but focus on understanding the possibly different model implications. From an economic point of view, we analyze the macroeconomic effects that have to be expected if financial integration takes place between countries that differ in the level of financial development, i.e. in the amount of risk that remains with the entrepreneurs and in the tightness of the borrowing constraint. From a more technical point of view, we show under which conditions, i.e. under which restrictions on the model parameters, the presence of uninsurable risks and borrowing constraints contributes to explain the empirical findings and how these conditions may change with different model assumptions.

In order to keep track of the individual effects, we consider different scenarios, increasing the model complexity step by step. First, we consider a baseline scenario that focuses on the effects of the uninsurable capital risk, the risky profits and the riskless wage income. We find that in the baseline scenario, the model is in principle capable of contributing to explain the empirical findings, but may also come to very different conclusions. In particular, by comparing the steady state under financial autarky vis-à-vis financial integration, we find that it is in fact the financially less developed and initially poor country that builds up a positive net foreign asset position under financial integration and that the domestic
capital stock and output may increase despite an increase of the interest rate.\footnote{In the following, we also refer to the steady state under financial autarky (integration) as the autarchic (integrated) steady state.} However, we also find that the results may be exactly reverse; the outcome strongly varies with the underlying parametrization. Therefore, in a second step, we derive two rules of thumb that describe the parameter restrictions with high accuracy. In line with Angeletos (2007) and Angeletos and Panousi (2011), our two rules show that the elasticity of intertemporal substitution (EIS) has to exceed a certain threshold level in order to explain the empirical findings. For our baseline scenario, we find that the parameter restrictions remain moderate and are easily satisfied by empirically plausible values of the elasticity of intertemporal substitution. In other words, in the baseline scenario, we find a quite robust pattern of international capital flows from poor to rich countries.

In the second scenario, we increase the tightness of the debt limit, which means that borrowing constraints occasionally bind. On the one hand, borrowing constraints make it more difficult for agents to smooth consumption and lead to an increase in aggregate demand for the safe asset. On the other hand, borrowing constraints restrict the access to external funds for scaling up production and, even if not currently binding, discourage risky investment. The latter effects dampen the upward trend of the aggregate capital stock associated with lower interest rates and mean that the overall effect of borrowing constraints is generally ambiguous. However, we find that the saving effect is the dominant effect and that tighter borrowing constraints lead to tighter parameter restrictions, i.e. tighter restrictions on the elasticity of intertemporal substitution compared to the first scenario. In particular, we find that in times of strong turmoil in financial markets, with an almost collapsing lending channel, the parameter restrictions become too tight in order to be satisfied by empirically plausible values. Put differently, we find that in the presence of severe borrowing constraints, financial integration may easily become an impediment for domestic economic development.

In the third and final scenario, we increase the persistence of shocks while keeping the unconditional variance at a constant level. A higher persistence of shocks increases the demand for the riskless asset and therefore, amplifies the effects of the financial market imperfections. In almost all exercises, we find that a higher persistence of shocks again leads to tighter parameter restrictions compared to the first two scenarios. This especially applies to moderate levels of the borrowing constraint. Hence, as an overall result, we find that with increasing model complexity, it becomes more difficult to contribute to explain the empirical pattern solely with the difference in financial development. From a more economic point of view, our results show that in the presence of different types of financing constraints and persistent risks, the response of international capital flows becomes more volatile and strongly reacts to even small deteriorations in financial market performance. This may also affect the society’s support for financial liberalization, which we will discuss more detailed when analyzing the welfare implications.

The remainder of this paper is organized as follows. Section 2 provides a brief review of the relevant literature. Section 3 describes the model and Section 4 describes the
benchmark parametrization. Section 5 introduces the different scenarios that compare the steady state under financial autarky vis-à-vis financial integration. Section 6 presents more details on transitory dynamics and discusses the welfare implications. Section 7 concludes. The appendix collects relevant proofs.

2 Literature review

The overview summarizes some of the opposing results arising due to variations in the source of the underlying risk and in the specification of borrowing constraints. We consider implications from both, closed- and open-economy settings. For a more detailed discussion of financial market imperfections and macroeconomic performance see, for example, Brunnermeier et al. (2012) or Gourinchas and Rey (2013).

Large parts of the analysis of financial market imperfections and financial integration build on the class of heterogeneous-agents, incomplete-markets models. In the standard incomplete-markets model, agents have to decide on an optimal consumption and savings path but face stochastic fluctuations in the income process.2 The set of instruments to insure against income risk is restricted to a riskless asset and agents can only borrow up to an exogenous debt limit. Due to the presence of uninsurable income risk and borrowing constraints, agents engage in precautionary saving, which finally leads to a lower interest rate in the autarchic steady state (see Huggett 1993, Aiyagari 1994).

Mendoza et al. (2009b) study the welfare implications of financial integration in the presence of uninsurable income risk and borrowing constraints.3 Differences in the level of financial development between countries are captured by differences in the tightness of the borrowing constraint. Mendoza et al. (2009b) show that if financial integration takes place, it is the financially less developed country that accumulates a positive net foreign asset position. However, since uninsurable income risk and borrowing constraints on the household side do not break the equality between the interest rate and the marginal product of capital, the financially less developed country is actually the rich country in terms of output under financial autarky, which means that capital flows from the rich to the poor country under financial integration. Mendoza et al. (2009b) assume exogenous differences in productivity levels between countries, which circumvent this result. However, these differences do not endogenously arise from the differences in financial market performance.

Contrary to the assumption of fluctuations in labor income, Angeletos (2007) emphasizes the importance of rate-of-return or capital risk and augments the neoclassical growth model to study the macroeconomic consequences of market incompleteness.4,5 Angeletos

2See Heathcote et al. (2009) or Ljungqvist and Sargent (2012).
3See also Mendoza et al. (2009a) where investment risk is additionally included but without aggregate capital accumulation.
4See Phelps (1962) and Levhari and Srinivasan (1969) for early discussions of the saving effect of risky returns and Sandmo (1970) for a comparison with income risk.
5See also Angeletos and Calvet (2005, 2006) for a discussion with CARA preferences and endowment risk.
(2007) shows that in the presence of capital risk, the financially less developed country may also be the initially poor country under financial autarky. The fundamental difference compared to income risk is that capital risk also affects the demand for investment, thereby breaking the equality between the interest rate and the marginal product of capital. Consequently, as Angeletos and Panousi (2011) show, capital may flow from the financially less developed and initially poor country to the financially more developed and initially rich country under financial integration. However, as emphasized by Angeletos and Panousi (2011), even in the presence of capital risk, the deep structural model parameters have to satisfy certain conditions in order to explain the empirical findings.

Our approach most closely relates to Angeletos (2007) and Angeletos and Panousi (2011) by sharing the feature that entrepreneurs face capital risk and receive riskless wage income. However, entrepreneurs in our model also earn risky profits and we consider the case with occasionally binding borrowing constraints and with persistent effects of shocks. As we will show, these features may strongly affect the results from the baseline scenario.

Level effects of uninsurable investment risk in a closed-economy setting are also studied by Covas (2006) and Meh and Quadrini (2006). Meh and Quadrini (2006) consider different risk-sharing environments and find that the capital stock is lower if markets are incomplete. In contrast, Covas (2006) finds that the capital stock is higher under incomplete markets and shows that tighter borrowing constraints may further increase the difference between the complete and the incomplete markets case. Covas (2006), however, abstracts from any type of riskless income component beyond the safe asset.6

Finally, our approach also relates to the literature that focuses on the effects of financing constraints on the production side of the economy, e.g., Gertler and Rogoff (1990), Boyd and Smith (1997), Matsuyama (2005), Aoki et al. (2009), Buera and Shin (2009), Song et al. (2011), Benhima (2013), Clemens and Heinemann (2013), von Hagen and Zhang (2014), and Bacchetta and Benhima (2015).7 As shown by Buera and Shin (2009) and Clemens and Heinemann (2010, 2013), financing constraints on the production side may help to overcome the result that the financially less developed country is also the rich country in traditional incomplete-markets models.8 Furthermore, in the presence of financing constraints, domestic output may increase under financial integration despite an increase of the interest rate. In our model, tight borrowing constraints also restrict the access to external financing, but increase the demand for the riskless asset as well. As we will show, this combination may lead to very different implications compared to an isolated consideration of financing constraints.

6See also Covas and Fujita (2011) for a discussion of idiosyncratic and aggregate risk and Goldberg (2013) for a discussion of a credit crunch.
7von Hagen and Zhang (2014) additionally distinguish between financial capital and foreign direct investments and von Hagen and Zhang (2011) compare the effects of limited commitment and incomplete markets.
8See also Buera and Shin (2011) for a discussion of the effects of increasing shock persistence, Buera et al. (2011) for a multi-sector analysis and Buera and Shin (2013).
3 The model

We analyze the implications of financial liberalization in an incomplete-markets economy with two sectors of production and heterogeneous entrepreneurs. The economy structure can be outlined as follows.

Time is discrete and indexed by \( t \in [0, ..., \infty] \). In the final good sector, a large number of perfectly competitive firms produce a homogeneous good which can be used for consumption and investment. Input factors are intermediate goods and labor. In the intermediate goods sector, firms operate under monopolistic competition and each firm, producing a single intermediate good, is owned and managed by one entrepreneur.

The economy is populated by a continuum \([0,1]\) of infinitely-lived households. Each household consists of one entrepreneur and is endowed with one unit of labor supplied inelastically to the perfectly competitive labor market. Since we assume perfect consumption sharing on the household level, we refer to the household and to the entrepreneur interchangeably. The entrepreneur invests in the own firm and can trade a riskless bond subject to a borrowing constraint. Idiosyncratic risk is introduced by stochastic fluctuations in the entrepreneur’s productivity; a shortcut to capture different kinds of business risk. The model structure leads to the existence of capital risk since investment has to be chosen before the idiosyncratic shock is realized, of risky profits, and of a riskless income component given by the riskless wage income. Markets are incomplete so that full insurance against idiosyncratic risk is not obtainable. Furthermore, the borrowing constraints entrepreneurs face on bond holdings simultaneously impede consumption smoothing and restrict the access to external funds for scaling up individual production.

Under financial autarky, the bond market has to clear on the country-wide level, whereas under financial integration, bonds can be traded on the international level. We assume that the small economy we consider only differs with respect to the level of financial development from the rest of the world. In the baseline scenario, the level of financial development is determined by the amount of risk that cannot be insured through financial markets, and thus, remains with the entrepreneurs. In the second scenario, the level of financial development is also determined by the tightness of the borrowing constraint.\(^9\)

Depending on the scenario, a lower level of financial development means a larger amount of risk remaining with the entrepreneurs and/or a more tight debt limit. Note that we assume that financial integration takes place without financial development and that agents cannot simply bypass the domestic borrowing restrictions under financial integration.

3.1 Final good sector

In the final good sector of the small economy, the representative firm produces the homogeneous good, \( Y_t \), under perfect competition. Input factors are labor, \( L_t \), and intermediate goods, \( x_{it}, i \in [0,1] \). Production takes place according to the following generalized production function

\[ Y_t = L_t^{1-\alpha} \int_0^1 x_{it}^\alpha di, \quad 0 < \alpha < 1. \]  

(1)

Since \( \alpha \) is assumed to be less than one, intermediate goods are close but imperfect substitutes. The profit of the representative firm is given by

\[ \Pi_t^F = Y_t - w_t L_t - \int_0^1 p_{it} x_{it} \, di, \]

(2)

where \( p_{it} \) denotes the price of intermediate good \( i \) and where the price of the final good is normalized to unity. Optimization yields the standard result that each input factor is paid according to its marginal product

\[ w_t = (1 - \alpha) \frac{Y_t}{L_t} \]

(3)

\[ p_{it} = \alpha x_{it}^{\alpha-1} L_t^{1-\alpha}. \]

(4)

### 3.2 Household sector

Each household has preferences over consumption and maximizes discounted expected lifetime utility

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(c_{it}). \]

(5)

\( E_0 \) is the expectation operator conditional on information at date \( t = 0 \), and \( 0 < \beta < 1 \) is the discount factor. Preferences regarding momentary consumption are standard and display constant relative risk aversion

\[ U(c) = \begin{cases} 
\frac{c^{1-\rho} - \rho}{1-\rho} & \rho > 0, \ \rho \neq 1 \\
\ln(c) & \rho = 1.
\end{cases} \]

(6)

The risky technology available to produce the intermediate good is given by

\[ x_{it} = \theta_{it} k_{it}, \]

(7)

where \( k_{it} \) denotes the capital stock and \( \theta_{it} \) denotes the entrepreneur’s stochastic productivity. \( \theta_{it} \) is assumed to be uncorrelated across agents but may be correlated over time. The household’s budget constraint is given by

\[ k_{it+1} + b_{it+1} + c_{it} = p_{it} x_{it} + (1 - \delta) k_{it} + R_t b_{it} + w_t, \]

