

Matching Market Design

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Class Hours and Room: TBA

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Course Description

This is a Ph.D. level course on Matching Market Design. The aim of the course is to provide PhD students with the core knowledge on the field so that they can immediately start their own (theoretical or experimental) research projects. For this reason, I will cover the most important results and go over the proofs line by line, which enables students to understand the latest papers in the field.

Market Design concerns the creation of a venue in which buyers and sellers can meet and a format through which they can interact. There are several applications for which market design has been quite fruitful, the most notable ones being auction design and matching markets. As one of the most important functions of markets, matching focuses on who gets what, particularly when the scarce goods are indivisible and heterogeneous: for example, who gets which job at which firm, which buyer transacts with which seller, which student goes to which school, or which patient gets which transplantable organ. Matching Market Design aims to design a desirable matching procedure based on reported preferences in terms of incentive compatibility, efficiency, and fairness.

Course Materials

Matching theory consists roughly of two-sided matching theory and indivisible goods resource allocation problem (or house allocation problem).

- Roth and Sotomayor (1990). *Two-Sided Matching: A Study on Game-Theoretic Modelling*. Cambridge University Press. The book covers two-sided matching theory before 1990.
- Sönmez and Ünver (2011). Matching, Allocation, and Exchange of Discrete Resources. In Behhabib, Bisin, and Jackson (Eds.), *Handbook of Social Economics*, Volume 1A, pp. 781-852. North Holland. You should get its working paper version which is freely available online. The recent development of matching theory is mainly on indivisible goods resource allocation problem and its interaction with two-sided matching theory. This survey paper covers the recent literature in this regard.
- Roth (2002). The Economists as Engineer: Game Theory, Experimentation, and Computation as Tools for Design Economics. *Econometrica* 70, 1341-1378. This is for bed-time reading.
- Articles and working papers.
- Lecture notes in class.

Course Prerequisites

- Mathematical maturity: no measure theory nor graph theory is needed, but students should understand how to read and write formal proofs.
- A “basic” understanding of micro theory and game theory.

Grading Requirements

- Attendance (20%)
- Weekly assignments (40%): There will be an homework assignment weekly. Basically it is to prove some results. I strongly recommend students to work together with your classmates. If so, please write their names explicitly.
- Presentation (20%): I will list up recent papers. A student is supposed to choose one of them and present it once.
- Research proposal (20%): Every student needs to write a research proposal in Matching. It can be on theory or experiments. Please discuss your ideas with me.

Meeting 1: One-to-one matching problem (or marriage problem) and stable matchings

We introduce a model of marriage markets, or one-to-one matching problems. We introduce several definitions, and study the mathematical properties of stable matchings. We also introduce Gale and Shapley’s deferred acceptance mechanism. We cover some of Roth and Sotomayor (1990).

Meeting 2: Strategy-proofness on the deferred acceptance mechanism in one-to-one matching problems

We investigate the incentive compatibility of strategy-proofness of the deferred acceptance mechanism. We cover some of Roth and Sotomayor (1990).

Meeting 3: Many-to-one matching problem (or college admissions problem)

We consider the extension of the notions and results from one-to-one matching markets. We cover some of Roth and Sotomayor (1990).

Meeting 4: House allocation problem as indivisible goods resource allocation problem

We introduce a model of house allocation problem (a part of indivisible goods resource allocation problems). We introduce several axioms of incentive compatibility, efficiency, and fairness for a deterministic mechanism. In particular,

- We study the representative mechanism in theory and practice - the serial dictatorship mechanism.
- We cover the paper by Svensson (1999) “Strategy-proof Allocation of Indivisible Goods,” *Social Choice and Welfare*, 16, 557-567.

Meeting 5: Housing markets as indivisible goods resource allocation problem

- We introduce and study housing markets and the representative (deterministic) mechanism - Gale's top trading cycles mechanism that first appeared in Shapley and Scarf (1974) "On Cores and Indivisibility," *Journal of Mathematical Economics*, 1, 23-37.
- We cover the above paper by Svensson and the one by Sönmez (1999) "Strategy-Proofness and Essentially Single-Valued Cores," *Econometrica*, 67, 677-690.