(8)

where \( b_{it+1} \) denotes investment in the safe asset, \( R_t \equiv (1 + r_t) \) is the gross riskless interest rate, \( w_t \) is the wage rate, and \( p_{it} x_{it} \) describes the income part from selling the intermediate good at chosen price \( p_{it} \). The monopolistic optimization problem the household solves in each period is simple in this case since the capital stock is already installed at the beginning of period \( t \). Consequently, the amount of the intermediate good produced and sold to the
The representation in (9) indicates that the household essentially solves a consumption/savings problem as well as a portfolio choice problem between a riskless asset and a risky asset. To see the latter part more clearly, note that we can decompose the household’s income part from production, $\alpha \bar{L}_t \tilde{\theta}_t k_{it}^\alpha$, into its two components capital income and profits. This separation follows from the fact that capital is the crucial input factor in production and that profits arise due to the monopolistic structure in the intermediate goods sector. Capital income is given by $r_{it}^c k_{it}$ where the net return, $r_{it}^c$, measures the contribution of an additional marginal unit of capital, i.e. $r_{it}^c \equiv \alpha^2 \bar{L}_t \tilde{\theta}_t k_{it}^\alpha$. The net return will show up in the Euler equation for capital holdings in period $t+1$ and is risky because it depends on the entrepreneur’s productivity which is subject to idiosyncratic shocks. The residual term are profits that follow from the monopolistic structure in the intermediate goods sector. Profits are given by $\pi_{it} \equiv (1-\alpha) \alpha \bar{L}_t \tilde{\theta}_t k_{it}^\alpha$ and shrink to zero if intermediate goods become perfect substitutes which can be seen from the fact that $\pi_{it} \rightarrow 0$ if $\alpha \rightarrow 1$. Furthermore, profits are risky as well since they also depend on the entrepreneur’s stochastic productivity. Using this separation of capital income and profits, the household’s budget constraint can finally be written as

$$k_{it+1} + b_{it+1} + c_{it} = \alpha \bar{L}_t \tilde{\theta}_t k_{it}^\alpha + (1-\delta)k_{it} + R_t b_{it} + w_t,$$

(10)

where $R_t^c = (1 - \delta + r_{it}^c)$ is the gross return of capital. The representation in (10) highlights that the household essentially solves a portfolio choice problem between a riskless asset (bond) and a risky asset (capital). Furthermore, it shows that, in terms of Sandmo (1970), entrepreneurs do not only face capital risk but also income risk where the latter is induced by the existence of risky profits. Clearly, capital and income risk in our model are not independent because both, the risky return and profits depend on the same stochastic process. However, this separation plays an important role later on since only capital risk in addition to the borrowing constraint also affects the household’s investment decisions, whereas income risk only leads to precautionary saving. This will become more clear from the household’s first-order conditions.

Let the household’s period $t$ net worth, $\omega_{it}$, be defined as $\omega_{it} \equiv R_t^c k_{it} + \pi_{it} + R_t b_{it} + w_t$. Furthermore, let $V_t(\omega_t, \tilde{\theta}_t)$ be the associated optimal value function. Then, the single household’s optimization problem can be specified in terms of the following program\(^{10}\)

$$V_t(\omega_t, \tilde{\theta}_t) = \max_{c_t, b_{it+1}, k_{it+1}} \left\{ U(c_t) + \beta E \left[ V_{t+1}(\omega_{t+1}, \tilde{\theta}_{t+1}) | \tilde{\theta}_t \right] \right\}$$

(11)

\(^{10}\)The subscript $i$ is dropped in this definition for notational ease. Note that the time subscript attached to the value function indicates that the household’s program is not only defined at steady states.
s.t. \[ c_t + b_{t+1} + k_{t+1} = \omega_t \] \hspace{1cm} (12)
\[ k_{t+1} \geq 0 \] \hspace{1cm} (13)
\[ b_{t+1} \geq -\tilde{b}. \] \hspace{1cm} (14)

The constraint in (14) is the borrowing constraint the household faces on the safe asset. The tightness of the borrowing constraint is determined by the debt limit, \( \tilde{b} \). A lower value of \( \tilde{b} \) means a lower amount the household can borrow to either smooth consumption or to scale up individual production.

The first-order conditions of the individual problem are given by

\[ U'(c_t) = \beta R_{t+1} E_t \left[ U'(c_{t+1}) \right] + \lambda_t \] \hspace{1cm} (15)
\[ U'(c_t) = \beta E_t \left[ R_{t+1} U'(c_{t+1}) \right], \] \hspace{1cm} (16)

where \( \lambda_t \) is the nonnegative Lagrange multiplier associated with the borrowing constraint and where \( R_{t+1} = 1 - \delta + \alpha^2 \tilde{L}_{t+1} \tilde{\theta}_{t+1} k_{t+1}^{\alpha-1} \) is the gross return of capital in period \( t+1 \). Combining the two equations yields

\[ E_t R_{t+1} - R_{t+1} = \frac{\text{Cov}(U'(c_{t+1}), R_{t+1}^c)}{E_t U'(c_{t+1})} + \frac{\lambda_t}{\beta E_t U'(c_{t+1})}. \] \hspace{1cm} (17)

Equation (17) shows that the presence of uninsurable capital risk and potentially binding borrowing constraints drives a wedge between the expected return of capital and the riskless interest rate. The first term on the right-hand side describes the risk premium the household demands for bearing the uninsurable capital risk. Since \( \text{Cov}(U'(c_{t+1}), R_{t+1}^c) \) is negative, this expression is positive. The second term on the right-hand side additionally appears if the borrowing constraint binds in period \( t \). Since \( \lambda_t \) is nonnegative, both terms positively contribute to the wedge and will play a key role in the further analysis.

Definition 1 below summarizes the equilibrium under financial autarky and financial integration from the perspective of the small economy. Under financial autarky, the bond market has to clear on the country-wide level, whereas under financial integration bonds can be traded on the international level. Note that the time index indicates that the equilibrium is not only defined at steady states where aggregate prices are constant over time, but also takes account of transitory dynamics.

**Definition 1** Given the initial distribution of households, \( \Psi_0(\omega, \tilde{\theta}) \), a general competitive equilibrium under financial autarky is defined by

a) a sequence of policy functions \( \{c_t(\omega, \tilde{\theta}), k_{t+1}(\omega, \tilde{\theta}), b_{t+1}(\omega, \tilde{\theta})\}_{t=0}^{\infty} \), b) a sequence of value functions \( \{V_t(\omega, \tilde{\theta})\}_{t=0}^{\infty} \), c) a sequence of prices \( \{R_t, w_t, p_t(i)\}_{t=0}^{\infty} \), and d) a sequence of distributions \( \{\Psi_t(\omega, \tilde{\theta})\}_{t=1}^{\infty} \) such that, for all \( t \)

1. The policy functions described above solve the household’s decision problem.
2. Intermediate goods and labor are paid according to their marginal product.
3. Aggregate quantities of consumption, capital, labor and bonds are the aggregation of individual quantities. For given prices markets clear, especially $B_t = 0$ and $L_t = 1$.

4. The sequence of distributions is consistent with the initial distribution, the policy functions and the stochastic process for productivity.

A competitive equilibrium under financial integration is defined in a similar fashion. However, bonds can be traded on the international level given the world interest rate $R^*$. $B_t$ then represents the net foreign asset position of the small economy.

4 Parametrization

In this section, we describe the benchmark parametrization, identify the financial parameters of the model and explain the differences between the three scenarios. In total, we have to assign seven parameter values, $\{\alpha, \beta, \rho, \delta, \overline{\theta}, \rho_\theta, \sigma\}$. We mainly choose standard values that are commonly considered in the literature.

In accordance with our discussion in the previous section, $\sigma$ and $\overline{\theta}$ are the formalization of a country’s level of financial development. $\sigma$ measures the level of uninsurable risk and is defined as the standard deviation of $\ln(\tilde{\theta})$. We directly target the properties of $\tilde{\theta} = \theta^\alpha$ since it shows up as the relevant term in the household’s budget constraint. As generally shown by Angeletos and Calvet (2006), a higher value of $\sigma$ means a higher portion of risk that cannot be insured through financial markets and thus remains with the entrepreneurs. $\overline{\theta}$, the debt limit, defines the tightness of the borrowing constraint. A more tight borrowing constraint means a stronger impediment for entrepreneurs to either smooth consumption or to scale up individual production. A higher amount of risk remaining with the entrepreneurs and/or a more tight borrowing constraint means a lower level of financial development.

In each scenario, we consider the same values of $\{\alpha, \beta, \rho, \delta, \sigma\}$ for our small and financially less developed benchmark economy. The parameter values are standard and commonly considered in the literature. The discount factor, $\beta$, is set to 0.96 and $\alpha$ is set to 0.4, implying a labor income share of 0.6. The elasticity of intertemporal substitution, $\vartheta = 1/\rho$, is set to 2/3 which means that the parameter of relative risk aversion, $\rho$, equals 1.5. The depreciation rate, $\delta$, is set to a standard value of 0.08. In general, the productivity process is first-order Markov and defined as

$$\ln \theta_{t+1} = -\frac{\alpha \sigma^2}{1 + \rho_\theta} + \rho_\theta \ln \theta_t + \epsilon_{t+1}, \quad \epsilon \sim N(0, \sigma^2_\epsilon),$$

where $\rho_\theta$ is the serial correlation parameter and where the specification of the constant term in (18) leads to the normalization $E(\tilde{\theta}) = 1$. $\sigma^2_\epsilon$ is adjusted accordingly to ensure that

$^{11}$Note that the interest rate under financial integration is determined by the large and financially more developed country that represents the rest of the world. Hence, interest rate differentials between countries under financial autarky are endogenously explained by differences in financial development.

$^{12}$See also Corneli (2009) and Angeletos and Panousi (2011).
σ is equal to 0.4 which is comparable to Covas (2006), Angeletos (2007) and Angeletos and Panousi (2011).

The three scenarios only differ with respect to the values of ρ, θ, and b. In the baseline scenario, we focus on uninsurable risk only, assuming away tight borrowing constraints and persistent effects of shocks. That means, ρ controlling the persistence of shocks is set to zero and b is equal to the Natural Debt Limit (NDL) defined as the maximum amount of repayable debt that is consistent with nonnegative consumption. Differences in financial development between countries in the baseline scenario are solely captured by differences in the level of uninsurable risk. In the second scenario, we increase the tightness of the borrowing constraint and in the third scenario, we increase the persistence of shocks. Table 1 below summarizes the benchmark parameter values that are equal in all three scenarios.

Table 1: Benchmark Parameter Values

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<th>Parameter</th>
<th>Value</th>
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<td>discount factor</td>
<td>β</td>
</tr>
<tr>
<td>curvature of production (final good sector)</td>
<td>α</td>
</tr>
<tr>
<td>depreciation rate</td>
<td>δ</td>
</tr>
<tr>
<td>elasticity of intertemporal substitution</td>
<td>ϑ = 1/ρ</td>
</tr>
<tr>
<td>standard deviation of ln(θ)</td>
<td>σ</td>
</tr>
</tbody>
</table>

5 Results

5.1 Overview

In this section, we describe our three scenarios. In the baseline scenario, we focus on uninsurable risk, in the second scenario we increase the tightness of the borrowing constraint, and in the third scenario we additionally allow for persistent effects of shocks. Apart from these differences, however, the focus remains the same and relates to the questions of interest, i.e. explaining the direction of international capital flows and accompanying changes in domestic economic development. In particular, we are interested in two main features a model should have in order to contribute to explain the empirical pattern. First, the financially less developed country should not only display a lower interest rate in the initial equilibrium under financial autarky, but also lower levels of capital and output, i.e. it should also be the economically less developed country. This result ensures that when financial integration takes place, it is in fact the initially poor country that builds up a positive net foreign asset position, which explains the Lucas paradox. Second, the increase

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13Since x_{it} = θ_{it}k_{it} is close to zero for bad realizations of θ_{it}, we define the NDL in steady state as NDL ≡ w/(R − 1).
in the interest rate from the perspective of the financially less developed country should lead to a higher aggregate capital stock and a higher output level in the steady state under financial integration rather than to lower levels as predicted by standard theory. This result ensures that, at least from a steady state comparison, we observe a positive correlation between economic growth, higher interest rates and capital outflows. We will analyze under which conditions the presence of uninsurable risk and borrowing constraints contributes to explain these findings and how the conditions may change with different model assumptions. In the baseline scenario, where our model closely relates to Angeletos and Panousi (2011), we derive two rules of thumb that describe the parameter restrictions with high accuracy.

5.2 Scenario 1: Uninsurable risk

Figure 1 shows the main results for our baseline scenario. The solid blue lines in Figure 1 show the long-run relationship between selected macroeconomic variables and the interest rate for our small and financially less developed benchmark economy. The steady state under financial autarky, where the bond market has to clear on the country-wide level, is indicated by point A and the steady state under financial integration, where bonds can be traded on the international level, is indicated by point I. The horizontal and vertical dashed lines in Figure 1 indicate the autarchic steady state of the large and financially more developed country that represents the rest of the world. Entrepreneurs in the financially more developed country face a lower level of uninsurable risk (\( \sigma = 0.2 \)) which reflects the superior insurance opportunities provided by the financial sector. Since the financially more developed country is assumed to be sufficiently large, it determines the interest rate under financial integration (\( R^{Int} = 1.04163 \)).

Inspecting Figure 1 leads to a number of interesting results. First, Figure 1 shows that an equilibrium like point A is exactly the starting point that is needed in order to explain the empirical findings. Point A means that our financially less developed benchmark economy does not only display a lower interest rate in the initial equilibrium under financial autarky, but also lower levels of the domestic capital stock and output, i.e. it is also the economically less developed country. This can be seen from comparing the position of point A (autarchic steady state financially less developed country) with the positions of the horizontal and vertical dashed lines (autarchic steady state financially more developed country). The lower interest rate in the financially less developed country can be traced back to the higher level of risk that entrepreneurs have to bear. The higher level of risk leads to a higher precautionary saving demand for the riskless asset which forces the interest rate to fall in order to clear the bond market under financial autarky. This risk-induced saving effect is well-known in the literature and occurs under both, capital and income risk (see Aiyagari 1994; Angeletos 2007). However, the lower capital stock and the lower output level observed in the financially less developed country crucially depend on the existence of capital risk (see Angeletos 2007). As Equation (17) in Section 3 shows, capital risk drives a wedge between the expected return of the risky asset and
Figure 1: Main Results Baseline Scenario

Note: Point A shows the autarchic steady state of the financially less developed benchmark economy and point I shows the steady state under financial integration. The horizontal and vertical dashed lines show the autarchic steady state of the large and financially more developed country representing the rest of the world. Point A’ and A” show two possible alternative equilibria for the financially less developed country that, in a similar fashion, are obtained under different parametrizations.

the riskless interest rate since the entrepreneurs demand a risk premium if full insurance is not provided. This wedge makes it possible to observe a lower interest rate and a lower capital stock in the financially less developed country. This result is exactly what is needed in order to explain the Lucas paradox, i.e. the fact that it is the initially poor country that accumulates a positive net foreign asset position so that capital flows from the poor to the rich country under financial integration. To see this more clearly, consider a financial market liberalization reform that removes the trading barriers between the two countries after both countries have reached the autarchic steady state. The financially less developed country will converge to its new steady state under financial integration that is indicated by point I in Figure 1. Inspecting the position of point I in the lower panel shows that the financially less developed country features a positive net foreign asset position in the integrated steady state, implying that capital flows from the less to the financially more developed country under financial integration. This result is driven

\[\text{The general pattern of transitory dynamics presented in Section 6 shows that the evolution of the net}\]
by the fact that the interest rate increases from the perspective of the financially less
developed country. The interest rate increases because it is determined by the financially
more developed country that displays a higher interest rate in the autarchic steady state
due to the lower level of risk. Consequently, since capital flows from the financially
less developed to the financially more developed country under financial integration, the
financially less developed country has to be the initially poor country in order to explain
that capital does indeed flow from the poor to the rich country. That is exactly the case
in the initial equilibrium indicated by point A.

The second important property that can be inferred from point A refers to the conse-
quences of financial integration for domestic economic development. Although the riskless
bond is the only asset that is traded on the international level, the change in the interest
rate also affects the domestic capital stock and output. According to standard theory,
the increase in the interest rate from the perspective of the financially less developed
country should lead to a lower capital stock and a lower output level in the integrated
steady state. This follows from the usual opportunity-cost effect stating that investing
in one asset becomes less attractive if the return of the other asset increases. However,
the upper panels in Figure 1 show that the financially less developed country displays
a higher capital stock and a higher output level in the integrated steady state (point I)
compared to the autarchic steady state (point A). This can be seen from the fact that
point A is located on the increasing part of the long-run capital and output functions,
i.e. on the increasing part of the blue lines. The result that the capital stock and output
may increase with the interest rate is driven by a second effect that exists in the presence
of capital risk and that relates to the agents’ willingness to take risk (see Angeletos and
Panousi 2011). Due to diminishing absolute risk aversion, entrepreneurs are willing to
increase investment in the risky asset, i.e. to build up the capital stock, when they become
richer. Since entrepreneurs become richer under financial integration by building up their
positive net foreign asset position, the wealth effect stimulates capital accumulation when
the interest rate increases above its autarchic steady state level. Though the accumulation
of wealth is a gradual process, which means that the capital stock may initially fall during
the transition, the wealth effect may finally dominate the opportunity-cost effect so that
the capital stock and output are higher in the integrated steady state. That is exactly the
case when starting from an equilibrium like point A.

In summary, the effects of uninsurable capital risk that are described by Angeletos
and Panousi (2011) may also be preserved in the presence of income risk that is induced
by the existence of risky profits in our model. However, as indicated by point A’ and A”

\begin{itemize}
  \item foreign asset position towards the integrated steady state is a gradual and monotone process. In particular,
        in all exercises considered, we find that the financially less developed country facing an increase in the
        interest rate under financial integration runs a persistent series of current account surpluses along the
        transition path. See also Angeletos and Panousi (2011).
  \item in a two-country framework with similar weight on each country, one would expect that the common
        interest rate under financial integration settles at a level between both autarchic steady state interest rates.
        However, qualitatively, the effect that the interest rate increases from the perspective of the financially less
        developed country remains the same as in our exercise.
  \item See Section 6 for a discussion of the transitory dynamics.
\end{itemize}
in Figure 1, the model predictions in the baseline scenario may also be quite different. First, assume that the autarchic steady state of the financially less developed country is given by point A’ rather than by point A. We will show in the next section that such an equilibrium exists under different parametrizations.\footnote{Note that the blue lines in Figure 1 themselves change with different parameter values. However, the main characteristics we refer to, i.e. the U-shaped form and the fact that the blue line may lie below the horizontal dashed line, are preserved.} Point A’ in the upper panels in Figure 1 means that a small increase in the interest rate leads to a lower long-run capital stock and a lower long-run output level, because in the close neighborhood of point A’, the opportunity-cost effect dominates the wealth effect. Consequently, an equilibrium like A’ means that the model fails to explain a boost in long-run domestic economic development from the perspective of the financially less developed country. Furthermore, point A” shows that the model may even fail to explain the Lucas paradox. This follows from the fact that in point A”, the financially less developed country is the initially rich country under financial autarky which means that capital flows from the rich to the poor country under financial integration.

Given these opposing outcomes, how can we find the conditions under which the model is capable of contributing to explaining the empirical findings and under which the model may fail? In principle, this is a cumbersome task since the model has no closed-form solution and numerous simulations have to be conducted. However, we partly overcome this problem by deriving two rules of thumb that explain the required parameter restrictions with high accuracy.\footnote{We refer to our rules as rules of thumb since their derivation is partly based on a model comparison. See Appendix A for details.} We will present the two rules in the next section.

5.2.1 Two rules of thumb

The first rule of thumb relates to the Lucas paradox. The rule describes the condition guaranteeing that the financially less developed country is also the initially poor country in the autarchic steady state so that it is in fact the initially poor country that accumulates a positive net foreign asset position under financial integration. According to Figure 1, the first rule may lead to an equilibrium like point A but does not yet rule out an equilibrium like point A’. Therefore, we derive a second rule that describes the condition guaranteeing that the long-run domestic capital stock and long-run output necessarily increase with the interest rate. If this condition is satisfied, then the financially less developed country is not only the poor country in the autarchic steady state, but an increase in the interest rate implied by financial integration also leads to a higher capital stock and higher output in the integrated steady state. In order to derive our first rule, we assume that entrepreneurs in the financially more developed country can completely insure against idiosyncratic risk, i.e. markets in the financially more developed country are assumed to be complete. However, as shown in Appendix B, the complete markets assumption is not restrictive in this case so that both rules can also applied to the general case where entrepreneurs in both countries suffer from incomplete markets as in Figure 1. In short, our two rules can be stated as follows.
Rule of Thumb 1 In the autarchic steady state, levels of the aggregate capital stock and output are lower in the economy with incomplete markets than in the case of complete markets if and only if
\[ \vartheta > \frac{\hat{\phi}}{2 - \hat{\phi}}, \quad \hat{\phi} \equiv \frac{\alpha - \delta \frac{K^*}{Y^*} \alpha Y^*}{1 - \delta \frac{K^*}{Y^*}}, \quad (19) \]
where production is approximated by \( Y^* = (K^*)^\alpha \).

Rule of Thumb 2 Based on the autarchic steady state of the economy with incomplete markets, the long-run capital stock and long-run output necessarily increase with any increase of the interest rate if and only if
\[ \vartheta > \frac{\hat{\phi}}{1 - \hat{\phi}}, \quad \hat{\phi} \equiv \frac{\alpha - \delta \frac{K^*}{Y^*}}{1 - \delta \frac{K^*}{Y^*}}, \quad (20) \]
where production is approximated by \( Y^* = (K^*)^\alpha \).

In both rules, \( K^* \) denotes the aggregate capital stock and \( Y^* \) denotes aggregate output, and both are evaluated at the autarchic steady state. We provide a detailed description of how to derive the two rules in Appendix A. The general idea is to start with a simpler two-period model version and to use the results provided by Angeletos (2007) and Angeletos and Panousi (2011) in order to understand how the condition changes between the two-period model and the infinite-horizon model.

What does the first rule of thumb show us? The first rule shows that the structural parameters of the model have to satisfy a certain condition in order to guarantee that the financially less developed country is also the initially poor country in the autarchic steady state. More specifically, the first rule shows that the elasticity of intertemporal substitution, \( \vartheta \), has to exceed a certain threshold level. The fact that the elasticity of intertemporal substitution is a key parameter of the model is well-known from Ak-type models that capture the effects of uninsurable capital risk.\(^{19}\) Intuitively, the presence of capital risk leads to a lower risk-adjusted return and agents’ response to this change crucially depends on their attitude towards intertemporal substitution. In fact, our model comes close to an Ak-model if \( \alpha \) is close to unity and our first rule of thumb confirms the well-known result that the elasticity of intertemporal substitution has to be greater than unity in order to ensure that a lower level of financial development also leads to a lower level of economic development (c.f., Weil 1990; Obstfeld 1994). However, if \( \alpha \) is less than unity, there exist two additional effects in our model. First, entrepreneurs earn riskless wage income and second, they also face income risk due to the existence of risky profits. The income risk tends to tighten the parameter restrictions by increasing the precautionary demand for saving. In contrast, the riskless wage income tends to loosen the parameter restrictions by reducing the percentage drop of consumption in times when

\(^{19}\)The fact that the elasticity of intertemporal substitution rather than the parameter of relative risk aversion is the final key parameter is also discussed intensively by Angeletos (2007) and Angeletos and Panousi (2011).
the risky asset pays off poorly, thereby weakening the need for precautionary savings. In order to assess the overall effect, and to evaluate the performance of our first rule, we can compute the final threshold level for the elasticity of intertemporal substitution.\textsuperscript{20} Table 2 shows the results for our financially less developed benchmark economy. The numbers in the second column are the predicted threshold levels that are derived from our first rule and the numbers in the third column are the actual threshold levels that are derived from simulations. The numbers in parentheses indicate the prediction errors.

Table 2: Under-accumulation of Capital: Threshold Levels

<table>
<thead>
<tr>
<th>Depreciation rate</th>
<th>First rule of thumb</th>
<th>Model solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0.244</td>
<td>0.238</td>
</tr>
<tr>
<td>0.04</td>
<td>0.125</td>
<td>0.122</td>
</tr>
<tr>
<td>0.08</td>
<td>0.083</td>
<td>0.076</td>
</tr>
</tbody>
</table>

Table 2 shows two main results. First, the second column shows that our first rule is able to predict the actual threshold levels with high accuracy. This follows from the fact that the prediction errors that are reported in the second column are rather small. Further simulations suggest that this result is also robust to changes in $\alpha$ and in the discount factor $\beta$. Second, and equally important, the third column shows that the threshold level for the elasticity of intertemporal substitution is very low for plausible values of the depreciation rate, which is generally in line with Angeletos (2007). For $\delta = 0.08$, a value of the elasticity of intertemporal substitution slightly larger than 0.076 is already sufficient to ensure that the financially less developed country is the initially poor country in the autarchic steady state and that capital flows from the poor to the rich country under financial integration.\textsuperscript{21} This means that even in the presence of income risk, the combination of uninsurable capital risk and riskless wage income explains the Lucas paradox quite well. Introducing capital risk makes it possible to observe a lower capital stock and a lower interest rate in the financially less developed country and the riskless wage income ensures that the restrictions on the structural parameters are quite loose. Even if we reduce the share of the riskless wage income to 0.4, the threshold level for the elasticity of intertemporal substitution does not exceed 0.15 in case the depreciation rate equals 8 percent. Since large parts of the empirical literature suggest a value of the elasticity of intertemporal substitution close to unity (see Angeletos 2007), the parameter restrictions remain very

\textsuperscript{20}According to our first rule, we compute the values of $\vartheta$ satisfying $\vartheta = \hat{\phi}/(2 - \hat{\phi})$, i.e. condition (19) holds with equality. For these critical values, the capital stock should be exactly the same under complete and incomplete markets. Larger values of $\vartheta$ then lead to $\vartheta > \hat{\phi}/(2 - \hat{\phi})$, such that, according to our first rule, the capital stock and output should be lower in the economy suffering from incomplete markets.

\textsuperscript{21}In terms of the parameter of relative risk aversion, $\rho$, we obtain the restriction $\rho < 13$. 

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Table 3: Long-run Effects Capital and Output: Threshold Levels

<table>
<thead>
<tr>
<th>Depreciation rate</th>
<th>Second rule of thumb</th>
<th>Model solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0.645</td>
<td>0.556</td>
</tr>
<tr>
<td></td>
<td>(0.089)</td>
<td></td>
</tr>
<tr>
<td>0.04</td>
<td>0.287</td>
<td>0.286</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>0.08</td>
<td>0.184</td>
<td>0.182</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td></td>
</tr>
</tbody>
</table>

Next, we consider how the restrictions on the elasticity of intertemporal substitution change if we additionally demand that the long-run capital stock and long-run output necessarily increase with the interest rate. If this condition holds, then the financially less developed country is not only the poor country in the autarchic steady state but the increase in the interest rate implied by financial integration also leads to a higher capital stock and a higher output level in the integrated steady state. Inspecting the second rule in (20) shows that the threshold level for the elasticity of intertemporal substitution will most probably increase compared to our first rule. This result is intuitive since the second condition rules out an equilibrium like point A’ in Figure 1 which is more restrictive than what is required under our first rule. In order to assess the overall effect, we compute the corresponding threshold levels for the elasticity of intertemporal substitution that are predicted by our second rule and that are obtained from simulations. The results appear in Table 3 and numbers in parentheses indicate the prediction errors.