Meeting 6: House allocation with existing tenants

- We study house allocation problems with existing tenants and the top trading cycles for this.
 - We cover the paper by Abdulkadiroğlu and Sönmez (1999) "House Allocation with Existing Tenants," *Journal of Economic Theory*, 88, 233-260.
 - If time permits, we will cover kidney exchange. An immediate application is the one by Roth, Sönmez, and Ünver (2004) "Kidney Exchange," *Quarterly Journal of Economics*, 119, 457-488.
 - If times permits, we also cover pairwise kidney exchange by Roth, Sönmez, and Ünver (2005) "Pairwise Kidney Exchange," *Journal of Economic Theory*, 125, 151-188.
- (optional) House allocation with overlapping generations
 - If time permits, we cover my paper: Kurino (2013) "House Allocation with Overlapping Generations," *American Economic Journal: Microeconomics*, forthcoming.

Meeting 7: Lottery mechanisms and the random serial dictatorship mechanism in a house allocation problem

The mechanisms we discuss in meetings 4 to 6 are deterministic. In this meeting we consider a lottery (stochastic) mechanism in a house allocation problem. We introduce some results and several axioms for lottery mechanisms, and then the random serial dictatorship mechanism. We cover some of Bogomolnaia and Moulin (2001) "A New Solution to the Random Assignment Problem," *Journal of Economic Theory*, 100, 295-328 which is useful for the study of lottery mechanisms.

Meeting 8: The probabilistic serial mechanism

Before around 2000, the random serial dictatorship mechanism (RSD) had been considered as the most desirable lottery mechanism in a house allocation problem in terms of incentive compatibility, efficiency, and fairness. However, Bogomolnaia and Moulin (2001) pointed out the clear efficiency loss of RSD, and then proposed a new mechanism, called the *probabilistic serial mechanism* (PS). The PS mechanism has triggered a growing body of the literature. We cover Bogomolnaia and Moulin (2001). Axiomatic characterizations have been provided for example by Hashimoto, Hirata, Kesten, Kurino, and Ünver (2012) "Two Axiomatic Approaches to the Probabilistic Serial Mechanism," *Theoretical Economics*, forthcoming.

Meeting 9: School choice problem or priority-based indivisible goods resource allocation problem

A school choice problem is an indivisible goods resource allocation problem where additionally schools have priorities over students, not preferences. For this reason, it can be seen as an application of many-to-one matching theory. We apply Gale-Shapley's two-sided matching mechanism and Gale's top trading cycles housing market mechanism to this problem. We cover the following two papers.

- Balinski and Sönmez (1999) "A Tale of Two Mechanisms: Student Placement," *Journal of Economic Theory*, 84, 73-94.
- Abdulkadiroğlu and Sönmez (2003) "School Choice: A Mechanism Design Approach," *American Economic Review*, 93, 729-747.

Meeting 10: Matching with contracts

The most general matching model is Matching with Contracts, which is introduced by

- Hatfield and Milgrom (2005) "Matching with Contracts," *American Economic Review*, 95, 913-935.

A non-trivial application of the model has been discovered by

- Sönmez and Switzer (2013) "Matching with (Branch-of-Choice) Contracts at the United States Military Academy," *Econometrica*, 81, 451-488.

We cover the above two as well as the following.

- Hatfield and Kojima (2010) "Substitutes and Stability for Matching with Contracts," *Journal of Economic Theory*, 145, 1704-1723,

which is technically useful.

Meetings 11 to 13: (to be determined)

I am afraid that the materials up to meeting 10 might not be covered in class. In this case, we discuss the remaining topics in the next class. The meetings 11 to 13 are reserved for this purpose. If time permits, we will discuss kidney exchange problems:

- Roth, Sönmez, and Ünver (2004) "Kidney Exchange," *Quarterly Journal of Economics*, 457-488.
- Roth, Sönmez, and Ünver (2005) "Pairwise Kidney Exchange," *Journal of Economic Theory*, 151-188.

Meeting 14: Student Presentation

I will list up some of recent papers in matching.