First, we observe that the prediction errors reported in the second column are again rather small. Consequently, also our second rule is able to describe the properties of the model with high accuracy. Second, the numbers in the third column confirm the conjecture that the threshold level is higher compared to our first rule. For \( \delta = 0.08 \), the threshold level more than doubles from 0.08 to 0.18. However, since the threshold level is still below 0.2, and large parts of the empirical literature suggest a value of the elasticity of intertemporal substitution close to unity, we can conclude that in the baseline scenario, an equilibrium like point A in Figure 1 is in fact the most likely outcome for a broad range of plausible parameter values. Nevertheless, it is important to know that the empirical estimates of the elasticity of intertemporal substitution are far from being uniform. Dacy and Hasanov (2011), for example, find a much smaller value of the elasticity of intertemporal substitution of around 0.2. Their finding raises the issue of how the parameter restrictions may change when tight borrowing constraints and persistent effects of shocks are taken into account. We address these issues in the next sections.
5.3 Scenario 2: Borrowing constraints

In the second scenario, we relax the assumption that entrepreneurs may in principle borrow up to the Natural Debt Limit (NDL) and consider more tight borrowing constraints. As it can already be inferred from the first-order conditions derived in Section 3, borrowing constraints affect entrepreneurs in two different ways. Since, in bad times, agents need to borrow to finance both, consumption and the capital stock of the individual firm, borrowing constraints impede consumption smoothing and limit the access to external funds for scaling up individual production. Since the corresponding effects tend to work in opposite directions as discussed below, the ultimate effect of borrowing constraints is generally ambiguous and borrowing constraints may either tighten or weaken the parameter restrictions compared to the baseline scenario.

Figure 2: Aggregate Results: Borrowing Constraints

![Graphs showing long-run relationships for capital stock, output, and NFA position with interest rate on the x-axis.]

Note: Point A shows the autarchic steady state of the financially less developed benchmark economy and point I shows the steady state under financial integration. Compared to the baseline scenario, entrepreneurs in the benchmark economy considered here face a borrowing limit of $b = 1.2$. The horizontal and vertical dashed lines show the autarchic steady state of the large and financially more developed country that represents the rest of the world. The financially more developed country is identical to the baseline scenario.

Figure 2 provides first insight into the effects of borrowing constraints. Similar to Figure 1, the blue lines show the long-run relationship between selected macroeconomic
variables and the interest rate for the financially less developed benchmark economy. The steady state under financial autarky is indicated by point A and the steady state under financial integration is indicated by point I. However, different from the baseline scenario, entrepreneurs in the benchmark economy considered here face a borrowing limit of $\delta = 1.2$, which roughly corresponds to the average net income in the steady state under financial autarky.\footnote{Huggett (1993) suggests a debt limit of one year’s average endowment to be reasonable.}

At first view, Figure 2 shows that the main characteristics of the baseline scenario carry over to the case where borrowing constraints play a significant role. In particular, we observe that it is still possible to obtain an equilibrium like point A in Figure 2 that leads to the same implications as in the baseline scenario, i.e. of capital flowing from poor to rich countries and a boost in long-run domestic economic development despite an increase of the interest rate. However, Figure 2 also shows that there are some differences compared to the baseline scenario that reflect the saving and investment effect of borrowing constraints. We investigate these differences in the following.

The saving effect of borrowing constraints refers to the households’ demand for the riskless asset and can be inferred from the lower panel in Figure 2. In particular, by comparing the position of point A with the corresponding position in the baseline scenario, we find that the autarchic steady state interest rate in our financially less developed benchmark economy decreases with tight borrowing constraints. Though the difference is quantitatively moderate, the lower interest rate indicates that tighter borrowing constraints lead to a higher aggregate demand for the riskless asset.\footnote{In the baseline scenario, the autarchic steady state interest rate in the financially less developed benchmark economy is equal to 4.15%. If the debt limit is equal to $\delta = 1.2$, the interest rate decreases to 4.04%.} The higher aggregate demand for the riskless asset can be explained by two effects of borrowing constraints on the individual level. In a mechanical way, borrowing constraints reduce the supply of the riskless asset on the individual level by limiting the amount households can borrow. Furthermore, households who are currently not constrained but whose level of net worth is rather low save more compared to the case where only the NDL is imposed. This higher individual demand for the riskless asset mainly follows from a stronger necessity to self-insure against the idiosyncratic risk. Since both, the lower supply and the higher demand on the individual level lead to an increased demand for the riskless asset on the aggregate level, the interest rate has to decrease with tighter borrowing constraints in order to clear the bond market under financial autarky. The observed saving effect of borrowing constraints is similar to standard incomplete-markets models in which tight borrowing constraints as well lead to a lower interest rate in the autarchic steady state (e.g., Huggett 1993). Furthermore, if a lower interest rate is the only effect of borrowing constraints, we can expect that tighter borrowing constraints lead to tighter parameter restrictions compared to the baseline scenario. To see this more clearly, note that a lower interest rate level pushes the location of the corresponding autarchic steady state capital stock to the left on the long-run capital function, i.e. to the left on the blue line in the upper left panel.
of Figure 2. Consequently, it becomes less likely that the financially less developed country ends up being the poor country under financial autarky, which is crucial to explain that it is the poor country accumulating a positive net foreign asset position under financial integration. Moreover, it becomes also less likely that the autarchic steady state capital stock of the financially less developed country is located on the increasing part of the long-run capital function so that financial integration necessarily leads to a boost in long-run domestic economic development. Hence, if the only effect of borrowing constraints is to lead to a lower interest rate, we can expect that tighter borrowing constraints lead to tighter parameter restrictions in order to maintain an equilibrium like point A. However, in our case, borrowing constraints also affect entrepreneurs’ investment decisions, which changes the relation between aggregate capital accumulation and the interest rate. We analyze this effect in the following.

One of the main changes in the relation between aggregate capital accumulation and the interest rate that is implied by borrowing constraints can be seen with the help of the following example. In the baseline scenario, the capital stock in the financially less developed country reaches the same level as in the financially more developed country if the interest rate is equal to 3.68%. However, if we introduce tight borrowing constraints, the financially less developed country remains the poor country until the interest rate decreases by more than 8%. This can be seen in Figure 2 by the intersection point of the blue line and the dashed horizontal line. Consequently, we observe that tight borrowing constraints dampen aggregate capital accumulation so that a lower interest rate under tight borrowing constraints does not necessarily mean a higher aggregate capital stock. From this we can conclude that the investment effect of borrowing constraints weakens or even outweighs the saving effect of borrowing constraints: Even if the interest rate is lower in the steady state under financial autarky, the financially less developed country may still be the poor country under financial autarky and the capital stock may still be located on the increasing part of the long-run capital function. In other words, borrowing constraints do not necessarily lead to tighter parameter restrictions compared to the baseline scenario since the saving and investment effect work in opposite directions. The reason behind this opposing effect of capital accumulation can be again most clearly seen on the individual level. Since borrowing constraints do not only affect consumption smoothing, but also limit the options to scale up individual production, entrepreneurs who are currently constrained invest less compared to the case where only the NDL is imposed. Furthermore, entrepreneurs who are currently not constrained but whose level of net worth is rather low, choose a lower capital stock as well. Intuitively, the riskless asset is more suitable than the risky asset to transfer resources to those states where the borrowing constraint may become binding and entrepreneurs are willing to adjust their portfolio in favor of the riskless asset.24 Consequently, investment levels are lower in case of tight borrowing constraints, either because borrowing constraints are binding today or are expected to become binding in the near future.

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24 Angeletos and Panousi (2011) also briefly mention this effect of borrowing constraints, but borrowing constraints do not play any role in their theoretical model.
In sum, what can be inferred from Figure 2 are two opposing effects of borrowing constraints. On the one hand, borrowing constraints lead to a lower interest rate in the autarchic steady state compared to the case where only the NDL is imposed. On the other hand, borrowing constraints also affect investment and a lower interest rate does not necessarily mean a relatively higher capital stock. Therefore, the overall effect of borrowing constraints is generally ambiguous and borrowing constraints may either tighten or weaken the parameter restrictions compared to the baseline scenario. In order to assess the overall effect, we consider several debt limits and compare the corresponding threshold levels for the elasticity of intertemporal substitution.

Table 4: Under-accumulation of Capital - Borrowing Constraints

<table>
<thead>
<tr>
<th>Debt Limit</th>
<th>Threshold Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDL</td>
<td>0.076</td>
</tr>
<tr>
<td>twice average net income</td>
<td>0.111</td>
</tr>
<tr>
<td>average net income</td>
<td>0.154</td>
</tr>
<tr>
<td>20 percent of average net income</td>
<td>0.417</td>
</tr>
</tbody>
</table>

Table 4 shows the results that again refer to the Lucas paradox. Specifically, Table 4 shows the threshold levels the elasticity of intertemporal substitution has to exceed in order to ensure that the financially less developed benchmark economy is also the poor economy in the autarchic steady state compared to the financially more developed economy where markets are assumed to be complete. The threshold level in the first row of Table 4 is known from the baseline scenario, while the other threshold levels refer to more tight borrowing constraints. The tightest borrowing constraint in Table 4 means that agents can only borrow up to approximately 20 percent of the average net income in the autarchic steady state.

Though the discussion above has shown that the overall effect of borrowing constraints is generally ambiguous, Table 4 shows that the threshold level monotonically increases with the tightness of the borrowing constraint. That means, we find that the saving effect of borrowing constraints dominates the investment effect so that tighter borrowing constraints lead to tighter parameter restrictions in order to explain the Lucas paradox.\(^{25}\) Compared to the baseline scenario, the threshold level is twice as large if the debt limit is equal to the average net income in the autarchic steady state and becomes roughly 5 times larger when entrepreneurs can only borrow up to approximately 20 percent of the average net income in the autarchic steady state.\(^{26}\) Similarly, by setting \(\alpha\) to 0.6, we find that the threshold level monotonically increases from 0.147 when only the NDL is imposed to 0.455 when the debt limit corresponds to 20 percent of the average net income in the autarchic steady state. However, are these results just driven by the fact that markets are

\(^{25}\)The observation of tighter parameter restrictions is in line with Covas (2006).

\(^{26}\)If the borrowing limit is approximately equal to 10 percent of the average net income in the autarchic steady state, the threshold level is even around 0.8.
assumed to be complete in the financially more developed country?

Figure 3: Threshold Levels, Changes in Risk and Debt Limits

Figure 3 shows that this is not the case and that the same pattern also occurs if entrepreneurs in both countries suffer from uninsurable risk and borrowing constraints. The left panel of Figure 3 shows the relationship between the elasticity of intertemporal substitution and the corresponding autarchic steady state capital stock for several combinations of uninsurable risk and borrowing constraints. The key insight is that the intersection point of any two lines determines the threshold level the elasticity of intertemporal substitution has to exceed in order to ensure that the financially less developed country is also the poor country in the autarchic steady state. For example, Figure 3 shows that the elasticity of intertemporal substitution has to be larger than 0.2 in order to ensure that the financially less developed country represented by the blue line displays a lower capital stock compared to the financially more developed country represented by the red line. Comparing the green and the blue line shows that the corresponding threshold level is even around 0.9. Recall that, in the baseline scenario, a value of the elasticity of intertemporal substitution slightly larger than 0.076 already ensures that the financially less developed country is the poor country in the autarchic steady state so that capital flows from the poor to the rich country under financial integration. Hence, we find that borrowing constraints may lead to significantly tighter parameter restrictions in order to explain the Lucas paradox.

Next, we consider how borrowing constraints affect the parameter restrictions from the baseline scenario if we additionally demand that the long-run domestic capital stock and long-run output necessarily increase with the interest rate. Table 5 shows the corresponding threshold levels for the financially less developed benchmark economy. If the elasticity of intertemporal substitution is larger than the threshold level, the financially less developed benchmark economy is not only the poor economy in the autarchic steady state, but the increase in the interest rate implied by financial integration also necessarily leads to a higher capital stock and a higher output level in the integrated steady state.
Based on the same borrowing limits considered before, we observe that the threshold level monotonically increases with the tightness of the borrowing constraint. In particular, we find that for the tightest debt limit presented in Table 5, the elasticity of intertemporal substitution has to be larger than 2.5, which constitutes a drastic increase compared to the baseline scenario. Hence, borrowing constraints may also lead to significantly tighter parameter restrictions in order to explain higher levels of capital and output in the integrated steady state.\textsuperscript{27}

In summary, our results show that if borrowing constraints do not only affect the production side of the economy, but also affect households through the consumer credit channel, we observe stronger parameter restrictions compared to the baseline scenario. Whether these restrictions may turn out to be too strong depends on the question of interest and the tightness of the borrowing constraint. By exclusively focusing on the Lucas paradox, uninsurable risk and borrowing constraints may still explain capital flows from poor to rich countries as long as the borrowing constraints are not too tight in both countries. For example, the intersection point of the blue line and the red line in Figure 3 shows that the threshold level for the elasticity of intertemporal substitution remains around 0.2. However, as the green line shows, the threshold level easily rises to 0.9 or higher when borrowing constraints become very tight. Turning to the question of higher levels of capital and output in the integrated steady state, we observe a similar effect of borrowing constraints. When only the NDL is imposed, a value of the elasticity of intertemporal substitution slightly larger than 0.182 already ensures that the financially less developed economy necessarily features higher levels of capital and output in the integrated steady state compared to the autarchic steady state. However, Table 5 shows that the threshold level already increases to 0.56 if entrepreneurs face a debt limit approximately equal to the average net income in the autarchic steady state. Tighter debt limits may even require that the elasticity of intertemporal substitution is larger than 2 and, thus, at least twice as large as suggested by main parts of the empirical literature. Hence, we can conclude that especially very tight borrowing constraints strongly affect the parameter restrictions to explain the observed pattern of international capital flows and accompanying changes in domestic economic development. From a slightly different perspective, our results show

\textsuperscript{27}Similarly, if we set $\alpha$ to 0.6, we find that the threshold level monotonically increases from 0.385 when only the NDL is imposed to 2.5 when the debt limit approximately equals 20 percent of the average net income in the autarchic steady state.
that if borrowing constraints are very tight in financially less developed countries, it becomes less likely that these countries experience an increase in capital and output under financial integration. Hence, understanding the effects of borrowing constraints is crucial for understanding the implications of financial integration.

### 5.4 Scenario 3: Persistence of shocks

In the last scenario, we focus on the effects associated with changes in the properties of the underlying productivity process. So far, productivity is described by a simple i.i.d. shock and current levels of the individual return and of profits do not affect entrepreneurs’ expectations regarding future developments. However, though this assumption is widely used for analytical reasons, the empirical literature emphasizes the existence of substantial persistence in labor earnings risk and in business income risk. For example, Storesletten et al. (2004) analyze the properties of labor earnings and find idiosyncratic risk to be highly persistent with an annual autocorrelation coefficient of 0.95. Similarly, DeBacker et al. (2012) find strong persistence in business income risk from privately held businesses.

In order to take account of the empirical facts, we relax the assumption of $\rho = 0$ in the following and allow shocks to have persistent effects. We are especially interested in the question of how a higher level of persistence affects the parameter restrictions from the previous scenarios. Therefore, we consider several levels of the persistence parameter, $\rho$, and compute the corresponding threshold levels for the elasticity of intertemporal substitution. We show the results first and then provide some intuition.

**Table 6: Under-accumulation of Capital - Persistence**

<table>
<thead>
<tr>
<th>Persistence parameter ($\rho_\theta$)</th>
<th>Threshold Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.076</td>
</tr>
<tr>
<td>0.3</td>
<td>0.110</td>
</tr>
<tr>
<td>0.5</td>
<td>0.159</td>
</tr>
<tr>
<td>0.7</td>
<td>0.278</td>
</tr>
</tbody>
</table>

Table 6 shows the results that again refer to the Lucas paradox. Specifically, Table 6 shows the threshold levels the elasticity of intertemporal substitution has to exceed in order to ensure that the financially less developed benchmark economy is also the poor economy in the autarchic steady state compared to the financially more developed country where markets are assumed to be complete. The threshold level in the first row of Table 6 refers to the baseline scenario, while the other threshold levels refer to higher levels of the persistence parameter. The highest level of $\rho_\theta$ is equal to 0.7 and leads to substantial persistence of productivity shocks.

Inspecting the results in Table 6 shows that a higher level of persistence leads to a higher threshold level for the elasticity of intertemporal substitution. In other words, we observe that a higher level of persistence leads to tighter parameter restrictions in order
to explain the Lucas paradox. Compared to the baseline scenario, the threshold level doubles if $\rho_\theta$ is equal to 0.5 and becomes more than three times larger if $\rho_\theta$ is equal to 0.7. Although the increase remains moderate in absolute terms, the strong relative increase of the threshold level indicates a strong impact of persistent effects of shocks.

In order to understand the above results, it is useful to consider the saving effect associated with a higher level of the persistence parameter. Similar to tighter borrowing constraints, a higher level of persistence leads to a higher demand for the riskless asset in the financially less developed benchmark economy. This follows from the fact that a higher level of $\rho_\theta$ means that recovering from bad shocks takes longer and agents try to protect themselves by increasing their financial wealth. However, since the bond market has to clear on the country-wide level under financial autarky, the interest rate decreases in the financially less developed country. The lower interest rate then finally stimulates capital accumulation so that tighter parameter restrictions are needed in order to ensure that the financially less developed country remains the poor country under financial autarky. Hence, similar to the case of tight borrowing constraints, the saving effect of a higher level of persistence leads to tighter parameter restrictions in order to explain the Lucas paradox. Furthermore, the effects become even more pronounced if we consider the joint impact of persistence and borrowing constraints. For example, in the previous scenario, the threshold level for the elasticity of intertemporal substitution is equal to 0.15 if entrepreneurs face a debt limit equal to the average net income in the autarchic steady state. However, if we set $\rho_\theta$ to 0.7, the threshold level more than doubles and equals 0.37. Hence, a higher level of persistence strongly interacts with the effects of financial market imperfections and leads to tight parameter restrictions even at moderate levels of the borrowing constraint. The result of tighter parameter restrictions is also in line with the intuition that the average agent, who does not hold bonds in equilibrium under financial autarky, increases his investment in the risky asset in order to account for the higher precautionary saving demand (see, Covas 2006).

Finally, we address the question of how a higher level of persistence affects the parameter restrictions if we additionally demand that the long-run capital stock and long-run output necessarily increase with the interest rate. If this condition is satisfied, then the financially less developed country is not only the poor country under financial autarky, but the increase in the interest rate implied by financial integration also leads to a higher capital stock and higher output in the integrated steady state.

Table 7: Long-run Effects Capital and Output - Persistence

<table>
<thead>
<tr>
<th>Persistence Parameter ($\rho_\theta$)</th>
<th>Threshold Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.182</td>
</tr>
<tr>
<td>0.3</td>
<td>0.278</td>
</tr>
<tr>
<td>0.5</td>
<td>0.417</td>
</tr>
<tr>
<td>0.7</td>
<td>0.625</td>
</tr>
</tbody>
</table>
Table 7 shows the results for the financially less developed benchmark economy. Based on the same values of $\rho_0$ considered before, we observe that the threshold level for the elasticity of intertemporal substitution increases with the level of persistence. If $\rho_0$ is equal to 0.5, the elasticity of intertemporal substitution has to be larger than 0.4 to ensure that the financially less developed country features a higher capital stock and higher output in the integrated steady state. The threshold level associated with $\rho_0 = 0.7$ is even equal to 0.625 and highlights the strong impact of persistent effects of shocks. Recall that, in the baseline scenario, a value of the elasticity of intertemporal substitution slightly larger than 0.182 already ensures that financial integration leads to a boost in long-run domestic economic development from the perspective of the financially less developed country.

In summary, our results from the second and third scenario show that both, borrowing constraints and persistent effects of shocks may strongly affect the parameter restrictions that are required in order to explain the Lucas paradox and a boost in long-run domestic economic development. Though not all combinations of borrowing constraints and persistence reverse the results from the baseline scenario, we observe significantly tighter parameter restrictions at very tight borrowing constraints and at high levels of persistence. Therefore, our results show that in a quantitative application, a more careful consideration of the country-specific characteristics is needed in order to assess the implications of financial integration. These stronger requirements are not well understood from an isolated consideration of capital risk that underestimates the existence of tight parameter restrictions. With these results at hand, we conclude the discussion of our three scenarios.

6 Transitory dynamics and welfare implications

In this section, we briefly consider the transitory dynamics between steady states from the perspective of the financially less developed country. First, it remains to show that the accumulation of financial wealth is a gradual and monotone process that also shapes the short-run dynamics of other variables towards the integrated steady state. Second, we want to assess the welfare implications of financial integration, which requires to take account of the entire transition path.

6.1 Transitory dynamics

Figure 4 shows the adjustment paths of selected macroeconomic variables between the steady state under financial autarky and financial integration from the perspective of the financially less developed country. Entrepreneurs in the economy considered here face a debt limit equal to the average net income in the autarchic steady state, $\sigma$ is equal to 0.4, $\rho_0$ is equal to 0.7 and the other parameter values are identical to the baseline scenario. Note that for these parameter values, we know that they satisfy the above derived parameter restrictions. Apart from the net foreign asset position, the dynamics of aggregate variables over time are shown as percentage deviations from the corresponding autarchic steady state level. The net foreign asset position is measured relatively to the
Figure 4: Transitory Dynamics, First Exercise

Note: Figure 4 shows the transitory dynamics for the first exercise in which financial integration leads to higher levels of capital, output and the wage in the steady state under financial integration. The steady state under financial integration, relative to financial autarky, is indicated by the solid black lines.

The interest rate under financial integration is determined by the large and financially more developed country that represents the rest of the world. We assume that entrepreneurs in the financially more developed country can borrow up to roughly twice the average net income in the autarchic steady state, which leads to a 5.7 percent higher net interest rate compared to the financially less developed country. When barriers to trading the riskless asset are removed this gap is closed immediately.

The lower right panel of Figure 4 shows that the financially less developed country immediately starts to accumulate a positive net foreign asset position once trading barriers are removed. The monotone increase of the net foreign asset position means that the financially less developed country runs a series of current account surpluses along the transition path and finally reaches its long-run asset position.\(^{28}\) As mentioned earlier,

\(^{28}\)Note that the considered example serves to show the general pattern and does not target a specific asset-to-GDP ratio.
this pattern is not specific to the exercise considered here, but generally arises if financial integration is associated with an increase of the interest rate. Hence, the direction of international capital flows between countries can directly be inferred from the initial differential in interest rates.

The dynamics of the net foreign asset position also influence the short-run dynamics of the domestic aggregate capital stock. When trading barriers are removed, the capital stock of the financially less developed country initially falls below its autarchic steady state level but immediately starts to recover. Since the elasticity of intertemporal substitution is just large enough to ensure that the long-run capital stock increases with the interest rate, the capital stock eventually reaches its higher level in the steady state under financial integration. The observed short-run behavior of the aggregate capital stock is similar to Angeletos and Panousi (2011) and is driven by the different timing of the opportunity-cost and the wealth effect. The higher interest rate under financial integration immediately reduces entrepreneurs’ incentive to invest in the own firm, while the accumulation of wealth, stimulating investment in the following, is a gradual process. The fact that the adjustment of the capital stock takes some time is known from the related literature (e.g., Angeletos and Panousi 2011; Clemens and Heinemann 2013) and shows that the potential benefits of financial integration do not occur instantaneously.

Finally, output and the wage rate largely follow the adjustment path of the aggregate capital stock, but both variables return to their autarchic steady state levels somewhat faster. The faster recovery is driven by the improvement in production efficiency that is implied by the process of financial integration. Specifically, the accumulation of wealth under financial integration helps entrepreneurs to partly overcome the borrowing constraint and also reduces the differences in the risk premium implied by uninsurable capital risk. In other words, we observe that financial integration leads to an increase in total factor productivity.

In order to facilitate the understanding of the following welfare analysis, we also briefly consider a second exercise in which financial integration leads to a lower long-run capital stock and a lower output level. We obtain such a long-run development by reducing the elasticity of intertemporal substitution from 0.67 to 0.25. Figure 5 shows the corresponding results. The key difference compared to the first exercise is that now, the wealth effect associated with a higher interest rate is not strong enough to finally outweigh the opportunity-cost effect. Consequently, from the perspective of the financially less developed country, the increase of the interest rate under financial integration does not lead to a boost in long-run domestic development. Instead, Figure 5 shows that the capital stock, output and the wage rate remain permanently below their autarchic steady state levels once the barriers to trading the riskless asset are removed. Since this development is quite different compared to the first exercise where the wage, capital and output reach higher levels in the integrated steady state, we will compare the corresponding welfare implications.

See also Buera and Shin (2009) and Clemens and Heinemann (2013) for a discussion of financial integration and productivity.
Note: Figure 5 shows the transitory dynamics for the second exercise in which financial integration leads to lower levels of capital, output and the wage compared to the situation of financial autarky. The steady state under financial integration, relative to financial autarky, is indicated by the solid black lines.

6.2 Welfare implications

We measure the individual welfare effect of financial integration as the proportional change in consumption that is required to leave each household indifferent between financial autarky and financial integration. Our measure takes account of the transition path between the two steady states. Formally, for each household with net worth, \( \omega \), and productivity, \( \tilde{\theta} \), the individual welfare effect, \( g(\omega, \tilde{\theta}) \), measures the change in consumption that equates the value functions under financial autarky and financial integration, i.e. \( g(\omega, \tilde{\theta}) \) solves

\[
(1 + g(\omega, \tilde{\theta}))^{1-\rho} = V^I(\omega, \tilde{\theta}) / V^A(\omega, \tilde{\theta}),
\]

where \( V^A(\omega, \tilde{\theta}) \) and \( V^I(\omega, \tilde{\theta}) \) are the value functions under financial autarky and financial integration, respectively.\(^{30}\) A positive value of \( g \) reflects a welfare gain of financial

\(^{30}\)In the case of log-utility, (21) reads \( 1 + g(\omega, \tilde{\theta}) = \exp((1-\beta)(V^I(\omega, \tilde{\theta}) - V^A(\omega, \tilde{\theta}))) \).
integration compared to the situation of remaining in the steady state under financial autarky.

Figure 6 shows the results for our first exercise in which financial integration leads to higher levels of capital and output in the integrated steady state. Specifically, Figure 6 shows the individual welfare effect of financial integration for households with average productivity but with different levels of net worth that defines the household’s total income. We observe that households with low net worth experience a welfare loss under financial integration while households with high net worth experience a welfare gain. Quantitatively, the welfare loss for poor households remains moderate with roughly 0.05 percent of consumption, whereas welfare gains are much more pronounced for households with high net worth. In short, Figure 6 shows that the poor lose while the rich win from a financial market liberalization reform. This result also applies to other productivity levels and is in line with Mendoza et al. (2009b), Angeletos and Panousi (2011), and depending on the tightness of the credit constraint, Clemens and Heinemann (2013).

Figure 6: Individual Welfare Effects, First Exercise

In order to understand the above results, it is useful to firstly consider the interest rate effect of financial integration for poor households. In the first exercise, the interest rate increases from the perspective of the financially less developed country once trading barriers are removed. The higher interest rate is beneficial for savers but simultaneously means an increase in the cost of borrowing. Since the poorest households borrow a substantial amount under financial autarky, they strongly suffer from the higher interest rate under financial integration and hence, experience a welfare loss.

Moving to higher levels of net worth, financial integration turns out to be beneficial. According to Figure 6, households with net worth larger than 1.1 are better off in the case of financial integration. This result is remarkable since not all households are savers and gain from the higher interest rate. In fact, up to a level of net worth equal to 2.65, households still borrow albeit at a decreasing rate. Consequently, the welfare gains of these households must be crucially influenced by the dynamics of the wage, eventually reaching higher levels under financial integration than under financial autarky. This result becomes
more evident if we consider our second exercise in which financial integration leads to lower levels of the capital stock, output and the wage compared to the situation of financial autarky. The corresponding welfare effects are presented in Figure 7 and, again, refer to households with average productivity.

Figure 7: Individual Welfare Effects, Second Exercise

Although a one-to-one comparison of individual numbers between the two exercises is restricted due to the change in the elasticity of intertemporal substitution, we observe some interesting differences between the groups of agents who gain from financial integration. First, Figure 7 shows that households in the second exercise have to be about four times richer in order to benefit from financial integration. Moreover, we do not observe households who borrow in the autarchic steady state and gain from financial integration, but we do observe households who save and lose from financial integration. This follows from the fact that in the second exercise, households with net worth larger than 2.86 already become savers, whereas a positive welfare effect of financial integration is only associated with net worth larger than 4. Hence, in the second exercise, only households who save a substantial amount benefit from financial integration while in the first exercise, where output and the wage reach higher levels under financial integration, a larger group of agents experience a welfare gain. Consequently, we can conclude that the effects of financial integration on domestic economic development are not only of interest to describe the empirical pattern, but also may have significant welfare implications, at least for some members of society.

Finally, we compute the aggregate welfare effect of financial integration to answer the question whether the individual welfare gains or losses dominate on the aggregate level. Following Mendoza et al. (2009b), we compute the aggregate welfare effect, $G$, according to

$$
(1 + G)^{1-\rho} = \frac{\int_{\omega, \tilde{\theta}} V^I(\omega, \tilde{\theta}) \Psi^A(\omega, \tilde{\theta})}{\int_{\omega, \tilde{\theta}} V^A(\omega, \tilde{\theta}) \Psi^A(\omega, \tilde{\theta})},
$$

where $\Psi^A(\omega, \tilde{\theta})$ is the stationary distribution over households under financial autarky. $G$ can be interpreted as the required proportional change in consumption that makes
a utilitarian welfare planner indifferent between financial autarky and financial market liberalization. In our first exercise, we find an aggregate welfare gain for the financially less developed country of 0.13 percent of consumption, while in our second exercise, we find an aggregate welfare loss for the financially less developed country of 0.11 percent of consumption. In other words, we find an aggregate welfare gain when the capital stock, output and the wage reach higher levels under financial integration while we observe an aggregate welfare loss when financial integration leads to permanently lower levels of the capital stock, output and the wage. Although we do not want to claim that the differences in aggregate welfare between the two exercises are completely determined by the observed differences in output and the wage, our results are clearly in line with the related literature. Angeletos and Panousi (2011), for example, find a positive aggregate welfare effect if financial integration leads to a boost in domestic economic development, whereas Mendoza et al. (2009b) find a negative aggregate welfare effect if financial integration impedes domestic economic development. Hence, understanding the conditions under which financial integration stimulates domestic economic development is important for understanding the welfare implications of financial integration.

7 Concluding remarks

In this paper, we examined under which conditions the observed pattern of international capital flows and accompanying changes in domestic economic development can be explained by the presence of uninsurable idiosyncratic risks and borrowing constraints. Motivated by the mixed results from the literature, we employed a heterogeneous-agent model that does not only encompass capital risk and a riskless income component, but also income risk due to the existence of risky profits. Furthermore, borrowing constraints simultaneously impede consumption smoothing and restrict the access to external funds for scaling up individual production. We considered different scenarios, increasing the model complexity step by step. In the baseline scenario, we focused on the effects of the capital risk, the risky profits, and the riskless wage income. We found that in the baseline scenario, the restrictions on the model parameters that are required to explain capital flows from poor to rich countries and a boost in long-run domestic economic development are moderate and easily satisfied by empirically plausible parameter values. As in Angeletos and Panousi (2011), we emphasized the combination of capital risk and the riskless wage income. Introducing capital risk makes it possible to observe a lower capital stock and a lower interest rate in the financially less developed country and the riskless wage ensures that the parameter restriction remain quite loose. This combination works quite well even in the presence of income risk that leads to additional precautionary savings. We derived our results with the help of two rules of thumb that describe the underlying parameter restrictions with high accuracy and that can also be applied to other types of models considered in the literature.

In the second scenario, we considered the case with tight borrowing constraints. On the one hand, borrowing constraints make it more difficult for agents to smooth consum-
tion and lead to an increase in aggregate demand for the safe asset. On the other hand, borrowing constraints restrict the access to external funds for scaling up production and, even if not currently binding, discourage risky investment. However, we find that the saving effect is the dominant effect and that tighter borrowing constraints lead to tighter parameter restrictions compared to the baseline scenario. In particular, we find that in times of strong turmoil in financial markets, with an almost collapsing lending channel, the model predictions drastically change and indicate that financial integration may easily become an impediment for domestic economic development. This result is not only important for explaining the empirical pattern, but also has significant welfare implications, at least for those members of society who do not benefit from an increase of the interest rate under financial integration.

In the third and final scenario, we increased the persistence of shocks. A higher level of persistence increases the demand for the riskless asset and therefore, amplifies the effects of the financial market imperfections. In almost all exercises, we find that a higher persistence of shocks again leads to tighter parameter restrictions compared to the first two scenarios. This especially applies to moderate levels of the borrowing constraint. Hence, as an overall result, we can conclude that a very careful consideration of the country specific characteristics is needed in order to fully understand the implications of financial integration. Even if two countries are characterized by a similar level of financial development, the implications of financial integration may be very different, depending on the dominant source of risk, the exact specification of borrowing constraints or the persistence of risk. This paper provides a basis for understanding these specific differences in financial market performance.

There are several interesting ways in which the current analysis could be extended with regard to future research. For example, we do not assume that the process of financial liberalization is accompanied by improvements in financial market performance, resulting in less tight borrowing constraints or better insurance opportunities. Such improvements seem likely to occur once countries get access to the advanced financial instruments provided by developed countries. Allowing for this interplay of financial market performance and financial integration may relax the required parameter restrictions and, thereby, may further improve the ability of incomplete-markets models to explain the empirical pattern. Furthermore, improvements in financial market performance may increase the political support for financial liberalization among the population, thereby reducing the need for implementing redistributing measures. We leave investigating this channel for future research.

31Financial market liberalization may also be accompanied by other types of reforms. See Buera and Shin (2009) for a discussion.
References


Appendix

Appendix A. Derivation of the two rules of thumb

In order to derive our two rules of thumb, we proceed along the following steps. First, we simplify our model by reducing the time horizon to two periods and by assuming that all households start with the same level of net worth. The first simplification relates to the discussion of Krusell and Smith (2006), emphasizing the usefulness of the two-period setup for conducting pilot studies within the class of incomplete-markets models. The second simplification means that choices of capital and bonds are identical across all households so that aggregation is easily obtained. Within the simplified framework, we compare the outcome under incomplete markets and complete markets and derive the condition guaranteeing that the capital stock and output are lower in the incomplete markets case. Subsequently, we derive a corresponding condition for a simplified model version sharing the main features of Angeletos (2007) and Angeletos and Panousi (2011), i.e. capital risk and riskless wage income, but no further risky income component. Based on the comparison of the two conditions and the conditions described in Angeletos (2007) and Angeletos and Panousi (2011), we finally derive the two rules of thumb that refer to the steady state of the underlying infinite-horizon model.

Step 1. Two-period model - incomplete markets

The simplified household’s optimization problem under incomplete markets is given by

\[
\max_{c_t, k_{t+1}, b_{t+1}} U(c_t) + \beta E_t U(c_{t+1}) \quad (23)
\]

\[
s.t. \quad c_t = \omega_t - k_{t+1} - b_{t+1} \quad (24)
\]

\[
c_{t+1} = \omega_{t+1} \quad (25)
\]

\[
\omega_{t+1} = b_{t+1} R_{t+1} + k_{t+1} R^r_{t+1} + w_{t+1} + \pi_{t+1}, \quad (26)
\]

where the subscript \( i \) is dropped whenever optimal choices are the same across households. By assumption, \( \omega_t \) is equal across all households, whereas the individual level of net worth in the final period \( t + 1 \) depends on the realization of the idiosyncratic shock. The decomposition of \( \omega_{t+1} \) is in line with the expression in (10) and again shows the existence of capital risk and profits constituting a risky income component. The risky return is given by \( R^r_{t+1} \equiv 1 - \delta + \alpha^2 \tilde{\theta}_{t+1} k_{t+1}^{\alpha - 1} \) and risky profits are given by \( \pi_{t+1} \equiv (1 - \alpha) \alpha \tilde{\theta}_{t+1} k_{t+1}^{\alpha} \), exploiting the fact that \( L_{t+1} = 1 \) holds in equilibrium.

The first-order conditions are given by

\[
U'(c_t) = \beta R_{t+1} E_t[U'(c_{t+1})] \quad (27)
\]

\[
U'(c_t) = \beta E_t[R^r_{t+1} U'(c_{t+1})], \quad (28)
\]

\[\text{32} \text{The specification of investment risk in Angeletos and Panousi (2011) slightly differs from Angeletos (2007), but the main results we refer to are comparable. See also Angeletos and Panousi (2009).}\]
where (27) is the Euler equation for bond holdings and (28) is the Euler equation for capital. Note that, since the optimal choice of risky investment is the same across all households, $k_{t+1}$ denotes the individual as well as the aggregate capital stock. Using this result and the fact that $E(\tilde{\theta}) = 1$, the expression for the wage reduces to

$$w_{t+1} = (1-\alpha) k_{t+1}^\alpha.$$

Finally, since market clearing of the bond market implies $\int_0^1 b_{t+1} di = 0$ and $b_{t+1}$ is the same across all households, the capital Euler equation becomes a function of $k_{t+1}$ alone.

$$U'(\omega_t - k_{t+1}) = \beta E_t[R^r_{t+1}U'(k_{t+1}R^r_{t+1} + w_{t+1} + \pi_{t+1})].$$

(29)

**Step 2. Two-period model - complete markets**

In the complete markets case, households can completely insure against idiosyncratic risk. That means, instead of earning a risky return and risky profits as under incomplete markets, households receive the expected value of the risky return and of risky profits in each state in period $t + 1$. This follows from the fact that the risk is purely idiosyncratic and complete markets allow for perfect risk pooling. Hence, the optimal choice of capital under complete markets is without risk and the capital Euler equation is given by

$$U'(\omega_t - k_{t+1}) = \beta E_t[R^r_{t+1}U'(k_{t+1}E_t[R^r_{t+1}] + w_{t+1} + E_t[\pi_{t+1}]),$$

(30)

where the subscript $i$ attached to the return and to profits is dropped in order to emphasize that the expected value of the return and of profits is identical across all households, i.e. $E_t[R^r_{t+1}] = E_t[R^r_{t+1}]$ and $E_t[\pi_{t+1}] = E_t[\pi_{t+1}]$. Equation (30) highlights the fact that the complete markets case corresponds to a deterministic setting where the riskless return and riskless profits are equal to their expected values. In other words, the comparison of the optimal choice of capital between (29) and (30) corresponds to the discussion of how agents respond to the introduction of risk.

**Step 3. Comparing the Euler equations**

In order to simplify the comparison between (29) and (30), the right-hand side of (29) is approximated using a second-order Taylor expansion around the point $(E_t[R^r_{t+1}], E_t[\pi_{t+1}])$.

For small risks, (29) simplifies to

$$U'(\omega_t - k_{t+1}) = \beta E_t[R^r_{t+1}] U'(\cdot) + \frac{1}{2} \beta [E_t[R^r_{t+1}] U''(\cdot) k_{t+1}^2 + 2 U''(\cdot) k_{t+1}] \sigma^2_{R^r_{t+1}} + \frac{1}{2} \beta E_t[R^r_{t+1}] U''(\cdot) \sigma^2_{\pi_{t+1}} + \beta [E_t[R^r_{t+1}] U''(\cdot) k_{t+1} + U''(\cdot)] \sigma_{R^r_{t+1}\pi_{t+1}},$$

with $(\cdot) \equiv (k_{t+1}E_t[R^r_{t+1}] + w_{t+1} + E_t[\pi_{t+1}]). \sigma^2_{R^r_{t+1}}$ is the conditional variance of the return.

---

33See Baiardi et al. (2014) for a related analysis.
\(\sigma_{\pi_t+1}^2\) is the conditional variance of profits and \(\sigma_{R_{t+1}^r}^2\) is the conditional covariance between the two. The term on the right-hand side in (31) is equal to the corresponding expression under complete markets in (30), and the additional terms in (32)-(34) capture the influence of risk. Based on this representation, the following relation between the optimal choice of \(k_{t+1}\) under complete and incomplete markets can be derived.

**Lemma 1** Let \(k_{t+1}^*\) denote the solution in the incomplete markets case. Then, the capital stock is the same under complete and incomplete markets if and only if the additional terms in (32)-(34) evaluated at \(k_{t+1}^*\) sum up to zero. Moreover, the capital stock in the incomplete markets case is lower (larger) than in the complete markets case if and only if the additional terms in (32)-(34) evaluated at \(k_{t+1}^*\) sum up to a value strictly lower (larger) than zero.

**Proof of Lemma 1.** For notational ease, we introduce the following definitions

\[
f(k_{t+1}) \equiv U'(\omega_t - k_{t+1}) \tag{35}
g(k_{t+1}) \equiv \beta E_t[R_{t+1}^r] U'(\cdot) \tag{36}
h(k_{t+1}) = \frac{1}{2} \beta \left[ E_t \left[ R_{t+1}^r \right] U''(\cdot) k_{t+1}^2 + 2 U''(\cdot) k_{t+1} \right] \sigma_{R_{t+1}^r}^2 \tag{37}
\]

with \((\cdot) \equiv (k_{t+1} E_t [R_{t+1}^r] + \omega_{t+1} + E_t[\pi_{t+1}])\) and where it is explicitly indicated that all functions depend on \(k_{t+1}\). Then, (31)-(34) can be paraphrased as

\[
f(k_{t+1}) = g(k_{t+1}) + h(k_{t+1}), \tag{38}
\]

whereas in the complete markets case, according to (30) we have

\[
f(k_{t+1}) = g(k_{t+1}). \tag{39}
\]

Let \(k_{t+1}^*\) denote the solution in the incomplete and \(\hat{k}_{t+1}\) denote the solution in the complete markets case.

If \(h(k_{t+1}^*) = 0\) then \(f(k_{t+1}^*) = g(k_{t+1}^*)\). Since \(\hat{k}_{t+1}\) satisfies \(f(\hat{k}_{t+1}) = g(\hat{k}_{t+1})\) as well, it follows that \(k_{t+1}^* = \hat{k}_{t+1}\).

If \(h(k_{t+1}^*) < 0\) then \(f(k_{t+1}^*) < g(k_{t+1}^*)\). Since \(f(k_{t+1})\) is continuous and strictly increasing in \(k_{t+1}\) and \(g(k_{t+1})\) is continuous and strictly decreasing in \(k_{t+1}\), it follows that \(f(k_{t+1}^*) < g(k_{t+1}^*)\) and \(f(\hat{k}_{t+1}) = g(\hat{k}_{t+1})\), it implies \(k_{t+1}^* < \hat{k}_{t+1}\).

If \(h(k_{t+1}^*) > 0\) then \(f(k_{t+1}^*) > g(k_{t+1}^*)\). Since \(f(k_{t+1})\) is continuous and strictly increasing in \(k_{t+1}\) and \(g(k_{t+1})\) is continuous and strictly decreasing in \(k_{t+1}\), it follows that \(f(k_{t+1}^*) > g(k_{t+1}^*)\) and \(f(\hat{k}_{t+1}) = g(\hat{k}_{t+1})\), it implies \(k_{t+1}^* > \hat{k}_{t+1}\).

The other way around it follows from \(k_{t+1}^* = \hat{k}_{t+1}\) that \(h(k_{t+1}^*) = 0\) since \(f(k_{t+1}^*) = g(k_{t+1}^*)\).
If \( k_{t+1}^* < \hat{k}_{t+1} \), it follows that \( f(k_{t+1}^*) < g(k_{t+1}^*) \) since \( f(k_{t+1}) \) is continuous and strictly increasing in \( k_{t+1} \), \( g(k_{t+1}) \) is continuous and strictly decreasing in \( k_{t+1} \) and \( f(\hat{k}_{t+1}) = g(\hat{k}_{t+1}) \). Consequently, since \( k_{t+1}^* \) satisfies \( f(k_{t+1}^*) = g(k_{t+1}^*) + h(k_{t+1}^*) \), it follows that \( h(k_{t+1}^*) < 0 \).

If \( k_{t+1}^* > \hat{k}_{t+1} \), it follows that \( f(k_{t+1}^*) > g(k_{t+1}^*) \) since \( f(k_{t+1}) \) is continuous and strictly increasing in \( k_{t+1} \), \( g(k_{t+1}) \) is continuous and strictly decreasing in \( k_{t+1} \) and \( f(\hat{k}_{t+1}) = g(\hat{k}_{t+1}) \). Consequently, since \( k_{t+1}^* \) satisfies \( f(k_{t+1}^*) = g(k_{t+1}^*) + h(k_{t+1}^*) \), it follows that \( h(k_{t+1}^*) > 0 \).  

\textit{End of proof.}

Before we proceed with the derivation of our final rule, we can use the expressions in (32)-(34) to briefly discuss the differences in the influence of the underlying sources of risk. Initially, we focus exclusively on the effect of income risk being captured by the expression in (33). In our model, income risk is implied by the existence of risky profits but the same expression also occurs if, for simplicity, \( \pi_{t+1} \) is treated as a stochastic endowment component with variance \( \sigma^2_{\pi_{t+1}} \). According to (33) and Lemma 1, the well-known result applies that uninsurable income risk generates precautionary savings and, consequently, leads to over-accumulation of capital if \( U''(\cdot) > 0 \) (see e.g., Leland 1968; Kimball 1990). Since this condition holds in our case, uninsurable income risk alone would necessarily lead to larger levels of capital and output in the financially less developed country with incomplete markets.

Next, we focus exclusively on capital risk so that only (32) shows up in the incomplete markets case and the model economy becomes closely related to Levhari and Srinivasan (1969) and Sandmo (1970).\(^{34}\) Inspecting (32) shows that the ultimate effect of uninsurable capital risk on the choice of \( k_{t+1} \) is ambiguous. On the one hand, a precautionary saving effect exists as well, but on the other hand, the term \( 2U''(\cdot) k_{t+1} \) is negative if agents are risk-averse.\(^{35}\) Intuitively, and in contrast to income risk, agents can reduce the extent to which their resources are exposed to potential losses by consuming more and saving less in case where risk is associated with risky returns. Consequently, capital risk leads to lower levels of capital and output in the economy with incomplete markets if the latter effect dominates.

Finally, we assume that both types of risk exist simultaneously and if \( \sigma_{R_{t+1}^1 \pi_{t+1}} \neq 0 \), all terms in (32)-(34) show up in the approximation. In general, the appearance of the covariance in (34) implies another ambiguous effect on the choice of \( k_{t+1} \) and the sign of the covariance reflects the extent to which the risky asset can be used to hedge against the underlying income risk.\(^{36}\) In our model, \( \sigma_{R_{t+1}^1 \pi_{t+1}} \) is positive and second moments,

\(^{34}\)See also Rothschild and Stiglitz (1971) and for a separate analysis of both types of risk see Eeckhoudt and Schlesinger (2008).

\(^{35}\)In general, the sign of \( 2U''(\cdot) k_{t+1} \) also depends on whether agents are savers \( (k_{t+1} > 0) \) or borrowers \( (k_{t+1} < 0) \). In our case, however, only \( k_{t+1} > 0 \) is relevant.

\(^{36}\)For a more rigorous and simultaneous treatment of both types of risk in cases where income and the return are exogenously given, see Li (2012) and Buiardi et al. (2014).
evaluated at \( k_{t+1}^* \), are given by

\[
\begin{align*}
\sigma_{R_{t+1}}^2 & = \alpha^4 (k_{t+1}^*)^{2\alpha - 2} \sigma_{\theta_{t+1}}^2 \\
\sigma_{x_{t+1}}^2 & = (1 - \alpha)^2 \alpha^2 (k_{t+1}^*)^{2\alpha} \sigma_{\theta_{t+1}}^2 \\
\sigma_{R_{t+1} x_{t+1}}^2 & = (1 - \alpha)^3 (k_{t+1}^*)^{2\alpha - 1} \sigma_{\theta_{t+1}}^2 ,
\end{align*}
\]  

(40)

where \( \sigma_{\theta_{t+1}}^2 \) denotes the variance of \( \bar{\theta}_{t+1} \).

Based on the expressions in (40) and Lemma 1 we can now derive the final condition guaranteeing that the capital stock and output are lower under incomplete markets than under complete markets. Specifically, according to Lemma 1, the capital stock and output are lower under incomplete markets than under complete markets if and only if

\[
[ E_t [R_{t+1}^r] U'''(\cdot) (k_{t+1}^*)^2 + 2 U''(\cdot) k_{t+1}^* | \alpha^4 (k_{t+1}^*)^{2\alpha - 2} \sigma_{\theta_{t+1}}^2 ]
\]

\[
+ E_t [R_{t+1}^r] U'''(\cdot) (1 - \alpha)^2 \alpha^2 (k_{t+1}^*)^{2\alpha} \sigma_{\theta_{t+1}}^2
\]

\[
+ 2[E_t [R_{t+1}^r] U'''(\cdot) k_{t+1}^* + U''(\cdot)](1 - \alpha)\alpha^3 (k_{t+1}^*)^{2\alpha - 1} \sigma_{\theta_{t+1}}^2 < 0,
\]

where \( (\cdot) \equiv (k_{t+1}^* E_t[R_{t+1}^r] + w_{t+1} + E_t[\pi_{t+1}]). \)

Since \( k_{t+1}^* > 0 \), getting rid of \( \alpha^2 (k_{t+1}^*)^{2\alpha} \sigma_{\theta_{t+1}}^2 > 0 \) and collecting terms yields

\[
E_t[R_{t+1}^r] U'''(\cdot) [\alpha^2 + 1 + 2\alpha + \alpha^2 + 2\alpha - 2\alpha^2] + 2U''(\cdot)(k_{t+1}^*)^{-1}[\alpha^2 + \alpha - \alpha^2] < 0, \quad (42)
\]

and finally

\[
- \frac{1}{\alpha} \frac{U'''(\cdot)}{U''(\cdot)} (E_t[R_{t+1}^r]k_{t+1}^*) < 2. \quad (43)
\]

Applying \( U''(x) = -\rho x^{-\rho - 1} \) and \( U'''(x) = \rho(\rho + 1)x^{-\rho - 2} \) leads to

\[
\left(\frac{1}{\vartheta} + 1\right) \frac{\alpha}{k_{t+1}^* E_t[R_{t+1}^r]} + w_{t+1} + E_t[\pi_{t+1}] < 2, \quad (44)
\]

where \( \vartheta \) is the elasticity of intertemporal substitution (\( \vartheta = 1/\rho \)). Since production of the final good simplifies to \( Y_{t+1}^* = (k_{t+1}^*)^\alpha \) and \( K_{t+1}^* \equiv \int_0^1 k_{t+1}^*di = k_{t+1}^* \), we get \( w_{t+1} + E_t[R_{t+1}^r]k_{t+1}^* + E_t[\pi_{t+1}] = Y_{t+1}^* + (1 - \delta)K_{t+1}^* \) and \( k_{t+1}^* E_t[R_{t+1}^r] = \alpha^2 Y_{t+1}^* + (1 - \delta)K_{t+1}^* \).

Plugging these expressions into (44) leads to the following final condition:

**Lemma 2** Levels of the aggregate capital stock and output are lower in the economy with incomplete markets compared to the complete markets case if and only if \(^{37}\)

\[
\vartheta > \frac{\phi}{2 - \phi} \quad \text{and} \quad \phi \equiv \frac{\alpha + (1 - \delta) K_{t+1}^*}{\alpha Y_{t+1}^* + 1 + (1 - \delta) K_{t+1}^*}, \quad (45)
\]

where production simplifies to \( Y_{t+1}^* = (K_{t+1}^*)^\alpha \).

\(^{37}\)Note that the condition described in (45) is defined for the interesting case of \( 2 - \phi > 0 \).

In this step, we derive a respective condition for the case when the two-period model shares the main features of Angeletos (2007) and Angeletos and Panousi (2011).\(^{38}\) Comparable to our model economy, each household owns a private firm, receives riskless wage income by supplying one unit of labor, and freely trades a riskless bond. However, no intermediate goods sector exists, but all firms produce the same final good using capital and labor as inputs. Each firm owner invests capital, and optimal employment is chosen after the capital stock has been installed and the contemporaneous idiosyncratic shock has been observed. Assuming that individual production in period \(t + 1\) takes place according to 

\[
y_{it+1} = \theta_{it+1}^\alpha k_{it+1}^{\alpha} l_{it+1}^{1-\alpha},
\]

capital income becomes linear in the capital stock.

With \(\ln(\theta) \sim N(-\sigma^2/2, \sigma^2)\) and identical initial conditions, the household’s maximization problem under incomplete markets reads

\[
\max_{c_t, k_{it+1}, b_{it+1}} U(c_t) + \beta E_t U(c_{it+1})
\]

s.t. \(c_t = \omega_t - k_{it+1} - b_{it+1}\)

\(c_{it+1} = \omega_{it+1}\)

\(\omega_{it+1} = b_{it+1} R_{it+1} + \pi_{it+1} + w_{it+1} + (1 - \delta)k_{it+1},\)

where the subscript \(i\) is dropped whenever optimal choices are the same across all households. \(\delta\) is the depreciation rate, \(k_{it+1}\) and \(b_{it+1}\) denote risky and riskless investment, respectively, and \(w_{it+1}\) is the wage rate. \(\pi_{it+1}\) is defined as

\[
\pi_{it+1} \equiv \max_{l_{it+1}} \theta_{it+1}^\alpha k_{it+1}^{\alpha} l_{it+1}^{1-\alpha} - w_{it+1} l_{it+1},
\]

where optimal employment maximizes \(\pi_{it+1}\) state by state, leading to \(l_{it+1} = \left(\frac{1-\alpha}{w_{it+1}}\right)^\frac{1}{\alpha} \theta_{it+1} k_{it+1}\).

Using this result, \(\omega_{it+1}\) simplifies to

\[
\omega_{it+1} = b_{it+1} R_{it+1} + k_{it+1} R_{it+1}^r + w_{it+1},
\]

where \(R_{it+1}^r \equiv 1 - \delta + \theta_{it+1} \alpha \left(\frac{1-\alpha}{w_{it+1}}\right)^\frac{1}{\alpha}\) is the risky return. Hence, in this case, households only face capital risk where capital income, \(k_{it+1} R_{it+1}^r\), is linear in the individual capital stock.

Bond market clearing implies \(\int b_{it+1} \, dt = b_{t+1} = 0\) so that the capital Euler equation in the incomplete markets case is given by

\[
U'(\omega_t - k_{t+1}) = \beta E_t [R_{it+1}^r U'(k_{it+1} R_{it+1}^r + w_{t+1})].
\]

Since labor market clearing implies \(\int l_{it+1} \, dt = 1\), we finally get \(w_{t+1} = (1 - \alpha) k_{t+1}^\alpha\).

In the complete markets case, households completely insure against the idiosyncratic

\(^{38}\) Regarding Angeletos (2007), we refer to the model without extensions (p.5f.).
risk and receive the expected value of the risky return in each state in period \( t + 1 \). The resulting capital Euler equation under complete markets is thus given by

\[
U'(\omega_t - k_{t+1}) = \beta E_t[R_{t+1}^r] U'(k_{t+1}E_t[R_{t+1}^r] + w_{t+1}),
\]

where the subscript \( i \) attached to the return is dropped in order to emphasize that the expected value of the individual return is identical across all households, i.e. \( E_t[R_{t+1}^r] = E_t[R_{t+1}^r] \).

Since only capital risk exists in the incomplete markets case, the right-hand side of equation (52) is approximated using a second-order Taylor expansion around the point \((E_t[R_{t+1}^r])\). For small risks this leads to

\[
U'(\omega_t - k_{t+1}) = \beta E_t[R_{t+1}^r] U'(\cdot) + \frac{1}{2} \beta [E_t[R_{t+1}^r] U''(\cdot) k_{t+1}^2 + 2 U''(\cdot) k_{t+1}] \sigma_{R_{t+1}^r}^2,
\]

with \( \cdot \equiv k_{t+1}E_t[R_{t+1}^r] + w_{t+1} \).

Let \( k_{t+1}^* \) denote the solution in the incomplete markets case. By analogy with Lemma 1, the capital stock and output are lower in the incomplete compared to the complete markets case if and only if

\[
[E_t[R_{t+1}^r] U''(\cdot) (k_{t+1}^*)^2 + 2 U''(\cdot) k_{t+1}^*] \sigma_{R_{t+1}^r}^2 < 0.
\]

Since \( \sigma_{R_{t+1}^r}^2 k_{t+1}^* > 0 \), this simplifies to

\[
-\frac{U''(\cdot)}{U''(\cdot)} E_t[R_{t+1}^r] k_{t+1}^* < 2.
\]

Applying \( U''(x) = -\rho x^{-\rho-1} \) and \( U''(x) = \rho(\rho + 1)x^{-\rho-2} \) leads to

\[
\left(\frac{1}{\vartheta} + 1\right) \frac{E_t[R_{t+1}^r] k_{t+1}^*}{E_t[R_{t+1}^r] k_{t+1}^* + w_{t+1}} < 2,
\]

where \( \vartheta \) is the elasticity of intertemporal substitution (\( \vartheta = 1/\rho \)). Since aggregate production simplifies to \( Y_{t+1}^* = (K_{t+1}^*)^\alpha \) and \( K_{t+1}^* \equiv \int_0^1 k_{t+1}^* di = k_{t+1}^* \), it follows that \( E_t[R_{t+1}^r] k_{t+1}^* = (1 - \delta)K_{t+1}^* + \alpha Y_{t+1}^* \) and \( E_t[R_{t+1}^r] k_{t+1}^* + w_{t+1} = (1 - \delta)K_{t+1}^* + Y_{t+1}^* \). Plugging these expressions into (58) yields the following final condition:

**Lemma 3** In the simplified two-period model sharing the main features of Angeletos (2007) and Angeletos and Panousi (2011), levels of the aggregate capital stock and output are lower in the economy with incomplete markets compared to the complete markets case if and only if

\[
\vartheta > \frac{\kappa}{2 - \kappa}, \qquad \kappa \equiv \frac{\alpha + (1 - \delta)K_{t+1}^* Y_{t+1}^*}{1 + (1 - \delta)K_{t+1}^* Y_{t+1}^*},
\]

where aggregate production simplifies to \( Y_{t+1}^* = (K_{t+1}^*)^\alpha \).
Comparing our condition in (45) with the condition in (59) shows that both share a similar structure, but φ and κ differ from each other when δ < 1. That means, in the relevant case without complete depreciation, the conditions to observe a lower capital stock and a lower output level in the financially less developed country differ between the models.

**Step 5. The first rule of thumb**

While the analysis so far has focused on the simplified model structure, the next step compares the results described in Lemma 3 with the condition explaining the behavior of aggregate variables in the autarchic steady state of the underlying infinite-horizon model. According to Angeletos (2007, p.11), and assuming the same Cobb-Douglas specification on the individual level as in the simplified two-period model version, the following condition applies for plausible parameter values:

\[ Y^* = (K^*)^\alpha \text{ in case of a Cobb-Douglas specification.} \]

Even though condition (60) refers to the steady state of the infinite-horizon model, whereas condition (59) explains the influence of uninsurable capital risk in the simplified two-period model, they share a similar structure.\(^{39}\) The only significant difference, which appears in the expressions of κ and \(\hat{\kappa}\), indicates a different role played by the depreciation rate δ. The difference, however, seems intuitively plausible due to the different nature of the steady state and of the final period in the two-period model. In the two-period model, aggregate consumption in the final period equals aggregate production and κ is equal to α if capital completely depreciates. The corresponding wage share equals \((1 - \alpha)\) and the income share of capital is equal to α. To obtain the same relation in the steady state of the infinite-horizon model and to get the same value of \(\hat{\kappa}\), the depreciation rate has to converge to zero so that no additional resources are needed to keep the aggregate capital stock at its steady state level. Hence, the ”natural” counterpart of δ = 1 in the two-period model is the case of δ = 0 in the steady state of the infinite-horizon model.

Since the comparison of (59) with (60) shows how the condition changes between the simplified two-period model and the underlying infinite-horizon model, we can use this information to finally derive our first rule of thumb. In the last step, we simply derive a new variable, \(\hat{\phi}\), such that the difference between \(\hat{\phi}\) and φ in (45) reflects the observed difference between \(\hat{\kappa}\) in (60) and κ in (59). Roughly speaking, we simply replace the term

\[^{39}\text{In Angeletos (2007), capital risk also includes depreciation risk. However, taking account of this feature in the two-period version does not change the condition in (59). Consequently, the implications derived from comparing (59) with (45) and (59) with (60) also remain unaffected.}\]
(1 − δ) in (45) with its counterpart −δ. Applying this last step leads to our first rule of thumb described in (19).

Step 6. The second rule of thumb

The derivation of the second rule builds on Angeletos and Panousi (2011) and the above described results. According to Figure 1, the second rule describes the condition guaranteeing that the long-run capital stock and long-run output necessarily increase with the interest rate. In the model considered by Angeletos and Panousi (2011), the following condition ensures that in the neighborhood of the autarchic steady state, the wealth effect of a higher interest rate dominates the opportunity-cost effect:

\[ \vartheta > \frac{\hat{\kappa}}{1 - \hat{\kappa}}, \quad \hat{\kappa} \equiv \frac{\alpha - \delta K^*}{1 - \delta K^*}, \tag{61} \]

where \( Y^* = (K^*)^\alpha \) in case of a Cobb-Douglas specification.

Comparing the condition in (60) with the condition in (61) shows that both share a similar structure. The only difference is that in the denominator of the first term in (60) there appears a '2', whereas in (61) there appears a '1'. Hence, we can use this similarity of the two conditions in order to derive our second rule. Specifically, we simply adjust our first rule in (19) to match the observed difference between (60) and (61). Applying this step leads to our second rule of thumb that is described in (20).

Appendix B. Baseline scenario: The general case

Figure 8: Threshold Level of the Elasticity of Intertemporal Substitution: The General Case

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[40] We slightly change the presentation compared to Angeletos and Panousi (2011, p.874) to facilitate the further discussion.
Figure 8 shows the general case for the baseline scenario where entrepreneurs in both countries suffer from uninsurable risk. The left panel of Figure 8 shows the relationship between the elasticity of intertemporal substitution, $\vartheta$, and the corresponding capital stock in the steady state under financial autarky for three economies that only differ in the level of uninsurable risk. The black line refers to the complete markets case, the red line refers to the economy with $\sigma = 0.2$ and the blue line refers to the benchmark economy with $\sigma = 0.4$. Apart from the differences in the level of uninsurable risk, the economies are identical to the benchmark economy.

The dashed vertical line in the left panel of Figure 8 shows the threshold level for the elasticity of intertemporal substitution predicted by our first rule of thumb. According to our first rule, the capital stock in the steady state under financial autarky should be lower in the benchmark economy than in the economy with complete markets if $\vartheta$ exceeds that threshold level. The actual threshold level, in turn, is determined by the intersection point of the blue line and the black complete markets line. As Figure 8 shows, our first rule of thumb is able to predict the actual threshold level with high accuracy.

The key insight from the left panel of Figure 8 is that the intersection point of the blue line and the red line, which determines the threshold level for the elasticity of intertemporal substitution in the general case where entrepreneurs in both economies suffer from uninsurable risk, is almost identical to the threshold level that is determined by the intersection point of the blue line and the black complete markets line. From this it follows that the predictions of our first rule of thumb that are derived under the assumption that markets are complete in the financially more developed country also carry over to the general case with frictions in both countries. In particular, in all cases considered, we find that if the elasticity of intertemporal substitution exceeds the threshold level that is predicted by our first rule, the financially less developed country features a lower capital stock and a lower output level in the autarchic steady state compared to any financially more developed country, and not only compared to the ideal case of complete markets.

Appendix C. Computational issues

*Steady state, financial autarky*

1. We start with an initial guess for the wage and for the interest rate.

2. We place a grid over the state space of net worth with more grid points being allocated to lower levels of net worth. The productivity process described in (18) is approximated by a five-state Markov chain using the method of Rouwenhorst (1995).

3. We compute the policy functions for consumption, capital holdings and bond holdings by value function iteration. Between grid points, we use linear interpolation.

4. The stationary distribution of households is computed using the policy functions and the transition matrix of the productivity process. Aggregate quantities are calculated by adding up weighted individual demands.
5. Within a first loop, the guess for the wage, and within a second loop, the guess for the interest rate are updated. The procedure is repeated until the market clearing conditions (labor and bond market) are satisfied, except for a tolerably small approximation error.

**Steady state, financial integration**

The procedure to compute the long-run equilibrium under financial integration for the small and financially less developed country is similar to the steps outlined above. However, bonds can be traded on the international level given a fixed interest rate that is determined by the large and financially more developed country.

**Transitional Equilibrium**

1. In a first step, we solve for the initial (autarchic) and the final (integrated) steady state, following the steps outlined above.

2. The number of transition periods, $T$, is chosen. $T$ is set sufficiently large to ensure that the integrated steady state is, approximately, reached in $T$ periods.

3. Based on an initial guess for the time path of the wage rate, the sequence of policy functions is computed by iterating backwards in time, starting from period $t = T-1$.

4. The sequence of distributions is computed by moving forwards in time, starting from the initial autarchic steady state. Aggregate quantities are calculated.

5. The initial guess for the time path of the wage rate is updated and the procedure is repeated from step 3 until the labor market clearing condition is satisfied at each point in time, except for a tolerably small approximation error.